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THE RESPONSE OF CUP AND PROPELLER ANEMOMETERS TO FLUCTUATING WIND SPEEDS

by

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SUMMARY

An experiment to investigate the generation of gusts in a wind tunnel is described and results of the performance of several cup and propeller anemometers under sinusoidally fluctuating wind conditions are presented.

Gusts have been generated in a 31.75cm x 31.75cm ($12\frac{1}{2}$in. x $12\frac{1}{2}$in.) wind tunnel by contra-rotating two rectangular plates that sinusoidally open and block the tunnel working section. By varying the plate design, tunnel speed and plate rotational speed, maximum gust frequencies, mean velocities and gust amplitudes of 2.5Hz, 16m/s and 0.98 respectively have been obtained.

The rotating plate gustgenerator was used to test 6 cup, 3 cup and propeller anemometers in a sinusoidally fluctuating air stream and demonstrated that the percentage overestimation of the mean wind speed varied from 3% (for a Gill 4-bladed propeller anemometer) to 11.5% for a 3 cup (lipless) anemometer of all polystyrene construction, at a gust amplitude of 0.5.

The 6 cup anemometer had a marked superiority in these tests over the 3 cup anemometer and also had lower starting speeds. The 4-bladed polystyrene propeller anemometers had the best performance of all the anemometers tested.

A way in which these tests together with the linear first order theory for such sensors may be applied to correct data from them is suggested.

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Introduction

It is well known that all propeller, vane and cup type anemometers over-estimate the average wind velocity under 'natural wind' conditions [Ower (1937), Deacon (1951), MacCready (1965, 1966, 1970), Gill (1966, 1967), Kondo (1970), Hyson (1972)]. Wind speed sensors are usually calibrated in wind tunnels by finding their outputs or rotational speed over a range of steady, horizontal wind speeds. Natural winds, in contrast are random and fluctuate about a mean value that may be derived by averaging the signal over some suitable length of time between 20 minutes and 1 hour. As a first order generalisation, the response distance (or distance constant) L, defined by

$$L = T \cdot U$$

(where T is the time constant of the sensor and U the wind speed) of a speed sensor can be considered constant for a step speed change to a final equilibrium speed value, while the response time T varies. The resulting effect is that the response time T decreases for accelerating flow and increases for decelerating flow. Cup and propeller anemometers consequently accelerate better than they decelerate, resulting in the indicated average wind speed being greater than the true average wind speed. If instantaneous velocity measurements are required, to compute a Reynolds stress for example, the propeller could be underestimating or overestimating the instantaneous velocity component depending on whether it is exposed to accelerating or decelerating signals, respectively, at the instant the sensor is being scanned.

MacCready (1966) has discussed those factors affecting the fidelity of a wind speed measurement and classifies them as the w, v, u and Data Processing errors. The particular error we are concerned with here is the 'u' error.

Instruments of light construction and very low bearing friction usually have lower Distance Constants (L) and are usually considered to be better able to respond to fluctuations in the input signal. They should, therefore, record a more accurate value of the instantaneous velocity signal and hence, yield a more accurate derivation of the average velocity.

It is obviously of vital importance to any wind sensing and data collection system to know the fidelity with which the recording is made and make corrections for any instrument errors. For the computation of one of the Reynolds stresses for example, the <u>instantaneous</u> fluctuating components of velocity u' and w' are required to derive $\tau = -\rho\, \overline{u'\, w'}$. Calculation of u' and w' in turn requires the integration of the signal from the anemometer to derive the mean velocity components $\bar{u}$ and $\bar{w}$ and their subtraction from the instantaneous velocity signal. In the case of the data aquisition system described by Lindley, Bowen and Morfee (1974) the original instantaneous data from the sensor will have been stored on magnetic tape so that this data manipulation can take place in a large digital computer. Thus an error in the instantaneous velocity measurement is involved at source, due to the response limitations of the instrument. A further error caused by the integration of the signal over the 20 minute to 1 hour averaging period results in an inaccurate computation of u' and w' (e.g. $u' = u - \bar{u}$).

A knowledge of the response characteristics of the sensor in fluctuating winds is therefore essential if accurate processing of the raw data is to be achieved.

Experiments to Determine Sensor Response to Fluctuating Winds

For the tests, a 31.75cm x 31.75cm (12½in. x 12½in.) working section open return wind tunnel, incorporating a 'gust generator' was used to produce a sinusoidally pulsating flow of variable speed, amplitude and frequency. The 'gust generator' shown in Fig. 1 comprised two contra-rotating plates, driven by a sewing machine motor through a system of gears and pulleys, that could cyclically block off or leave open the tunnel working section in a sinusoidal manner. By varying the speed of the tunnel fan and the speed of the sewing machine motor, a range of tunnel speeds and 'gust frequencies' of 0 to 16 m/s and 0 to 2.5Hz respectively, was possible. The amplitude of the 'gust' could be varied by using rotating plates having different 'porosities' (or opening ratios) and also by altering the position of the sliding doors at the exit of the tunnel. A DISA hot wire anemometer was placed at a section 0.55 m upstream of the plane containing the propeller or cup anemometer under test and both the hot wire and the propeller or cup signals were fed to a Hewlett-Packard storage oscilloscope. A set of such records are given in Fig. 2 and show a low, medium and high frequency gust from this set up: the solid line in each case being a propeller anemometer output, the other line being the hot wire signal. It can be clearly seen that the propeller is responding better to an accelerating flow than one which is decelerating. It was necessary to introduce a filter circuit between the hot wire signal and the storage oscilloscope to produce a signal that could be integrated with minimum error. The filter circuit is shown in Fig. 3. The time constant of this circuit is very dependent on the value of the capacitance in this circuit and a value of 0.15 μF was chosen such that the time constant was 5 msec.

The hot wire signal was used to obtain a true mean velocity (by integration) and the maximum amplitude of the signal so that the amplitude of the velocity fluctuation as a percentage of the mean wind velocity ($\Delta u/\bar{u}$) could be derived. The average velocity recorded by the sensor under investigation was found by integrating its output over the same period, so that the percentage overestimation of the mean wind velocity could be computed.

At the time of performing these experiments there was no frequency analyser available to us that would integrate signals with frequencies less than 2Hz. It was therefore necessary to resort to 'mechanical' integration of the curves using a planimeter on the photographic records from the storage oscilloscope. Values were obtained by analysing the records over at least 15 cycles for each test.

The first tests were performed on a $7\frac{1}{2}$ ins. (19.05cm) diam. Gill 2-bladed polystyrene propeller to investigate both the effects of gust frequency and amplitude on its response; the results of these tests are given in Figure 4 for gust frequencies of 0.67, 1.33 and 2.13 Hz. It can be seen that there is no marked difference in the percentage overestimation at the two higher frequencies. Similar trends have been established by Schrenk (1929), Deacon (1951) and Hyson (1972) for cup anemometers. For gust frequencies above 2 Hz all authors had detected very little increase in overestimation error; though the error had increased with increasing gust frequency for frequencies less than 2 Hz.

To obtain the range of gust magnitudes, four sets of (rotating) plates were used. They had opening ratios (i.e. the ratio of open area to total area of the plate) of 0, 14.1 percent (circular holes), 35.6 per cent (circular holes) and 35.4 per cent (rectangular slots). Fig. 5 reproduces the oscilloscope outputs at 2.13 Hz for the four different sets of vanes whilst Figure 6 gives a more detailed plot of the results covering the range $0 < \frac{1}{2}(\Delta u/\bar{u})^2 < 0.17$. The vanes with opening ratios not equal to zero had to be used in this low gust amplitude range.

The results for the 2-blade propeller confirmed that there was no further increase in the percentage overestimation of the mean wind speed for gust frequencies greater than 2 Hz. All subsequent tests on other cup and propeller anemometers were therefore performed at a frequency of 2 Hz which represents the condition at which the error can be expected to be a maximum. The other anemometers examined, included the Gill⁺ $7\frac{1}{2}$ins. dia. 4-bladed propeller anemometer, the RIMCO* $3\frac{1}{4}$ins. (8.26cm) 3 cup assembly (mounted on the light chopping spindle described by Lindley, Bowen and Morfee (1974) and our own design of 6-cup and 3-cup anemometers (Lindley 1974). The 3 and 6 cup assemblies had 5.35 cm (2.11ins) conical polystyrene cups; some had polystyrene arms, others had tubular metal arms; all these cup assemblies were set with cup arm radius to cup radius ratio (R/r) of 2.5. The staggered 6-cup assembly (Fig. 7) was set with a cup wheel spacing to cup radius ratio (L/r) of 2.5. These ratios had been found (Lindley 1974) to give optimum performance for this type of anemometer. The 3 cup anemometers tested were simply one of the wheel assemblies (instead of two) mounted on the same spindle.

The results of all these tests has been plotted in Figure 8, each line representing the best fit through all the experimental points for the sensor tested. The superiority of the Gill $7\frac{1}{2}$ ins diam. 4-bladed propeller anemometer over all the other anemometers tested can be clearly seen. Two of the staggered 6-cup wheel assemblies have a performance almost as good as the 4-bladed propeller and better than the Gill $7\frac{1}{2}$ ins. diam. 2-bladed propeller. It must be remembered however, that cup anemometers are omnidirectional and therefore not competitors where the propellers directional sensitivity is of importance.

Ower's (1937) prediction for the overestimation of vane anemometers in fluctuating winds as expressed by

$$E = \frac{1}{2}\left(\frac{\Delta u}{\bar{u}}\right)^2 \times 100 \text{ per cent}$$

is plotted and seen to be the upper bound to all the experimental results plotted in Fig. 8.

It is also interesting to note the improvement of the 4-bladed Gill propeller over its 2 bladed counterpart. Even though it has a larger distance constant, the 4-bladed Gill, which has a more even torque distribution during each revolution, has a better response to fluctuating winds. For the same reason the staggered 6 cup anemometer (with no lip to the cups), given as curve 5 in Figure 8, has a superior performance to a 3 cup assembly having identical geometry (curve 9). It is also evident that the lip on the conical cups has a beneficial effect. For comparison, Hysons (1972) results for a 3-cup anemometer designed by Sumner (1968), obtained at gust frequencies of 1.3 Hz and 2 Hz have been plotted; it is seen that his results lie very close to the RIMCO cup wheel tested (which in fact is the production model of Sumner's prototype).

⁺ Manufactured by R.M. Young Co. Ann Arbor, Michigan, U.S.A.

* Manufactured by Rauchfuss Instruments Ltd., Burwood, Victoria, Australia.

Schrenk's (1929) results for a wheel assembly of 4 hemispherical cups at a K value of 0.1 (where $K = 0.1$ is a result that corresponds with a frequency range of 2 Hz at velocities of the order 6 m/sec) are also plotted and lie within the range of the other 3-cup anemometers tested. Deacon's (1951) result (at a gust amplitude ($\Delta u/\bar{u}$) of 0.5) is also plotted for the British Meteorological Office pattern cup anemometers (again at $K = 0.1$) and seen to be on the upper boundary of the experimental results for the anemometers tested here. It should be emphasised that the lightweight cup and propeller anemometers tested are considerably lighter and faster in response than the more robust Meteorological Office type of instrument and one might expect better performance from them.

The absolute values of percentage overestimation error given in Fig. 8, should, however, be interpreted with care. It has to be remembered that all the instruments were tested in the artificial flow of a regular sinusoidally fluctuating stream of air of constant amplitude and frequency. A gust amplitude ($\Delta u/\bar{u}$) of 0.5 for example, might represent an extreme in nature. This would mean that the maximum overestimation error for the class of instruments tested would range from 3 per cent for the 4-bladed Gill propeller anemometer to 12.8 per cent for Deacon's (1951) M.O. Pattern cup anemometer.

When such instruments are used in the natural wind they are subjected to a range of velocities, gust amplitudes and gust frequencies, that for most of the time would be less severe 'a test' of the instrument than that used here, so that the results presented would represent the worst case and should only be used for comparative purposes. The results however, confirm that of all the cup or propeller anemometers tested, the 4-bladed propeller has the best performance in fluctuating winds.

Application of the Results and Theoretical Prediction

It is usual to assume that propeller and cup anemometers are first order (i.e. non-oscillatory) systems [MacCready (1965), (1966), (1970) and Gill (1966, 1967)]. This assumes that the sensors change toward a final equilibrium value depends only on the difference between the final value and its present value, and not on the sensor's present rate of change. The response characteristics can then be defined as we have already seen, by a single "time constant", T, or equivalently a "distance constant", L.

A first order system can be represented by

$$T\frac{dy}{dt} + y = f(t) \qquad (1)$$

where t denotes time, $f(t)$ represents the applied disturbance (forcing function), y is the sensor indication, and T is the time constant for the sensor.

The general solution of Eq. (1) for a step function input i.e. $f(t) = 0$ if $t \leqslant 0$ and $f(t) = A$ for $t > 0$ is

$$y = A\,(1 - e^{-t/T}) \qquad (2)$$

where T is the time taken for the variable to reach $(1 - 1/e)$, or 0.63 of the final value. For rotating cup or propeller anemometers, we have seen that T varies inversely with wind speed U. Thus, a distance constant, $L = UT$ is defined, L being constant at all speeds.

Thus if the wind speed changes abruptly from a constant U_o to $U_o + \Delta U$, then the rotor speed-up u, from its equilibrium value at U_o will be

$$u = \Delta U\,(1 - e^{-t/T}) = \Delta U\,(1 - e^{-x/L}) \qquad (3)$$

Fig. 9 (from MacCready 1970) gives a plot of the response in terms of T and L units. Note that the tangent to the curve at $x = 0$ intersects the ΔU step at $x = L$, the tangent at $x = L$ intercepts at $x = 2L$, etc. At $x = L$ the indication reaches 0.63 of the final value, at 2L it is 0.86, at 3L, 0.95 at 4L; 0.98 and so on.

The Distance constant (L) for both cup and propeller anemometers can easily be measured.

If we now consider the anemometers response to a sinusoidally varying wind

$$U = U_o + \Delta U\,(\sin\omega t) \qquad (4)$$

Then it can be shown that

$$y = A\,(1 + \omega^2T^2)^{-\frac{1}{2}}\,\sin\,(\omega t - \varphi) \qquad (5)$$

where $\varphi = \tan^{-1}(\omega T)$, the phase lag angle (in radians). The term $(1+\omega^2T^2)^{-\frac{1}{2}}$ is called the "dynamic gain" or "amplitude ratio" M, the ratio of output amplitude (X) to input amplitude (X_o)

for the case A = 1. Gill (1967) has shown that Eq. 5 can readily be expressed as

$$\frac{L}{\lambda} = \frac{\sqrt{\left({}^{X_o}/_{X}\right)^2 - 1}}{2\pi} \qquad (6)$$

where L = distance constant
λ = wave length of sinusoidal speed fluctuation or gust wave length
X_o = actual amplitude of speed change
X = indicated amplitude of speed change

Fig. 10 (from MacCready 1970) plots M and φ against several versions of the independent variables.

Now Eq. 6 does not predict the overestimation of mean wind speed in a fluctuating wind, having assumed in the first place that the sensor response to a step input did not depend on the sensors rate of change or on the direction of the change but only on the difference between the final value and its value at the time of the change. The function, as expressed by Eq. 6, to correct for amplitude ratio, is symmetrical for accelerating or decelerating winds so that the integrated effect would be zero over an averaging time of 20 mins. to 1 hour. This means that the linear first order theory assumed cannot predict an overestimation of mean wind speed in a fluctuating wind.

The oscilloscope traces shown in Fig. 2 demonstrate clearly that the propeller is responding in a non-linear fashion to the sinusoidal input signal (a linear response would result in an output wave form identical to the input wave form though it might be out of phase). The wind tunnel tests described in the previous section therefore demonstrate the departure of the anemometer characteristics from the linear first order behaviour often assumed. For practical purposes, Figure 8 demonstrates that the 4-bladed propeller has the best response to fluctuating winds, giving the least overestimation of the mean wind speed of all the sensors tested and in fact indicating that its response characteristics are closest to the assumed linear first order system.

The results presented here show that deviation from the first order system was a function of frequency up to 2 Hz and is also a function of gust amplitude. Having chosen the 4-bladed anemometer on these grounds, we can then correct data from it, in a relatively simple way by assuming it is behaving as a first order sensor, so that equation 6 can be applied. If for example a turbulent energy spectrum has been obtained with the 4-bladed propeller, the spectrum curve can be corrected to the true atmospheric spectrum by multiplying by $({}^{1}/M)^2$ at each frequency. This requires a Fourier analysis of the signal and a knowledge of the sensors distance constant (L) [from a simple step function response test in a wind tunnel to obtain a trace similar to Figure 9], so that equation 6 can be computed for each frequency.

Horst (1973) points out however, that for an orthogonal array of propellers, the procedure outlined above is somewhat complicated by the fact that the propeller's distance constant varies with angle of attack. i.e. The distance constant (L), which is normally determined when the wind is aligned with the propeller axis of rotation, varies as the wind angle to this axis changes. If in fact, the distance constant is a strong function of the angle between the wind direction and the propeller axis, the U and V axes of the array will, in general, have different time constants, characterising their response. The wind components along a rotated set of coordinates, such as that defined by the direction of the hourly mean wind, will therefore no longer have a simple response as described by eq. 1. The frequency response of a propeller in a non-axial, highly turbulent flow is still poorly understood and further work is continuing to improve this understanding.

Conclusion

The experiments have confirmed that cup and propeller anemometers overestimate the mean wind speed in sinusoidally fluctuating winds. Further, they also confirm that this tendency is frequency dependent for gust frequencies up to 2 Hz, when further increase in frequency has no further effect. Where directional sensitivity is required, the 4 bladed polystyrene propeller has the best response under these conditions, yielding a percentage overestimation of the mean wind speed of 3% at a gust amplitude of 0.5.

For omnidirectional cup anemometers, the results show that a staggered 6-cup design has a superior performance to an equivalent 3 cup array (having the same design and weight of cups). A lip on the cup has a beneficial effect on the performance in fluctuating winds. The results demonstrate that the performance of such sensors is not solely dependent on distance constant and that simply reducing the weight of cup or propeller assemblies has for example not necessarily produced the best performance. (The 4 bladed propeller being superior to the 2 bladed propeller and the 6 cup all polystyrene assembly being superior to the equivalent 3 cup all polystyrene assembly).

Space has not permitted a discussion of the merits of the 'gust generator' used in these experiments. Other experiments were made to investigate the performance of "scotch yoke" and 'siren' gust generators, and work is continuing on the simulation of gusts in wind tunnels. It is interesting to note that a recent experiment sponsored by the Building Research Establishment in the United Kingdom (B.R.E. 1974) has used a similar method of gust generation (to that used in the present tests) to provide quantitative information on how people respond to gusty winds. The tests took place in a reinforced concrete tunnel measuring 3.9 m x 2.7 m in section, at the National Physical Laboratory in the U.K.

The 'gust generator' comprised a series of vertical aluminium vanes which could be pushed backwards and forwards by a motor to give regular, large-scale gusts superimposed on the previously steady air flow. People were made to perform a range of tasks in these conditions and their performance was observed. There is evidently considerable scope for further work on gust generation as it applies to earth boundary layer tunnels and the effect of gusts on structures, people and transport systems.

Acknowledgements

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References

BRE 1974: "An ill wind" BRE News. No. 26 Winter 1974 p.6-7. Building Research Establishment England.

Deacon, E.L. 1951 (Aug): 'The overestimation error of cup anemometers in fluctuating winds'. Journ. of Scientific Insts., Vol 28, pp231-234.

Gill, G.C. 1966: 'Temperature and Wind Measuring Systems - Basic Principles and Dynamic Response' Seminar Series at White Sands Missile Range Oct. 24-28.
1967: 'On the Dynamic Response of Meteorological Sensors and Recorders' Proc. 1st Canadian Conference on Micrometeorology. Part I, Meteor. Service of Canada, Toronto, pp1-27.

Horst, T.W. 1973 (June): 'Corrections for Response Errors in a Three-Component Propeller Anemometer' Journ. of Applied Meteorology Vol. 12 pp716-725.

Hyson, P. 1972(Aug): 'Cup Anemometer Response to Fluctuating Wind Speeds' Journal of Appl. Meteor. Vol. 11, pp845-848.

Kondo, J., Naito, G., and Fujinawa, Y. 1971 (April): "Response of Cup Anemometer in Turbulence" Journ. of the Meteor. Soc. of Japan. Vol. 49, No. 2, pp63-74.

Lindley, D. 1974: "The Design and Performance of a 6 Cup Anemometer". To be published.

Lindley, D., Bowen, A.J. and Morfee, P. 1974(Dec). "A propeller anemometer for a digital wind data acquisition system" Fifth Australasian Conf. on Hyd. and Fluid Mechs. Christchurch, N.Z.

MacCready, P.B. 1965: "Dynamic response characteristics of Meteorological sensors". Bulletin Amer. Meteor. Soc., Vol. 46, No. 9, Sept. pp533-538.
1966: "Mean wind speed measurements in turbulence" Journ. of Appl. Meteor. Vol. 5 pp219-225.
1970: "Theoretical considerations in instrument design". Meteorological Monographs. Vol. 11, No. 33 pp202-210.

Ower, E. 1937 (May): 'On the Response of a Vane Anemometer to an Airstream of Pulsating Speed'. Philosophical Magazine, Series 7, Vol. 23, No. 157.

Schrenk, O. 1929: "Errors due to inertia with cup anemometers in fluctuating winds" Z. Tech-Phys., 10, 57-77.

Sumner, C.J. 1968: "A Low Torque Cup Anemometer" Australian J. Instr. Control. October, p39-40.

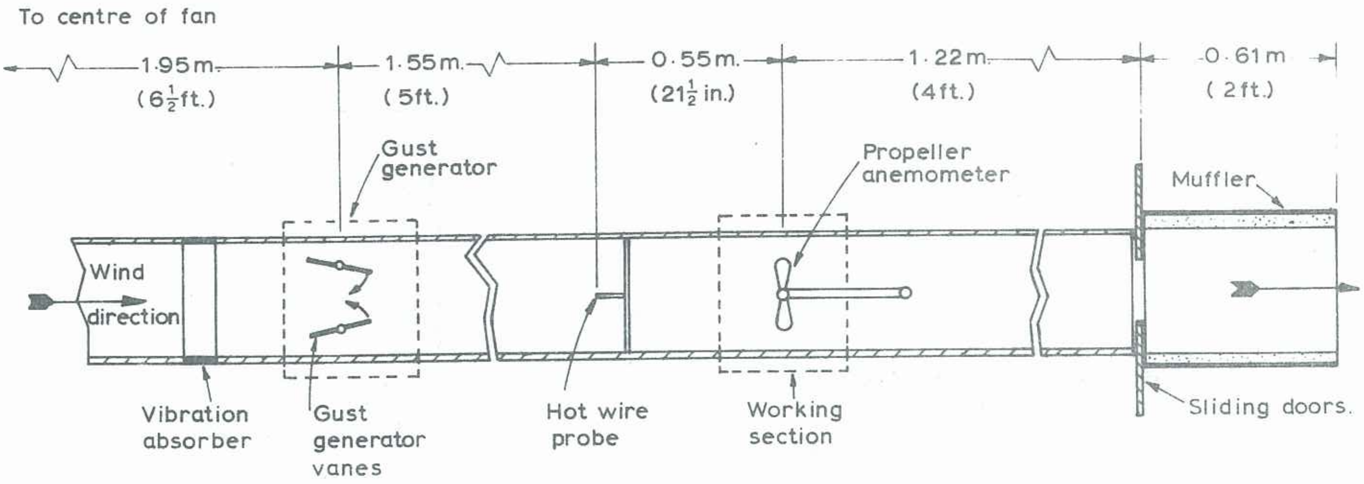
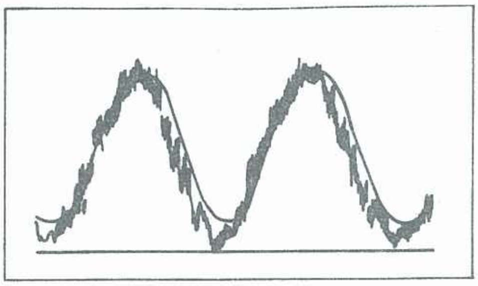
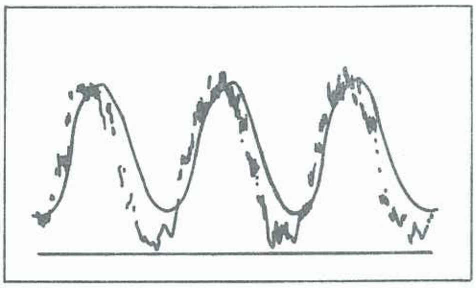


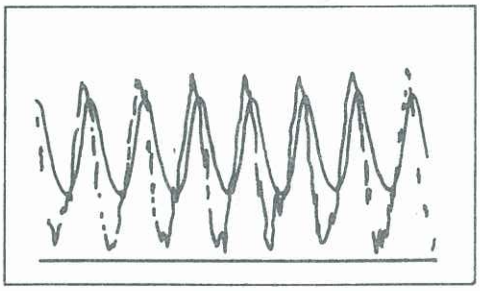
Fig. 1. GENERAL ARRANGEMENT OF THE "GUST GENERATOR" WIND TUNNEL.



(a) Low frequency gusts.



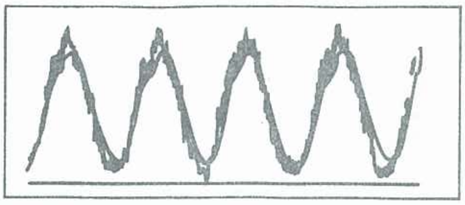
(b) Medium frequency gusts.



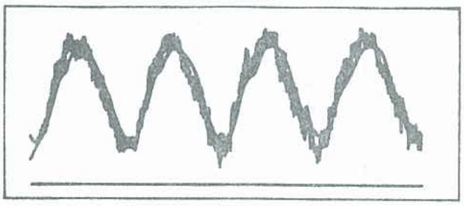
(c) High frequency gusts.

Key:
— Propeller anemometer output.
⁓ Hot wire anemometer output.

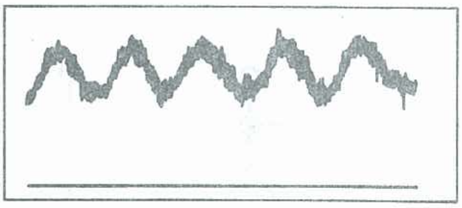
Fig. 2 OSCILLOSCOPE RECORDS OF SIGNALS AT DIFFERENT GUST FREQUENCIES.



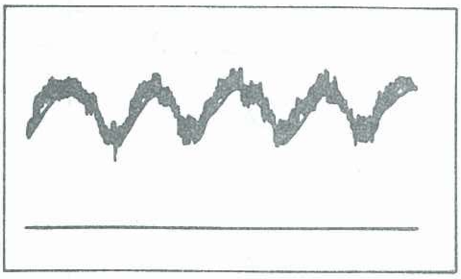
(a) Gust generated by "holeless" vanes.



(b) Gust generated by vanes with 14·1% opening ratio. (circular holes)



(c) Gust generated by vanes with 35·6% opening ratio. (circular holes)



(d) Gust generated by vanes with 35·4% opening ratio. (rectangular holes)

Fig. 5. GUSTS GENERATED BY DIFFERENT VANES. BIGGER OPENING RATIOS PRODUCED SMALLER GUST AMPLITUDES.

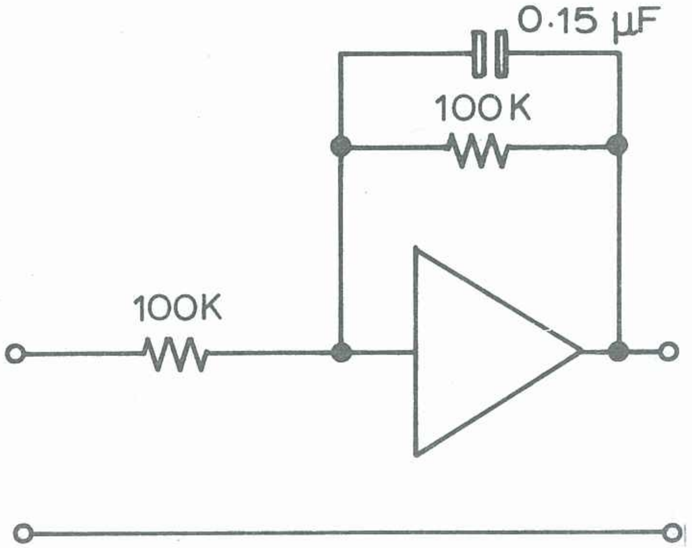


Fig. 3. FILTER CIRCUIT FOR THE HOT WIRE ANEMOMETER.

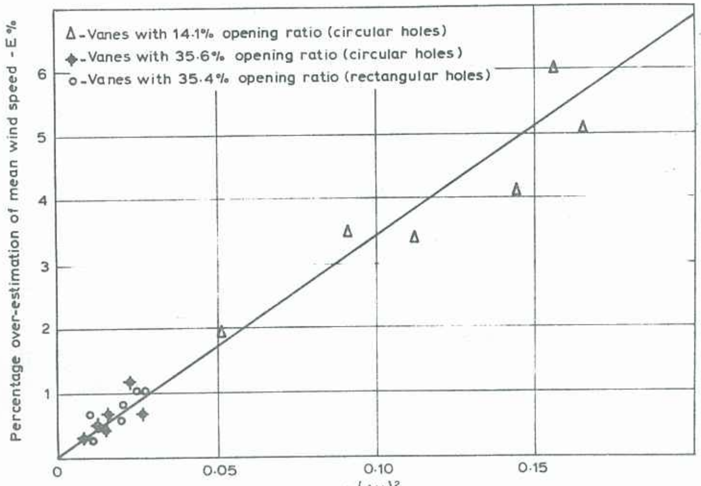


Fig. 6. PERCENTAGE OVER-ESTIMATION OF THE MEAN WIND SPEED FOR A 2 - BLADED $7\frac{1}{2}$ INS. DIAM. PROPELLER AT 2.13 Hz.

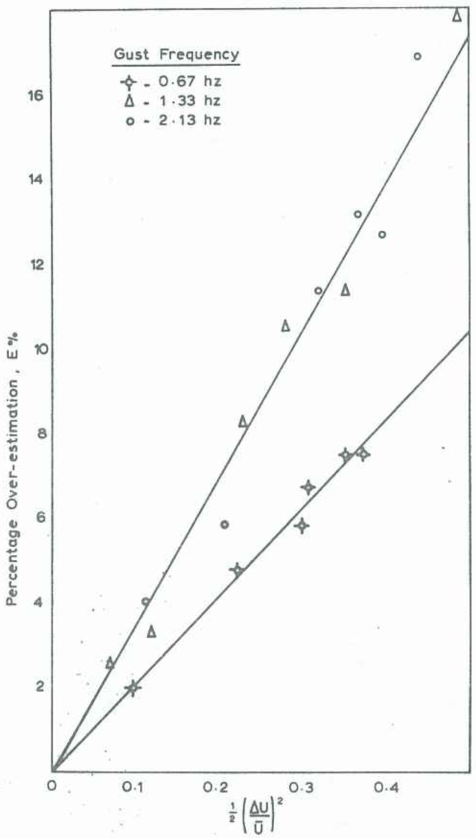


Fig. 4. THE EFFECT OF FREQUENCY ON THE OVER-ESTIMATION OF THE GILL 2-BLADE, $7\frac{1}{2}$ INS. DIAM. PROPELLER ANEMOMETER IN A SINUSOIDAL PULSATING WIND.

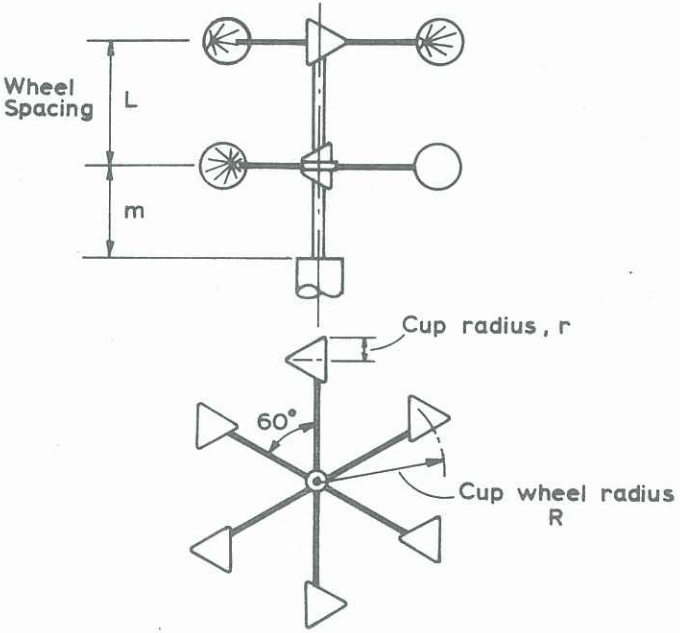


Fig. 7. SCHEMATIC OF 6 CUP ANEMOMETER

NOTE: Subtangents intersect 1.0 at T, 2T, 3T.

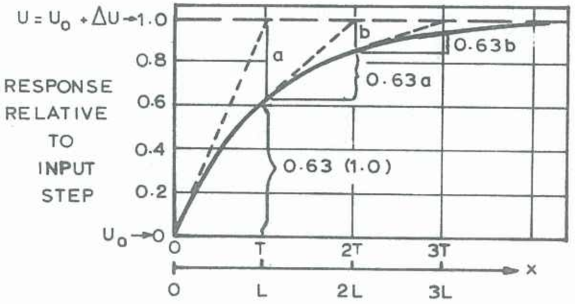
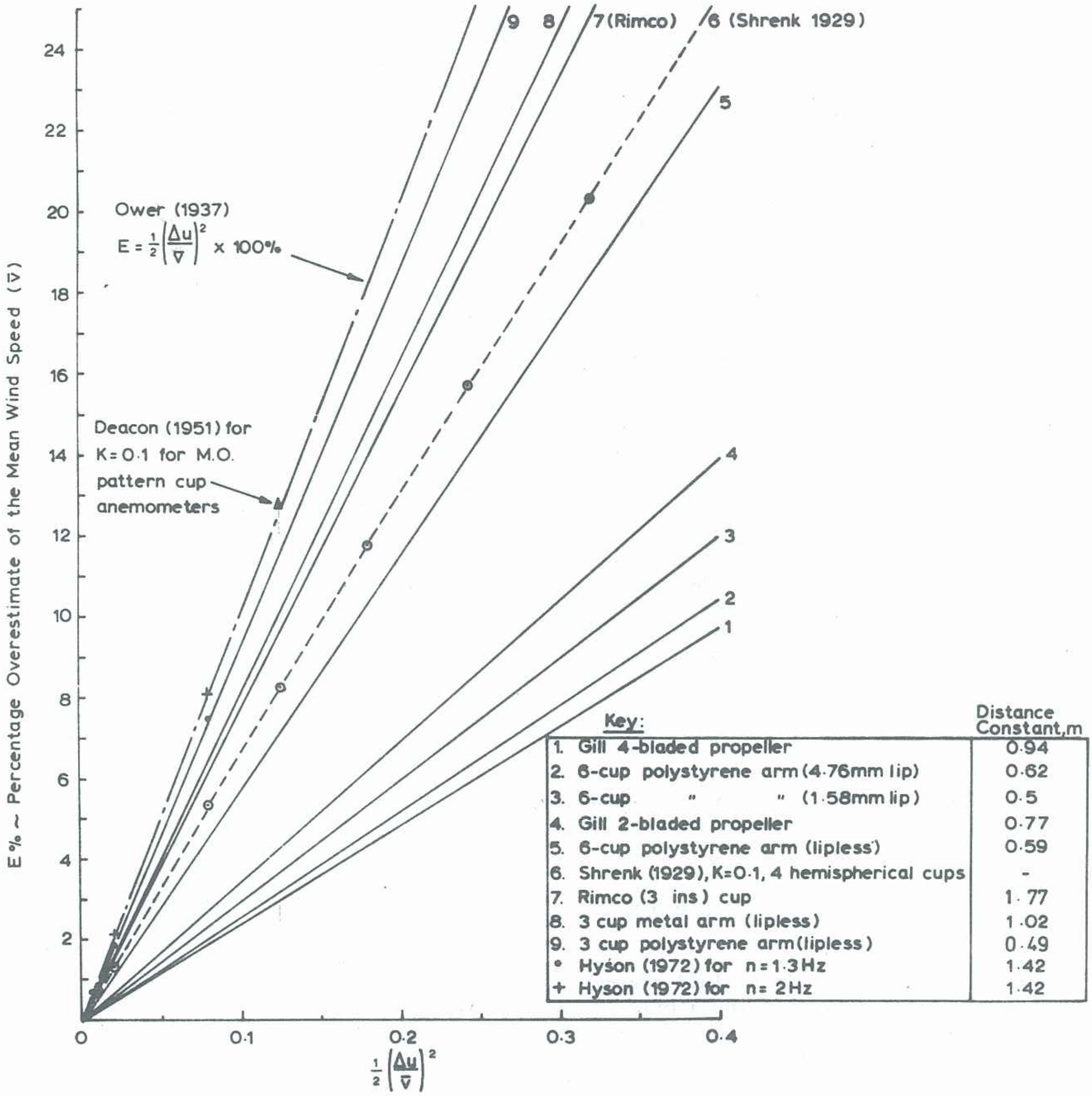


Fig.9. FIRST-ORDER RESPONSE TO A STEP INPUT.



Key:	Distance Constant, m
1. Gill 4-bladed propeller	0·94
2. 6-cup polystyrene arm (4·76mm lip)	0·62
3. 6-cup " " (1·58mm lip)	0·5
4. Gill 2-bladed propeller	0·77
5. 6-cup polystyrene arm (lipless)	0·59
6. Shrenk (1929), K=0·1, 4 hemispherical cups	-
7. Rimco (3 ins) cup	1·77
8. 3 cup metal arm (lipless)	1·02
9. 3 cup polystyrene arm (lipless)	0·49
• Hyson (1972) for n = 1·3 Hz	1·42
+ Hyson (1972) for n = 2 Hz	1·42

Fig. 8. PERCENTAGE OVERESTIMATION OF CUP AND PROPELLER ANEMOMETERS IN A FLUCTUATING WIND OF AMPLITUDE Δu AND MEAN SPEED $\bar{V}$.

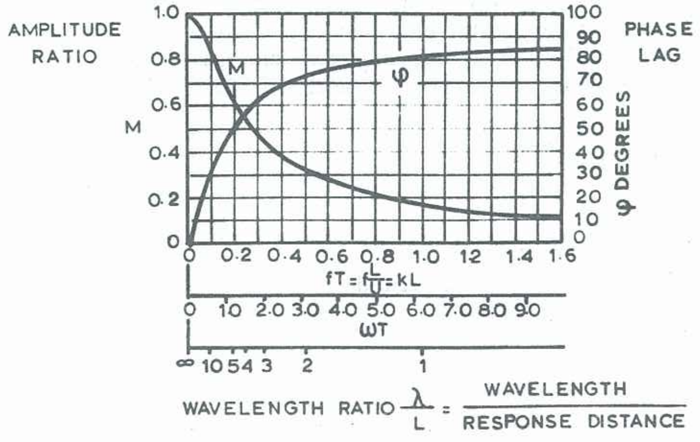


Fig. 10. FIRST-ORDER SYSTEM RESPONSE TO A SINUSOIDAL INPUT.