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THE TURBULENT BOUNDARY LAYER ON A YAWED FLAT PLATE

By

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SUMMARY

Measurements are presented of the velocity and shear stress profiles in a turbulent boundary on a yawed flat plate. Two-dimensional velocity profiles adequately represent the streamwise velocities. Shear stresses in the plane parallel to the plate are large, but there is no corresponding yaw in the velocity vector.

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In most of the experiments, reported in the literature, which relate to three-dimensional turbulent boundary layers, a two-dimensional boundary layer has been distorted by obstacles in the flow (e.g. Johnston (1960), Hornung & Joubert (1963), Perry & Joubert (1965).) The simplest case, that of a boundary layer on a flat plate, with its leading edge yawed at some angle to the flow, has received relatively little attention.

The earliest such measurements known to the authors, were reported by Ashkenas & Riddell (1955). Flat plates, with leading edges swept at 0°, 30° and 45° were used. Mean velocity and flow direction were found using a pitot tube and yaw probe. For yawed laminar flows the chord-wise flow is independent of the spanwise flow (the "independence principle"), and the rate of growth of the layer is reduced on yawed surfaces. It was concluded that this principle did not apply to turbulent flows; the measured thickness being greater on the yawed plate than at corresponding positions on the unyawed plate. However, the effect of yaw was found to be small, with angles of up to 2.5° between the free-stream direction and the local flow.

The experiment was repeated by Ashkenas (1958) on the plate with its edge yawed at 45° to the flow. A hot wire anemometer was used to measure velocity, shear stress and turbulence intensity profiles.

Swamy (1971) reported measurements in the boundary layer on a flat plate with its leading edge yawed at 30° and chamfered at 45°. In the resulting boundary layer, measurements using impact probes revealed velocity components perpendicular to the free stream of up to 15% of the free stream velocity, and flow yaw angles of 11°.

A prediction method for three-dimensional turbulent boundary layers, introduced by Bradshaw (1971), when applied to the layer on a yawed flat plate shows that the "independence principle" is not applicable, and also that the friction coefficient should be independent of yaw. A particularly significant suggestion is that all cross-flow originates at the leading edge.

Cross-flows in three-dimensional boundary layers are now usually plotted in the form of a hodograph. Johnston (1960) found that a triangular hodograph, with its apex at a value of $y^+ = 16$, adequately represent the flow pattern. Later workers, while finding the same basic shape for the hodograph, found that the apex occurred at much higher values of y^+ (e.g. Hornung & Joubert (1963) $y^+ \simeq 150$, Swamy (1971) $y^+ = 90$ to 220).

Wall shear stress under a two-dimensional boundary layer may be estimated using techniques depending on "near wall similarity" (e.g. Preston tubes, boundary layer fences, surface hot flows, semi-logarithmic plots of velocity versus distance from the wall), or by direct force measurement. In three-dimensions, Pierce and Zimmerman (1973) extended Clauser's (1954) method to include points in the laminar sublayer and transition region, using Schraub & Klines' (1965) method for estimating friction velocity from the data for each point in a measured profile, and concluding that the near wall flow of three-dimensional boundary layers is adequately represented by two-dimensional similarity correlations.

In the experiments reported here measurements were made of the boundary layers on two plates; (a) with the leading edge normal to the approaching flow, (b) with the leading edge yawed at 45°. In both cases the leading edge was chamfered, with an included angle of 20°. A matched pair of linearised hot wire anemometers was used, with perpendicular wires at 45° for the probe axis. Two sets of measurements were made on each plate, the first with the wires in planes perpendicular to the plate, and the second with the wires parallel to it. Static pressures were measured using tappings at selected points, and a movable static pressure probe. The plates were adjusted in the wind-tunnel until the observed variations in static pressure were less than 1% of the dynamic pressure: for the yawed plate the variations, well downstream of the leading edge were within 25%. The direction of surface flow at the surface was made visible using a suspension of titanium dioxide in water, with detergent added to reduce surface tension. Pierce and Zimmerman's method was used to evaluate the surface shear stress in the direction of the free stream, with Spaldings single formula law of the wall as the correlating equation. Friction velocity was estimated at each point; starting nearest the wall. If the estimations from the first two points were within 1%, the first was taken, as correct; if three of the first six were within 2½% the middle one of the three was used, otherwise the mean of the first six was calculated. Mean cross-flows and shear stresses were estimated using the differences between the mean voltages respectively, and distrances from the wall were measured to an accuracy of ± .012 mm.

Results

(a) The measured surface shear stresses were somewhat below those estimated from correlations. (see Fig. 1). It was therefore expected that a region of separated flow existed at the leading edge, and this was confirmed by the surface flow visualisation experiments discussed below.

(b) Streamwise velocity profiles were everywhere typical of those found in two dimensional turbulent boundary layers. Two profiles from the yawed plate are shown in Figs 2 (a) and 3 (a), with the Spalding single formula law of the wall, for comparison in Fig. 2 (a).

(c) Mean cross-flow components were everywhere very small, typically about 1% of the streamwise component. A printout of typical values is shown in Figs 2 and 3 for the yawed plate. Since the minimum value of y^+ was about 90 in all profiles, it was expected that the peak of a (triangular) hodograph would have been found, but only one of the hodographs resembled a triangle, and none of them showed cross-flow angles of more than 0.6^o.

(d) Normal Reynolds stresses in all cases were typical of those found for two-dimensional layers, with maximum values of uv/u_*^2 close to unity.

(e) The Reynolds stresses in planes parallel to the plate (transverse stresses) were measured using two wires which were separated in the direction normal to the plate. Transverse stress measurements were expected, therefore, to be in error, since the lower wire was subjected to a lower velocity, and, in general, a higher turbulence intensity than the upper wire.

Measurements of the transverse stresses were made in the two-dimensional layer for the purpose of estimating the size of the error, and to act as a correction for the measurements made in the three-dimensional boundary layer. The results of the measurements for the two-dimensional layer are shown in Fig. 4. The measurements in the layer on the yawed plate give a very different profile. In Figs 2 (b) and 3 (b) the transverse Reynolds stress profiles are plotted, both as they came "off the instruments", and corrected for the errors mentioned above. The corrected curve was obtained from the measured curve by subtracting the measured values at corresponding points on the normal plate from those on the yawed plate. The effect of the correction is to decrease the upstream values of transverse stress and to increase the downstream values. The boundary layer is seen to support transverse shear stresses, of a maximum magnitude of about 70% of the wall shear stress. The local shear stress vector may thus deviate from the local velocity vector by an angle as great as 35^o.

(f) The surface shear stress measurements and the values of cross-flow component measured on the plate prompted an investigation into the surface flow directions near the leading edge. Surface flow visualisation revealed that a separated flow region, of (streamwise) width 15 mm (2.4 x plate thickness) occurred on both plates. On the normal leading edge the flows, after reattachment, were parallel to the free stream, but on the yawed plate considerable surface cross-flow was exhibited, with traces making an angle of 30^o to the undisturbed stream direction. Further downstream the flow becomes more nearly parallel to the free stream direction, but still deviates from it by 5^o at the downstream limit of measurement.

The leading edge of the yawed plate was modified by adding modelling clay and chamfering to an included angle of 45^o. The length of the separated region was thereby increased to 40 mm (6.3 plate thicknesses) with considerably increased cross-flow angles. Curvature of the surface traces was still evident 200 mm downstream of the leading edge. The thickness of the plate used by Swamy (1971) is not known; all his measurements were made less than 200 mm downstream of the leading edge.

Two dimensional surface shear stress correlations are seen to adequately represent the streamwise surface shear stress in the alyer on a yawed flat plate, with the boundary layer thickness also being adequately represented.

It is also concluded that cross-flows in the layer are generated either at the leading edge or by separation from it, and, although "one-sided flat plate" conditions are not provided in the experiments reported here, yet the leading edge situation seems better than any previously reported.

A more fundamentally interesting conclusion is that boundary layers may support considerable transverse shear stress without a corresponding yaw in the flow pattern. In order to invalidate the above conclusion errors in estimating the transverse stress would need to be nearly three times as great on the yawed flat plate as on the normal plate, and would also need to take a very different form in the two layers.

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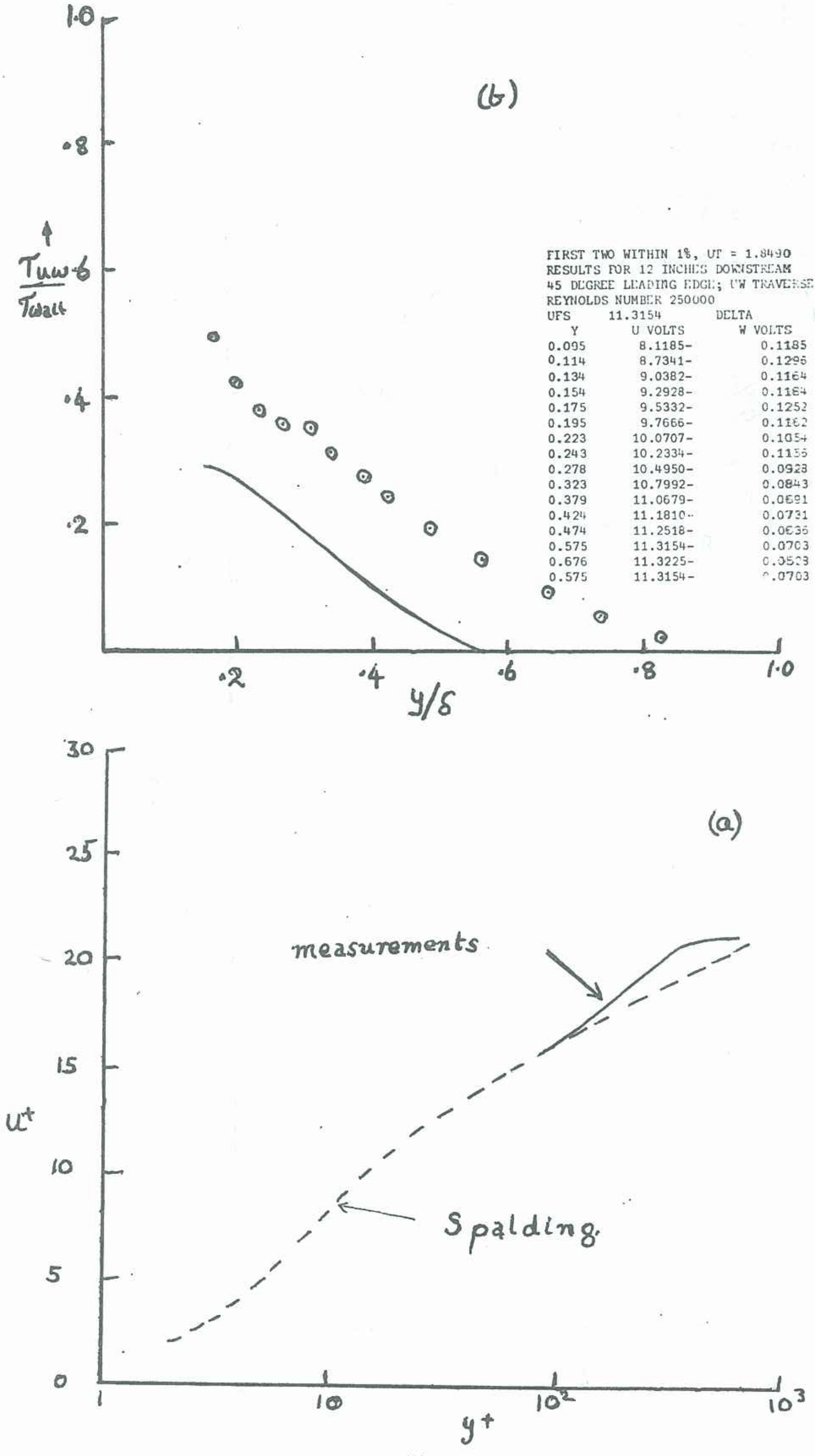

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FIRST TWO WITHIN 1%, UT = 1.8490
RESULTS FOR 12 INCHES DOWNSTREAM
45 DEGREE LEADING EDGE; UW TRAVERSE
REYNOLDS NUMBER 250000
UFS      11.3154        DELTA         0.5750        FRICTION VELOC        1.8490
  Y          U VOLTS          W VOLTS         U+            Y+          T/TW
0.095        8.1185-          0.1185        15.8751       92.6464       0.4954
0.114        8.7341-          0.1296        16.4892      111.1760       0.4250
0.134        9.0382-          0.1164        17.0634      130.6800       0.3832
0.154        9.2928-          0.1184        17.5440      150.1850       0.3600
0.175        9.5332-          0.1252        17.9980      170.6640       0.3493
0.195        9.7666-          0.1162        18.4386      190.1690       0.3137
0.223       10.0707-          0.1054        19.0127      217.4750       0.2750
0.243       10.2334-          0.1135        19.3198      236.9800       0.2441
0.278       10.4950-          0.0928        19.8138      271.1130       0.1960
0.323       10.7992-          0.0843        20.3879      314.9980       0.1461
0.379       11.0679-          0.0691        20.8953      369.6100       0.0927
0.424       11.1810-          0.0731        21.1089      413.4960       0.0535
0.474       11.2518-          0.0636        21.2424      462.2570       0.0232
0.575       11.3154-          0.0703        21.3626      560.7550       0.0000
0.676       11.3225-          0.0503        21.3759      659.2520       0.0000
0.575       11.3154-          0.0703        21.3626      560.7550       0.0000
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Fig. 2

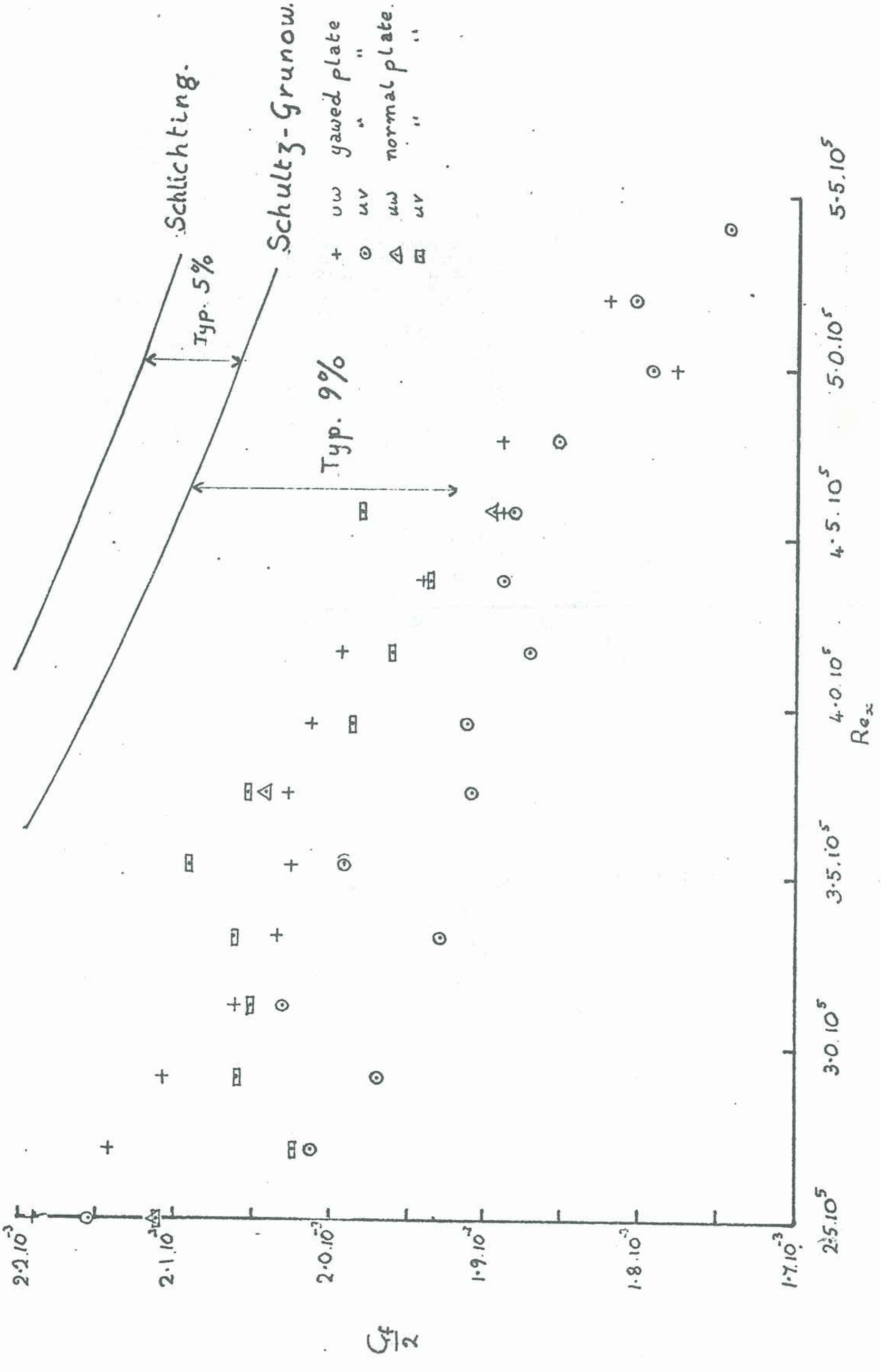

Schlichting.
Schultz-Grunow.
Typ. 5%
Typ. 9%
+ uw yawed plate
⊙ uv " "
△ uw normal plate
⊡ uv " "
2·2.10⁻³
2·1.10⁻³
2·0.10⁻³
1·9.10⁻³
1·8.10⁻³
1·7.10⁻³
$\frac{C_f}{2}$
2·5.10⁵
3·0.10⁵
3·5.10⁵
4·0.10⁵
4·5.10⁵
5·0.10⁵
5·5.10⁵
Re_x

Fig. 1

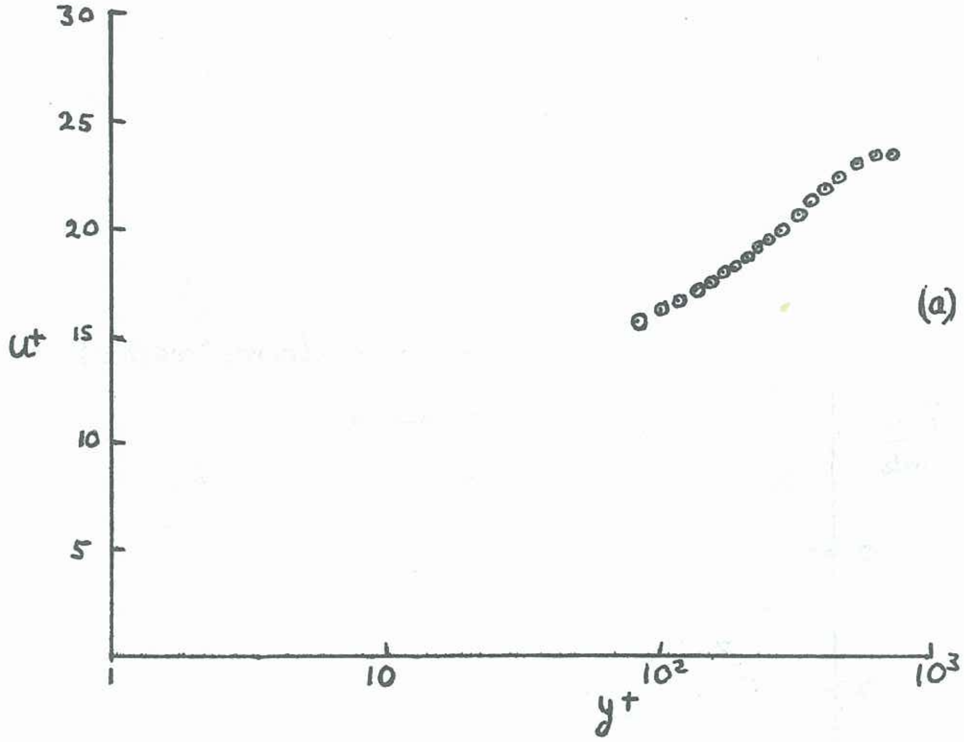
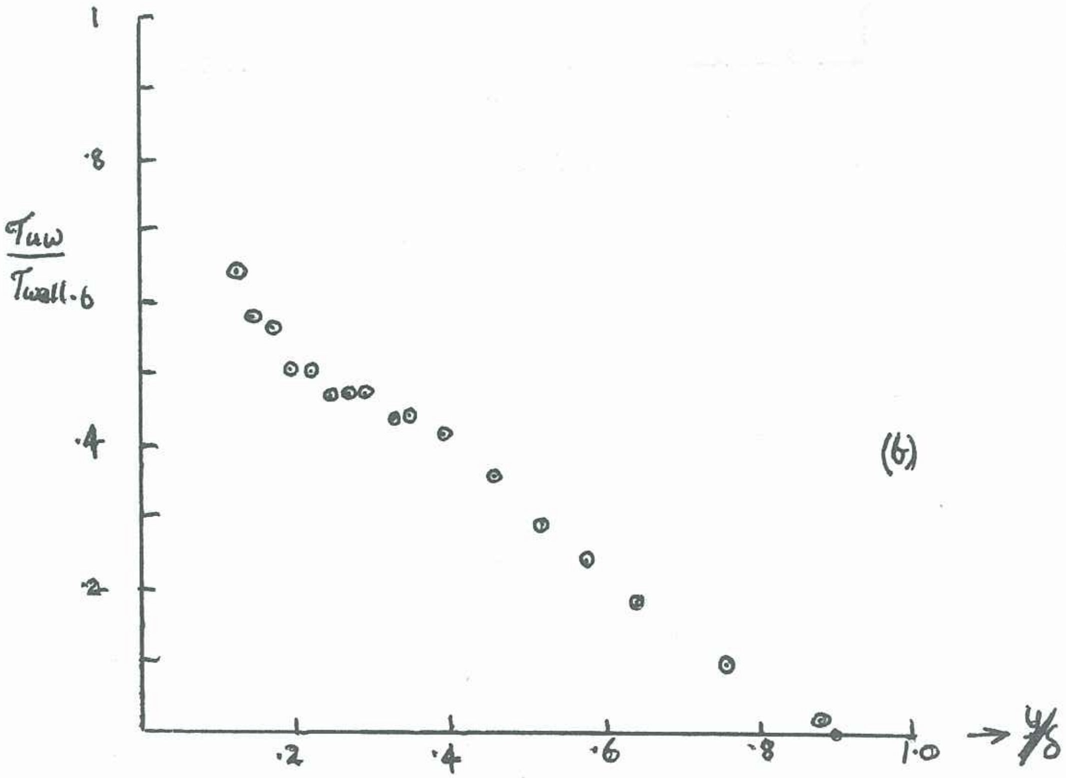


Fig 3. Velocity and shear stress 25 ins downstream.



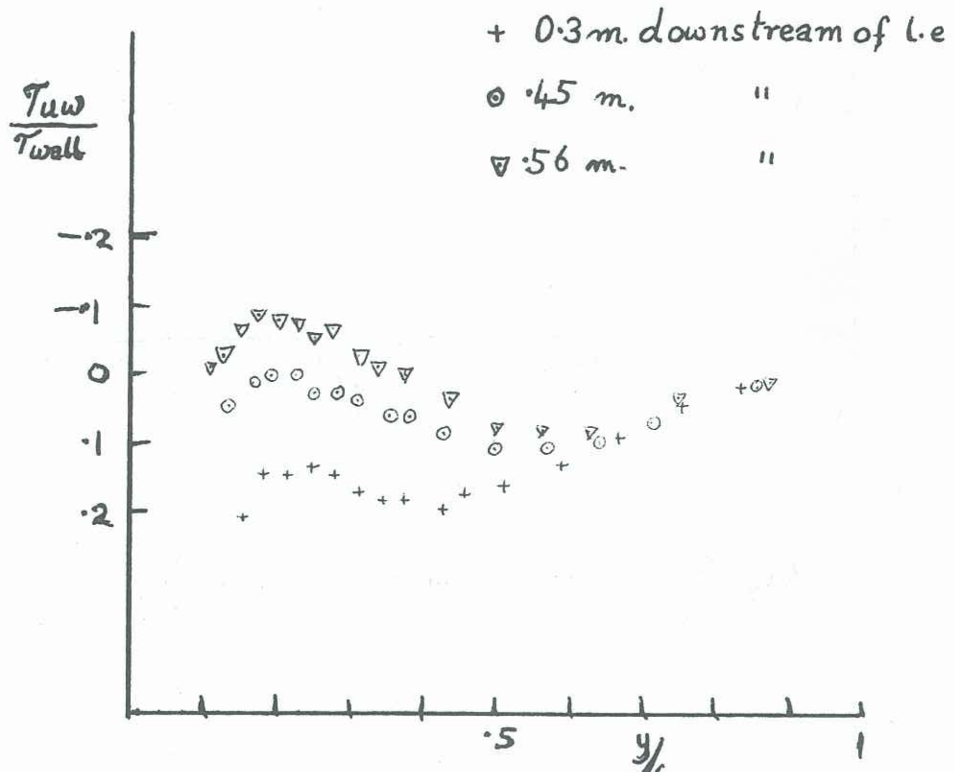


Fig 4