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A NUMERICAL STUDY OF KATABATIC WINDS AND THEIR EFFECT ON POLLUTANT DISPERSAL IN URBAN AREAS

by

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SUMMARY

This paper describes a series of numerical experiments designed to simulate the katabatic wind produced by nocturnal cooling of an isolated hill. It aims to explore the factors which determine the intensity, structure and extent of such winds. These factors are relevant not only to an understanding of the climatology of areas adjacent to hilly or mountainous terrain but also to an understanding of the flushing and dispersal of pollutant build-up in valleys and above plains bordering on hills. For this reason, an attempt is made to assess the effectiveness of the katabatic circulation in dispersing a pollutant emitted over the plain area adjacent to an isolated hill prior to the onset of cooling and to estimate the range over which pollutant concentration is significantly reduced.

The calculations provide a clear demonstration of the importance of the advection of momentum and heat in the flow, effects which have been ignored in most previous theoretical studies. We have also found that non-uniformities in ground cooling have a very large influence on the strength of the downslope wind when other factors are unchanged.

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<u>Introduction</u> Katabatic winds are gravity currents of relatively cold air which form adjacent to sloping ground as it is radiatively cooled, usually in the evening or at night. They are a ubiquitous feature of local weather in undulating or mountainous terrain and they occur at all latitudes and on horizontal scales ranging from a few hundred metres to many hundreds of kilometres. Physically, the effect of cooling the atmosphere from below produces a shallow, stably stratified layer of air in which, in the presence of orography, local horizontal pressure gradients are often significantly larger than the prevailing geostrophic gradients. In cases of intense nocturnal cooling, the stratification of the layer is often sufficient to inhibit the generation of turbulence by shear and at the top of the layer a strong inversion is formed. This provides an effective frictional decoupling of the surface layers from the air above and therefore modifications of the synoptic scale weather patterns by katabatic winds may produce significant local anomalies in climate in certain areas.

Traditionally, the major observational studies have been conducted in mountainous areas, especially in the United States, in the alpine regions of Europe and in Antarctica, where exceptionally strong katabatic winds are common. However, the realization that local orographic winds are an important factor in the persistence and/or dispersal of atmospheric pollution in many industrial or urban areas where the terrain is only moderately undulating or hilly has begun to stimulate wider interest and effort. A survey of early studies of katabatic winds forms part of an article by Defant (1951) and a succinct review of more recent work is given by Munn (1966). Most of the latter has been observational in character (see e.g. Ayer, 1961; Davidson and Rao, 1963; Buettner and Thyer, 1966; MacHattie, 1968; Mendonca, 1969 and Urfer-Henneberger, 1970) and there is still a paucity of theoretical work in the literature; in particular, the only numerical study in print the authors are aware of is that of Thyer (1966).

Of the few theoretical investigations since 1950, those of Fleagle (1950), Gleeson (1953) and Shieh (1971) all make a large number of assumptions to obtain tractable analytic problems, the most serious being the omission of advection effects. A somewhat more satisfactory analysis is given by Ball (1956), who draws on methods in hydraulic theory to study the behaviour of strong katabatic winds. Ball's theory has been particularly successful in application to the intense surface winds which occur over the vast ice caps of Antarctica (Ball, 1957, 1960).

A particular modelling difficulty is that in reality, local orography is exceedingly variable from place to place and "typical" geometries and orientations are hard to categorize. But local wind systems depend strongly on geometry and on such factors as orientation of the topography to the sun and to the geostrophic wind and during daylight hours on the inclination of the sun to the horizontal. Nevertheless, model studies of airflows induced by addition or subtraction of heat in the presence of simple configurations of valleys, hills and plains should yield significant insight and provide an invaluable basis for understanding the local wind circulations in more complex situations.

So motivated, we describe a preliminary series of numerical experiments which we believe simulate the broad physical features of the katabatic wind produced by ground cooling in a region including an isolated symmetrical hill. The hill is assumed to have an infinite straight crest and the flow is taken to be two-dimensional, although it is straightforward to adapt the numerical procedure to the corresponding axi-symmetric model. Ground cooling is simulated by prescribing the temperature on the hillside and on the adjacent horizontal plain as a decreasing function of time. In the model, the Boussinesq equations for a viscous, thermally diffusing fluid are integrated numerically in a sliced rectangular region (see figure 1) subject to certain prescribed boundary conditions (to be discussed later) and a prescribed initial state. In particular, the model is non-hydrostatic and advection effects are taken fully into account. Turbulent mixing is characterized by a constant eddy diffusivity, assumed the same for both momentum and heat and as the flows considered have relatively small vertical scales we have avoided the use of potential temperatures and densities at this stage. In addition to the dynamical and thermal fields, the model includes an equation representing the conservation of a pollutant aerosol which is used to monitor the dispersal of a shallow, uniform layer of pollutant initially over the plain and the lower part of the hillside.

Despite the geometrical simplicity of the model, it contains many parameters whose variation we are unable to explore fully here for reasons of space. Accordingly we describe a set of four experiments which give an impression of the principal features of katabatic winds and of some of the factors which govern their intensity and structure. A wider range of parametric behaviour will be discussed at the conference and details will be presented elsewhere in a subsequent paper.

The Model The momentum, continuity and heat equations for unsteady, two-dimensional convective motion of a viscous, heat conducting, Boussinesq fluid referred to rectangular co-ordinates (x, z) at time t are;

$$\frac{D\underline{v}}{Dt} = -\frac{1}{\rho_o}\nabla p + \left(\frac{T-T_o}{T_o}\right)\underset{\sim}{g} + K\nabla^2\underline{v} \quad , \tag{1}$$

$$\nabla.\underline{v} = 0 \quad , \tag{2}$$

$$\frac{DT}{Dt} = K\nabla^2 T \quad , \tag{3}$$

where $\underline{v}$ = (u, w) is the velocity, p the dynamic pressure, g = (o, g) the acceleration due to gravity, T the temperature, ρ_o and T_o are a reference density and temperature assumed here to be constant and K is a turbulent eddy diffusivity assumed also to be constant. Typical Rossby numbers for the flows we consider are large compared with unity and we neglect Coriolis effects although these may be important in the larger katabatic wind systems of Antarctica (Ball, 1960). The derivation of equations (1) - (3) are discussed in more detail by Smith et. al. (1974).

The equations are further simplified by introducing a streamfunction ψ and a horizontal vorticity ζ, defined by

$$u = -\frac{\partial\psi}{\partial z} \quad , \quad w = \frac{\partial\psi}{\partial x} \quad ,$$

and

$$\zeta = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = \nabla^2\psi \tag{4}$$

Equation (2) is then satisfied identically and the curl of Eq. (1) together with Eqs. (3) and (4) may be written in the following form suitable for numerical integration;

$$\frac{\partial\zeta}{\partial t} = -J(\psi, \zeta) + \frac{g}{T_o}\frac{\partial T}{\partial x} + K\nabla^2\zeta \quad , \tag{5}$$

$$\frac{\partial T}{\partial t} = -J(\psi, T) + K\nabla^2 T \quad , \tag{6}$$

$$\nabla^2\psi = \zeta \quad ,$$

where $J(\psi, \zeta)$ is the Jacobian notation for $\underline{v}.\nabla\zeta$.

In addition to these, our model includes an equation representing the concentration c(x, z, t) of a pollutant aerosol for which the governing equation is

$$\frac{\partial c}{\partial t} = -J(\psi, c) + K\nabla^2 c \quad , \tag{7}$$

where we have arbitrarily chosen the diffusivity of pollutant as equal to that for heat and momentum. This assumption is not necessary but since advection rather than diffusion governs pollutant dispersal at night when convective activity is at a minimum, the results and their significance should be insensitive to this choice.

Equations (4) - (7) are solved over the flow domain shown in Fig.1 for a hill of uniform slope, subject to the initial condition $\underline{v}(x, z, t)\equiv o$, $T(x, z, t)\equiv T_o$ and $c(x, z, t) = c_o(x, z)$ throughout the region of computation at $t=o$, the instant at which ground cooling commences and with the following boundary conditions:

(i) On the symmetry boundary BC ($x=o$, $h<z<H$);

$$u = o \ , \quad \frac{\partial w}{\partial x} = o \ , \quad \frac{\partial T}{\partial x} = o \ , \quad \frac{\partial c}{\partial x} = o \ ;$$

(ii) On the open lateral boundary EF ($x=L$, $o<z<H$); which is assumed to be sufficiently remote from the foot of the hill for variations in flow quantities to be neglected, we take

$$\frac{\partial u}{\partial x} = o \ , \quad w = o \ , \quad \frac{\partial T}{\partial x} = o \ , \quad \frac{\partial c}{\partial x} = o \ ;$$

(iii) On the upper boundary CF ($o<x<L$, $z=H$), a strong high-level inversion is assumed and we model this by taking

$$\frac{\partial u}{\partial z} = o \ , \quad w = o \ , \quad \frac{\partial T}{\partial z} = o \ , \quad \frac{\partial c}{\partial z} = o \ ;$$

(iv) On the round surface BDE ($o<x<\ell$, $z=h(1-x/\ell)$ and $\ell<x<L$, $z=o$) the no-slip condition

$$\underline{v} \wedge \underline{n} = o$$

is applied, where $\underline{n}$ is normal to the ground. It is also assumed that no pollution is lost to the ground

$$\underline{n} \cdot \nabla c = o$$

and ground cooling is modelled by specifying the ground temperature

$$T = T_g(x, t) \ ,$$

whose form is discussed later.

All the calculations were performed with $H=2$km, $L=8$km, $\ell=4$km, $\rho_o=1.2\text{g/m}^3$ and $T_o=300^oK$.

<u>Numerical Method</u> Equations (4) - (7) are replaced by finite-difference analogues on a uniform grid with a grid interval of 75 metres. The finite-difference scheme used is the Crowley (1968) second-order method. Early attempts to adapt a model (Smith et al, 1974) using centred time differencing and Arakawa conserving Jacobian operators led to irregularities in the temperature fields of the same type as those reported in the sea-breeze calculations of Fisher (1961) and the mountain wind computations of Orville (1964). Both of these authors found that the effect can be eliminated by upstream space differencing. Unfortunately, upstream space differencing introduces strong, "indefinite" diffusion into the numerical model, and Crowley's scheme was preferred.

A time step of 30 seconds is used in all calculations with $K=4\text{m}^2\text{sec}^{-1}$ and is found to be sufficiently small to avoid computational instability.

<u>Results and Discussion</u> The following commentary is based on four calculations, referred to as CTk, CTK, VTk, VTK; further calculations will be presented at the conference. In the CT experiments, the ground temperature is spatially constant with

$$T_g(t) = 300 - 5\sin(\pi t/18) \text{ degrees K} \ , \quad o<x<L \ ,$$

where t is measured in hours. In contrast, VT signifies that T_g varies along the hillslope, specifically,

$$T_g(x,t) = \begin{cases} 300 - (5-3x/\ell)\sin(\pi t/18) \text{ degrees K}, & o<x<\ell, \\ 300 - \ \ 2\sin(\pi t/18) \quad \text{"} & \ell<x<L, \end{cases}$$

Thus in CT experiments, the hillslope and ground cool simultaneously at the same rate whereas in VT experiments, the hillslope cools most rapidly near the summit, the rate of cooling diminishing towards the plain. There are many situations when the ground cooling rate is horizontally non-uniform, for example, due to changes in the covering vegetation, or the encroachment of urban areas, or simply because certain parts of the ground become shaded from the sun before others.

The third letter, k or K, refers to the value chosen for the diffusivity, either $4m^2/sec$ or $15m^2/sec$ respectively. We are well aware of the deficiencies of modelling turbulent transfer processes using constant diffusivities and even more so of the hazards involved in assigning a representative value appropriate to any particular atmospheric flow. The smaller value we choose is about the right order of magnitude and has been used in previous theoretical studies (see e.g. Gleeson, 1953, p.265). The larger value was chosen to permit comparisons to be made. In future studies we propose to include a more realistic formulation of frictional and heat diffusion processes but we believe the present calculations will serve as useful prototypes at this stage and we take the view that it is important to understand the behaviour of the constant K model first!

A selection of the computed velocity, temperature and concentration fields for experiments CTk and VTk are shown in Figs.2-7. Figures 2 and 3 show contours of equal horizontal and vertical velocity for experiment CTk after 9 hours when the minimum slope temperatures are attained. These sets of curves are typical in structure, but not in magnitude, of all the experiments performed. The principal features are a strong katabatic wind along the slope, whose depth of about 550m increases only slowly with downslope distance, together with a weaker and more extensive return circulation aloft. The calculated depth of the downslope wind is rather higher than is typically observed and we believe that this is due to the fact that our diffusivity is still rather high: we return to this point later. The katabatic wind suffers strong acceleration downslope, reaching a maximum velocity at about 800m upslope from the foot of the hill. Thereafter, this wind decelerates although it may be identified a kilometre or two along the plain and there is a small drift motion beyond this distance, extending through the lateral computational boundary.

The pattern of flow in experiment VTk is broadly similar, see Fig.4, but velocities are much higher even though the minimum ground temperature over most of the ground is significantly higher than in experiment CTk. Thus comparison of the two experiments shows clearly the supreme importance of horizontal temperature gradients, due to non-uniform ground cooling, in determining the strength of the katabatic wind.

The isotherms in experiment VTk are shown in Fig.5; these are markedly different from those in CTK (not shown) which are closely parallel to the ground. The nose-like features on the isotherms in Fig.5 demonstrate clearly the importance of horizontal advection in the heat equation when ground cooling is non-uniform. Nevertheless, horizontal advective accelerations are always important. Moreover, it appears that vertical accelerations must be taken into account in the heat equation. Thus comparison of the isotherms and contours of equal horizontal velocity between experiments CTk and CTK and between VTk and VTK suggest that although the reach of the inversion associated with the downslope wind increases with increasing diffusivity, the thickness of the downslope wind does not increase in proportion. This is presumably because the downward advection of heat near the hilltop and upward advection of heat deficit near the hillbottom, serve together to check the increase in horizontal pressure gradient with height which would otherwise be expected. This fact cautions us to explain the overestimate in calculating the depth of the downslope wind in such simplistic terms as "the thermal diffusivity is too high, therefore the depth of the induced horizontal pressure gradient is reduced, therefore the depth of the layer of induced downslope wind is lowered"! We must take into account vertical advection of heat as described above and it is likely that a further contributory factor to the large depth is our choice of constant diffusivity with a no-slip condition which overestimates surface frictional drag and consequently underestimates the strength of the katabatic wind. This would also underestimate vertical advection of heat and produce a too diffuse downslope wind.

To conclude this discussion we examine the effectiveness of the katabatic wind in dispersing a uniform layer of pollutant concentration ($c_o(x,z) \equiv 1$, $z<400$m; $c_o(x,z)=o$, $z>400$m). The results are displayed in Figs.6 and 7 which show a time sequence at three hourly intervals of the movement of the top of the layer (note that there is some diffusion present which serves to smooth out the sharp concentration interface but this is small compared with advection during the period of integration and the interface may be pictured as being sharp). In both experiments, pollutant is washed downslope and some is carried aloft. There is also a small drift out across the plain. Again, the patterns are broadly similar but the stronger circulation in experiment VTk is more effective in dispersing the pollutant.

We have yet to explore the effects of changes in aspect ratio H/L of the computational region but our experience suggests (Smith et al, 1974) that such changes will not unduly require us to change our conclusions.

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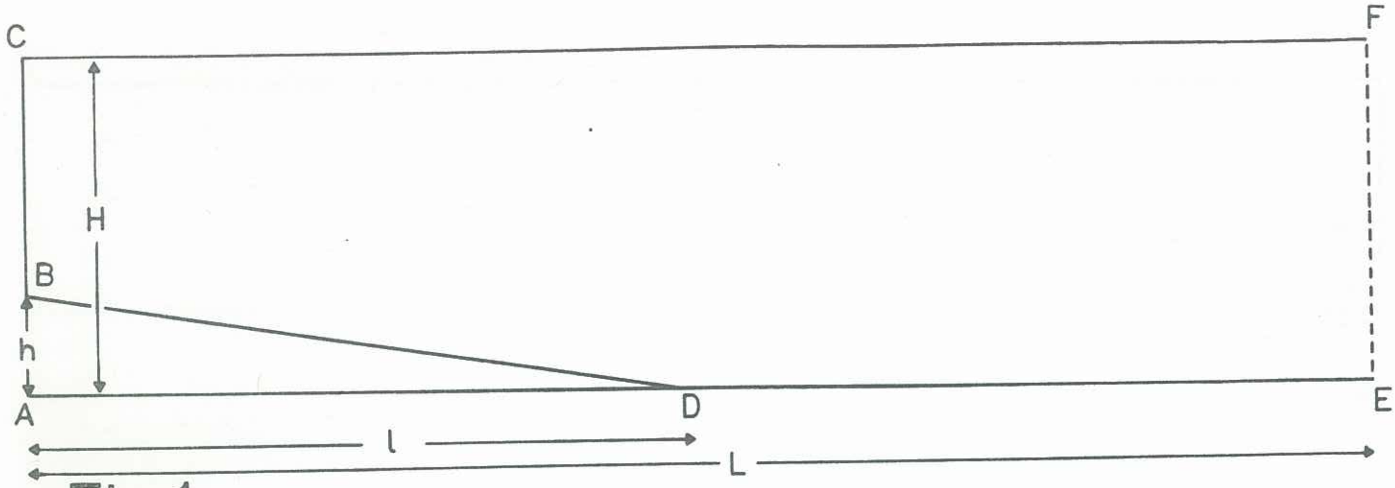

Fig.1

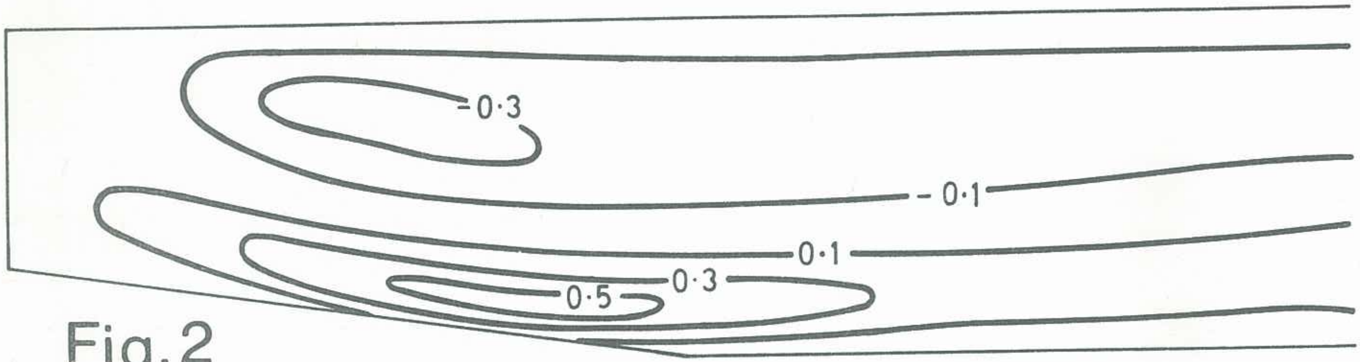

Fig.2

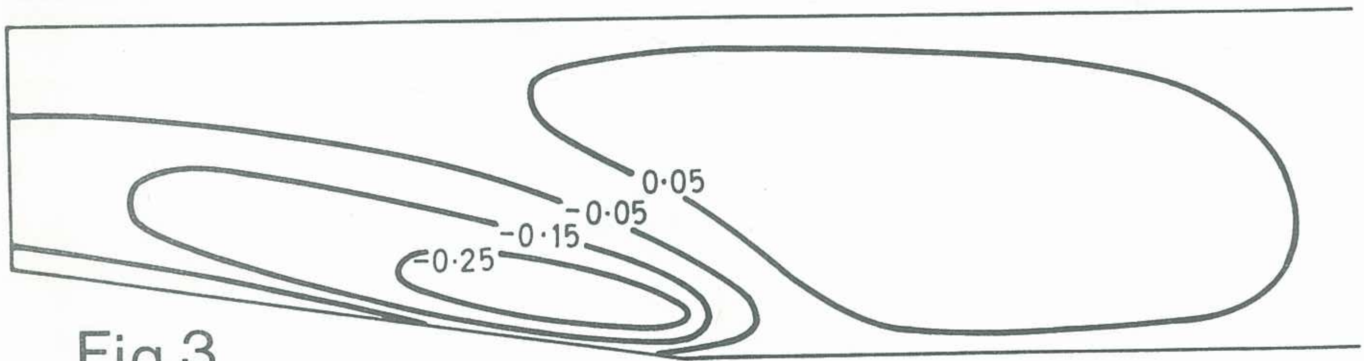

Fig.3

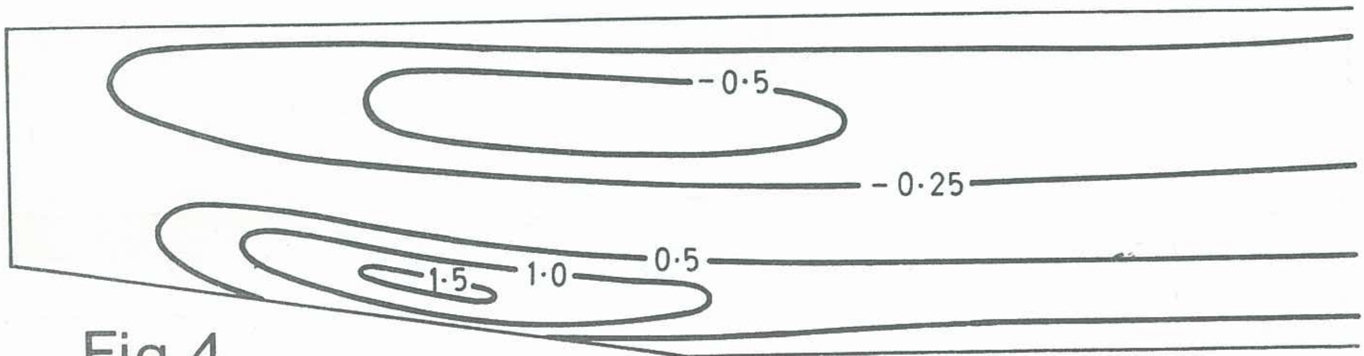

Fig.4

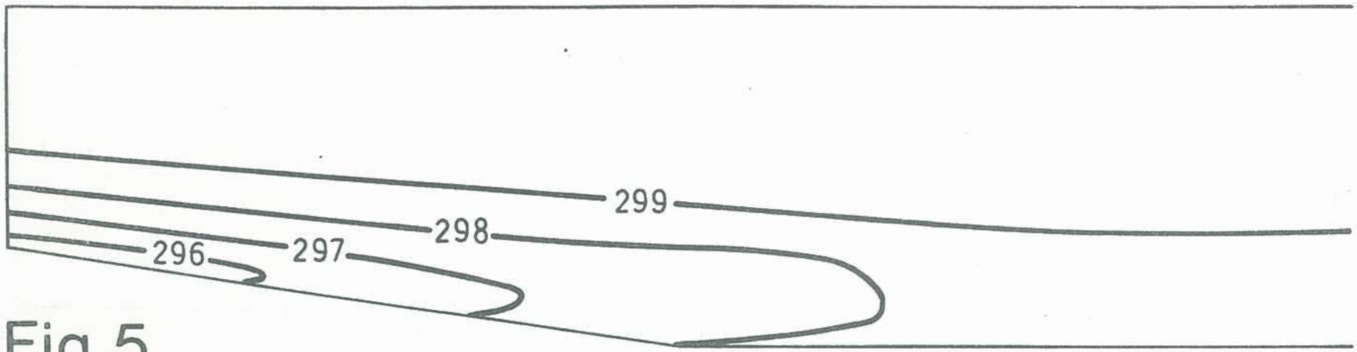

Fig.5

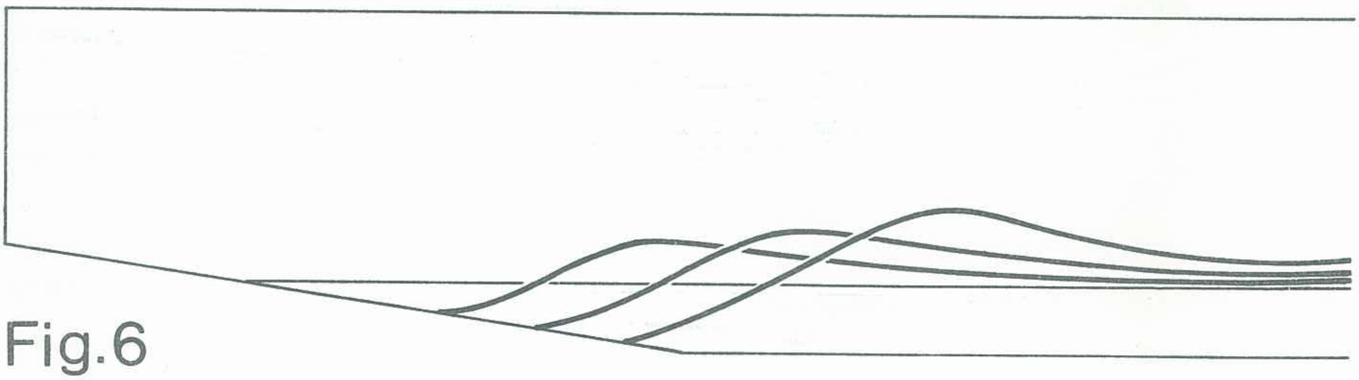

Fig.6

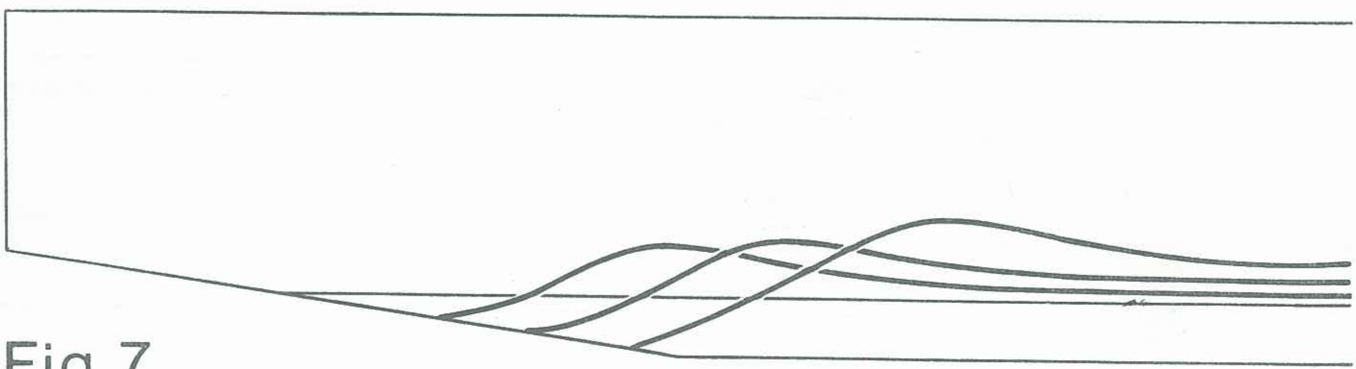

Fig.7