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A PERFORMANCE PREDICTION METHOD FOR HIGH SUBSONIC INLET MACH NUMBER CONICAL DIFFUSERS.

by

W. A. KAMAL　　A. O. ODUKWE　　J. L. LIVESEY

S U M M A R Y

A performance prediction method for high subsonic inlet Mach number conical diffusers, is developed which uses the entrainment principle in the calculation of the compressible turbulent boundary layer. A power law velocity profile is assumed together with Crocco's relation for the temperature distribution. Following Green, Morkovin's Hypothesis is invoked to extend to the compressible flow the existing relations for the entrainment function. Comparison with available experimental results shows good agreement.

W. A. Kamal — Research Student)
A. O. Odukwe — Lecturer) University of Salford,
J. L. Livesey — Professor of Fluid Mechanics) England

NOTATION.

x	axial distance
R	diffuser radius at station x
U, M_e	velocity and Mach number in the free stream
δ	Boundary layer thickness
H	boundary layer shape parameter $= \frac{\delta^*}{\theta}$
C_f	local skin friction coefficient
C_p	$= (\bar{p}_2 - \bar{p}_i)/(\bar{P}_i - p_i)$
C_L	$\bar{P}_2 - \bar{P}_i$/mean kinetic energy at diffuser inlet
y	distance normal to the wall
u, v	axial and radial velocity at a point
p, ρ, T, P	local pressure, density, temperature and total pressure
δ^*, θ	boundary layer displacement and momentum thickness
$H_1, \bar{H}, (H_1)_k, (H)_k$	shape parameters defined in text
u', v'	fluctuating velocity components
γ	ratio of specific heats

SUBSCRIPTS.

e	refers to the edge of the boundary layer
i,2	refer to diffuser inlet and outlet

a bar indicates mean values over diffuser cross-section
Other symbols are defined within the text

INTRODUCTION

A detailed survey of diffuser prediction methods revealed that only two procedures were available for compressible flow. These were: Method of Cocanower, Kline and Johnston (1) : The core flow was treated as compressible one-dimensional, but the boundary layer was considered as incompressible. Only the conditions of conservation of mass and momentum were satisfied; besides, the correlation for the shape parameter employed is known to be inadequate.

Method of Ashcroft (2): The great strength of Ashcroft's method was its fully ellyptic treatment of the compressible axisymmetric potential core flow. This was, however, combined by iteration with an inadequate compressible boundary layer calculation based on a primitive compressibility transformation.

In view of the shortcomings of these previous methods attempts were made to develop a prediction method based on the latest developments in compressible turbulent boundary layer prediction achieved over the last six years. After a detailed survey of such methods, it was decided that integral methods which are extensions to compressible flow of low speed techniques were most suitable for treatment of diffuser flows, and that at least initially one-dimensional core flows might be appropriate in view of the lengthy calculations involved in constructing performance maps. Finite difference methods were excluded on the following basis :-

(i) They are very temperamental and slight changes in geometry or flow conditions can sometimes result in the calculation becoming unstable.

(ii) None of them have been investigated under severe adverse pressure gradients, also their performance near separation has not be studied, while more than one integral method proved to be fairly accurate near the separation point.

(iii) McDonald (12) noticed no significant gain in accuracy over the less rigorous and much faster integral methods.

(iv) Their lengthy computer programming and execution time make their use uneconomical for the construction of performance maps.

Two methods were developed, the first was based on the entrainment principle while the second employed the kinetic energy deficit equation as an auxiliary boundary layer equation. The suitability of the entrainment method for solving the diffuser problem was disputed by Carmichael and Pustintsev (3), yet Nicoll and Ramparian (4) showed that good results can be obtained provided that realistic assumptions about the velocity profile were made and accurate correlations were used.

The entrainment method will be discussed in this paper. Comparison of the predictions with some of the available experimental results showed good agreement.

BASIC ASSUMPTIONS.

A. Core flow :- A steady, one-dimensional isentropic core is assumed to extend till diffuser exit(boundary layers do not merge.)

B. Boundary layers :-
1. Adiabatic, i.e. heat transfer to or from the diffuser is neglected.
2. Thin at inlet to diffuser.
3. Flow attached (or very slightly separated).

4. Boundary layers are growing under ordinary rates of strain (no shock waves or expansion fans).
5. Density fluctuations are neglected.
6. Diffuser walls are smooth and impermeable (no mass transfer).
7. Boundary layer is anisoenergetic, i.e., stagnation temperature is allowed to change within the boundary layer.
8. Axisymmetric

C. Flow generally :- 1. Fluid is a perfect gas and the recovery factor (r) is constant.
2. Static pressure is a function of axial distance only.

FORMULATION OF THE DIFFUSER PROBLEM.

Boundary Layer Momentum Integral Equation.

The basic equation is the axially symmetric momentum integral equation which is compressible flow, takes the form :

$$\frac{d\theta}{dx} + \theta\left[(2 + H - M_e^2)\frac{d(\ln U)}{dx} + \frac{d(\ln R)}{dx}\right] = \frac{C_f}{2} + \frac{1}{\rho_e U^2}\frac{d}{dx}\int_o^\delta (\rho\overline{v'^2} - \rho\overline{u'^2})\,dy \qquad (1)$$

The turbulence anisotropy term in equation (1), which accounts for the Reynolds normal stress is usually neglected in boundary layer calculations. Because of the large fluctuations usually associated with separated flow, it is very likely that this term should be included. However, in the present analysis, considering flows in unstalled diffusers, or diffusers with limited amount of stall, this term will be neglected.

For an external flow ($\frac{dU}{dx}$ known), equation (1) contains 3 unknowns, θ, H and C_f. To solve for the boundary layer an auxiliary relation between H and θ and an expression for C_f in terms of H and θ are needed. The entrainment equation is the required first relation, while for C_f a modified version of the Ludwig and Tillman correlation was employed. This takes the form

$$C_f = 0.246\,\frac{Te}{Tm}\,e^{-1.561\bar{H}}\,\bar{R}_\theta^{-0.268}$$

where $\bar{R}_\theta = \frac{\mu e}{\mu m} R_\theta$ $\qquad \mu m$ = viscosity at temperature Tm $\qquad (2)$

$$\frac{Tm}{Te} = 1 + 1.44\, r\, M_e^2 \quad \text{and} \quad \bar{H} = \frac{\int_o^\delta \frac{\rho}{\rho e}\left(1 - \frac{u}{U}\right)}{\theta}$$

Diffuser Overall Continuity Equation

In internal flows, the streamwise pressure distribution and consequently $\frac{dU}{dx}$ are determined not only by the frictionless flow external to the boundary layer but also by the development of the boundary layer itself. An additional relation should be provided for $\frac{dU}{dx}$ in terms of the boundary layer integral parameters, which in this case, considering the one dimensional core in the conical diffuser, becomes

$$\rho_e\, U\,(R - \delta^*)^2 = \text{Constant}$$

or in differential form

$$\frac{R - H\theta}{2}(1 - M_e^2)\frac{d(\ln U)}{dx} + \frac{dR}{dx} - H\frac{d\theta}{dx} - \theta\frac{dH}{dx} = 0 \qquad (3)$$

Entrainment equation

Head's (13) basic argument was that the distribution of mean velocity in the boundary layer and in particular, the velocity defect in its outer part (measured roughly by shape parameters such as H and H_1) should control the entrainment process and he further suggested that the density variation in the outer part of a compressible boundary layer should have little effect on the entrainment mechanism. Green (5) took up this suggestion by assuming that the entrainment F

$$F = \frac{1}{\rho_e U}\frac{d}{dx}\int_o^\delta \rho u\,dy \qquad (4)$$

and the kinematic shape parameter $(H_1)_k$

$$(H_1)_k = \int_o^\delta \frac{u}{U} dy \Big/ \int_o^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy \qquad (5)$$

satisfy the empirical relation obtained by Head for incompressible flow which could be fitted with the equation

$$F = 0.0306 \left[(H_1)_k - 3.0\right]^{-0.653} \qquad (6)$$

For an axisymmetric flow with a thin boundary layer, equation (4) becomes

$$F = \frac{1}{\rho_e UR} \frac{d}{dx} \left\{ R \int_o^\delta \rho u dy \right\}$$

Now using the definition of the mass flow thickness $\Delta = \int_o^\delta \frac{\rho u}{\rho_e U} dy = \delta - \delta^*$ (4')

equation (4') may be written as

$$\frac{d}{dx}(\rho_e U \Delta R) = \rho_e UFR$$

which on expansion and substitution of $\frac{d(\ln\rho e)}{dx} = - M_e^2 \frac{d(\ln U)}{dx}$ and $\Delta = H_1 \theta$ becomes

$$H_1 \frac{d\theta}{dx} + \theta \frac{dH_1}{dx} = F - H_1\theta \left[\frac{d(\ln R)}{dx} + (1 - M_e^2) \frac{d(\ln U)}{dx}\right] \qquad (7)$$

Now, with the empirical relation for the entrainment (equation 6) provided, a relation between the shape parameters H_1 and H is needed to achieve closure.
In the early approach of Green (5) complementary shape parameter relations were obtained in an indirect and fairly lengthy way through relations between $\bar{H}$ and H_1 and their kinematic equivalents $(H)_k$ and $(H_1)_k$. These relations were as follows :-

$$\bar{H} = \frac{H + 1}{1 + \frac{\gamma - 1}{2} r M_e^2} - 1 \qquad (8)$$

$$H_1 = 3.3 + \frac{0.9}{(\bar{H} - 1)^{4/3}} \qquad (9)$$

$$(H)_k = \bar{H} \left[1 + \frac{\frac{rM_e^2}{5} \frac{(\bar{H} + 1)\ (\bar{H} - 1)^2}{\bar{H}(3\bar{H} - 1)\ (2\bar{H} - 1)}}{1 + \frac{rM_e^2}{5}\left\{1 - \frac{\bar{H}(\bar{H} + 1)}{(3\bar{H} - 1)(2\bar{H} - 1)}\right\}} \right] \qquad (10)$$

$$(H_1)_k = 3.4 + 1.87 / ((H)_k - 0.5)^{3.8} \qquad (11)$$

In the present analysis the solution is to be advanced for the unknowns H and θ hence, the term $\frac{dH_1}{dx}$ in equation (7) has to be related to $\frac{dH}{dx}$. This relation was obtained as

$$\frac{dH_1}{dx} = Z_1 \frac{dH}{dx} + Z_2 \frac{d(\ln U)}{dx} \qquad \text{where}$$

$$Z_1 = \frac{-1.2 (\bar{H} - 1)^{-7/3}}{1 + 0.2 r M_e^2} \qquad Z_2 = -0.4 r Z_1 M_e^2 (1 + 0.2 M_e^2)(\bar{H} + 1) \qquad (12)$$

COMPUTATIONAL EQUATIONS AND SOLUTION PROCEDURE.

Rewriting equations (1), (3) and (7) in a non-dimensional form using the definitions $\tilde{\theta} = \frac{\theta}{R_o}$, $X = \frac{x}{R_o}$, $\tilde{R} = \frac{R}{R_o}$, $\theta' = \frac{d\tilde{\theta}}{dX}$, $H' = \frac{dH}{dX}$, $R' = \frac{d\tilde{R}}{dX}$ and $U' = \frac{d(\ln U)}{dX}$

then eliminating U' from equations (3) and (7) using equation (1) the system reduces to 2 simultaneous differential equations in the unknowns H and θ

$$a H' + b \theta' = c \qquad (13)$$

$$e H' + f \theta' = g \qquad (14)$$

where

$$a = \tilde{\Theta} Z_1 \qquad b = H_1 - A \qquad c = F - A\frac{C_f}{2} - (H_1 - A)\,\tilde{\Theta}\frac{R'}{\tilde{R}}$$

$$e = \tilde{\Theta} \qquad f = H + B \qquad g = R'\left(1 - B\frac{\tilde{\Theta}}{\tilde{R}}\right) + B\,\frac{C_f}{2}$$

$$A = \frac{Z_2 + H_1 (1 - M_e^2)}{2 + H - M_e^2} \qquad B = \frac{(\tilde{R} - H\tilde{\Theta})(1 - M_e^2)}{2\tilde{\Theta}\,(2 + H - M_e^2)}$$

The solution is obtained by simultaneous step-by-step integration of equations (13) and (14) starting from prescribed inlet conditions using a fourth order Runge-Kutta scheme. At the end of each step, new values of H and Θ are obtained. To proceed with the integration the local value of the Mach No. has to be calculated in order to evaluate local properties of the free stream and the various empirical functions. This is obtained by using the overall continuity equation in its integral form, which can be reduced to

$$(R - H\theta)^2 \cdot \frac{M_e}{(1 + \frac{\gamma - 1}{2} M_e^2)^3} = \text{Const.}$$

The value of the constant is evaluated at the diffuser inlet, then at the end of each step, values of H and θ are calculated which upon substitution in the above equation yield a value for the function

$$(M_e) = \frac{M_e}{(1 + 0.2\,M_e^2)^3}$$

This equation was solved numerically using the Bisection method.

Remarks on Computer Programme.

For the predictions to be accurate, the calculations must be terminated before the class A restrictions are violated, i.e., before the boundary layers merge and before the flow separates.
or

A - Boundary Layers Merging :

The programme was made to stop when $\varepsilon = o$ where

$$\varepsilon = R - \delta = R - (\delta^* + \Delta) = R - \theta\,(H + H_1)$$

B - Flow Separation :

More than one flow detachment criteria were employed to predict the point of first appreciable stall, e.g.,

$H = 3.1$ — Suggested by Moses and Chappel [6]

$\frac{d\theta}{dx} = 0.012$ — Suggested by Reneau and Johnston [7]

$\beta^* = 0.048$ — Suggested by Cocanower et al [1]

$$\text{where } \beta^* = \frac{X\,(1 - H_i\tilde{\theta}_i)^2\,(2\,H_i\tilde{\theta}_i)^{0.536}}{2\,H\tilde{\Theta}\tilde{R}^3}$$

However, the calculations were continued beyond the separation point to assess the capability of the analytical method to predict slightly separated diffuser.

COMPARISON WITH EXPERIMENTAL RESULTS.

Though a large amount of experimental data is available in the literature for the case of incompressible diffuser flow, only nine reports with compressible conical diffuser flow data are known to the authors. The diffusers have cone angles varying from 4^o to 31^o with inlet Mach numbers from 0.2 up to choking, the inlet Reynolds number varying from 2×10^5 to 7.5×10^6.

In some cases the authors did not report the position of the inlet measuring station. Consequently, in order for the inlet losses to be accounted to the diffuser, the inlet station is taken to be one diameter upstream of the sharp transition between the entry pipe and diffuser cone.

In the following, predictions of diffuser performance are compared with some of the available experimental data.

In Fig. (1), comparison between prediction of the growth of boundary layer parameters (θ and H) and experimental results of Little and Wilbur (8) are shown. As was observed with incompressible diffuser flow prediction methods, e.g. Carmichael and Pastinsev's (3) methods, the theory tends to underestimate the momentum thickness growth while the shape factor is overestimated, the deviations increasing for the thicker inlet boundary layer and/or the larger diffuser angle.

From the comparison between predictions of the present method and those of Cocanower et al with experimental data of Johnston (9) (in table 1) and Copp (10) (in Fig. (2)) it is obvious that the technique employed in this work leads to improved predictions, compared with these previous methods. It can be also concluded that, though predictions of the growth of boundary layer parameters may not be very accurate, yet predictions of overall quantities (e.g., pressure recovery) are sufficiently accurate, which justifies using this method for predicting diffuser performance characteristics.

In Fig. (3) theoretical predictions are compared with data of Scherrer and Anderson (11) for conical diffusers whose geometry was very slightly shaped to suit near sonic inlet velocities. From Fig. (2) and (3) it is noticed that though the predictions are accurate up to yet the theory does not predict the experimentally observed rapid deterioration of performance as choking is approached. This is attributed to the one dimensional treatment of the potential core flow which is a limitation especially in the inlet zone, where strong streamline curvature exists. Work is continuing to overcome this limitation.

No experimental data is available on the separation point in high subsonic Mach number diffusers and consequently incompressible flow data were employed to assess the accuracy of the theoretical predictions of the point of boundary layer separation. Cocanower's correlation seems to be the best detachment criterion available, though the theory always predicts a premature (early) separation.

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TABLE 1. COMPARISON OF PREDICTED PRESSURE RECOVERY VALUES WITH JOHNSTON'S EXPERIMENTAL DATA.

$\frac{N}{R_i}$	AR	B_i	R_e	M_{e_i}	C_p		
					Measured	Cocanower's	Present Method
15.25	4	0.0075	3.17×10^5	0.3	0.874	0.840	0.8599
15.25	4	0.0075	6.34×10^5	0.6	0.854	0.845	0.852
15.25	4	0.0075	8.97×10^5	0.85	0.840	0.838	0.838

N - Diffuser axial length B_i - Diffuser inlet blockage

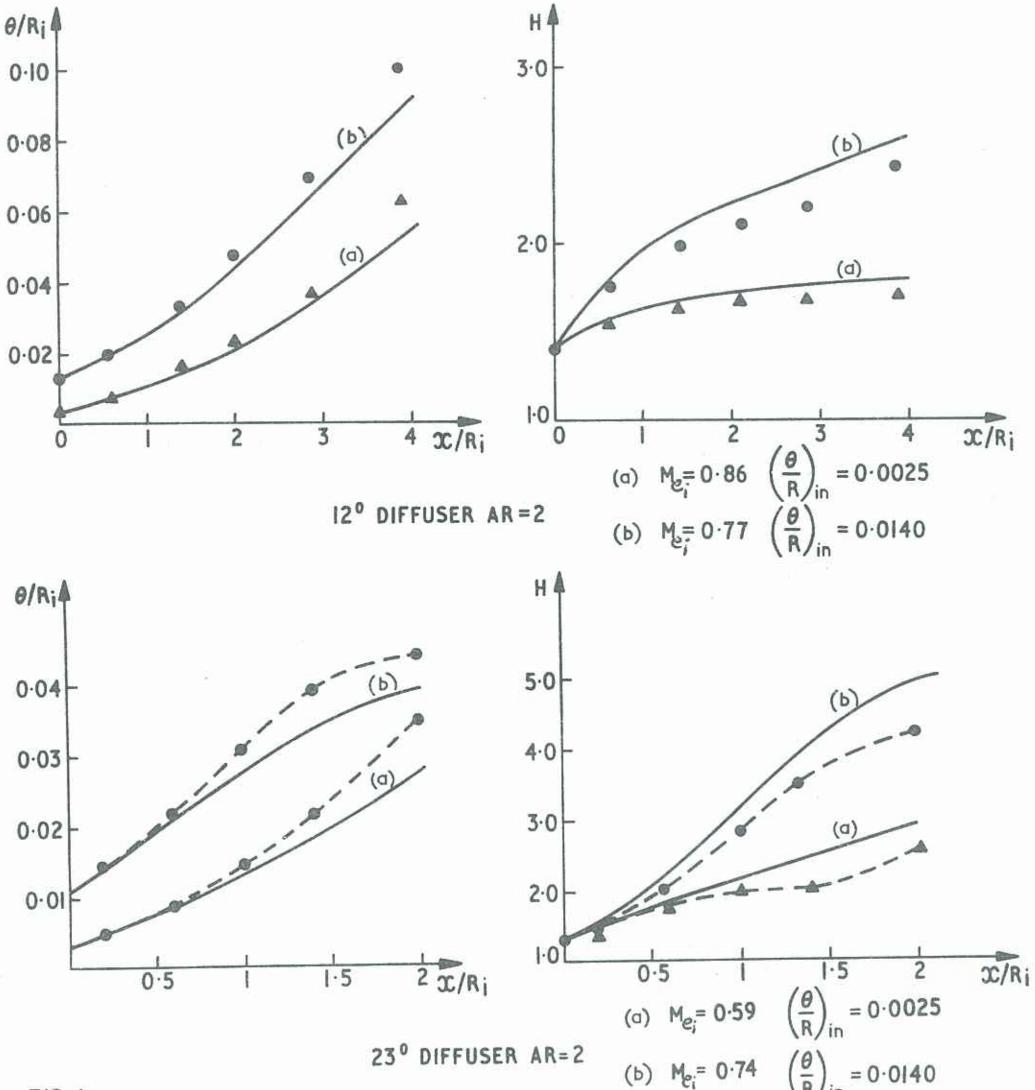

FIG. 1
GROWTH OF BOUNDARY - LAYER PARAMETERS IN 12° AND 23° DIFFUSERS. COMPARISON WITH EXPERIMENTAL RESULTS OF LITTLE AND WILBUR.

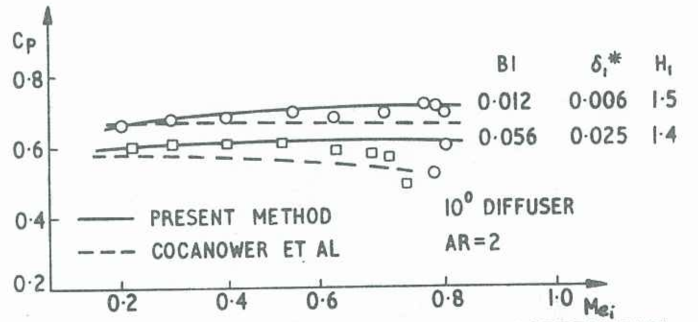

FIG. 2 DIFFUSER PRESSURE RECOVERY COEFFICIENT vs. INLET MACH No. COMPARISON WITH DATA OF COPP.

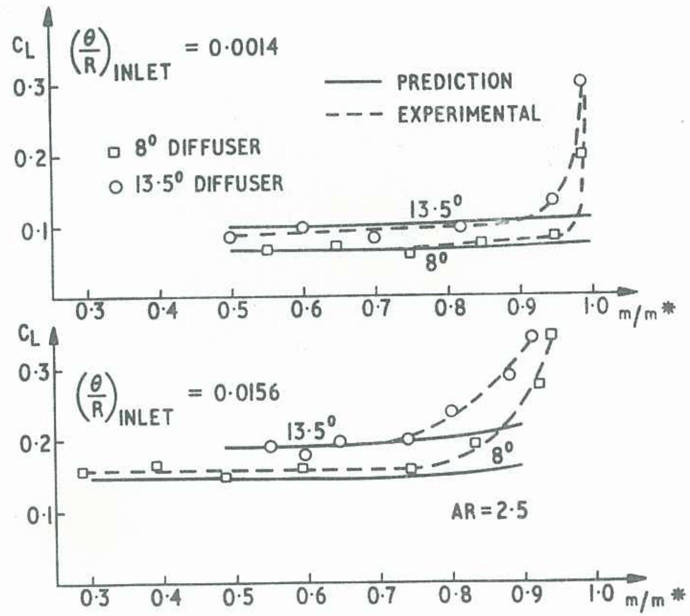

FIG. 3 VARIATION OF PRESSURE LOSS COEFFICIENT WITH MASS FLOW RATIO (MEASURE OF MACH No.) DATA OF SCHERRER AND ANDERSON.