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A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF FLOW THROUGH SHORT AXISYMMETRIC CONTRACTIONS

by

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SUMMARY

Investigations have been made into the effect of curtailing the length of contracting ducts upon their performance. These ducts had wall velocity distributions approximating to a step change in velocity/pressure. Each duct had an overall contraction ratio of approximately 4:1, with outlet diameters of about 103 mm. Generally good agreement was obtained between the theoretical and experimental results. The presence of the boundary layer, however, reduces the effective area ratio of the contraction. It was inferred that a contraction boundary with a monotonic wall velocity distribution can be terminated at inlet and outlet wall slopes of 2^{o} and 3.5^{o} respectively without impairing its performance. Also, adverse pressure gradients up to $\frac{2}{\rho q_2^2}\frac{dp}{dx} = -0.38$ can be permitted on the final portion of the contraction boundary without risking flow separation.

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GLOSSARY

a, β	-	Constants
q	-	Resultant velocity
x	-	Distance in the axial direction*
R	-	Distance in the radial direction*
V_x	-	Axial velocity component
V_R	-	Radial velocity component
C_p	-	Pressure coefficient $= (P-P_2)/(\frac{1}{2}\rho q_2^2)$
Ψ_s	-	Stokes stream function
χ	-	Flow direction
$\frac{2}{\rho q_2^2}\frac{dp}{dx}$	-	Non-dimensional pressure gradient
R_e	-	Reynolds Number
D	-	Diameter
$f_m(x)$	-	Functions of x
A, A_1, C and B	-	Contractions tested
Subscripts :-		1 - refers to inlet 2 - refers to outlet

INTRODUCTION

Normal practice in designing contractions is to obtain a boundary shape which is expected to give a uniform velocity distribution at the outlet and minimum flow disturbance. Theoretically designed boundary shapes have all been designed, to date, on the basis of potential flow theory. In general, the ducts so designed are of infinite length and are usually arbitrarily terminated at some point to produce a finite length contraction.

In this paper, both theoretical and experimental investigations have been carried out to examine the effect of curtailing the contraction length on the performance of the contraction. The effect of real flows on the potential theory design has also been assessed.

THE THEORETICAL DESIGN METHOD

The design approach used is to obtain a duct for a specified wall velocity distribution for two-dimensional flow using Lighthill's theory (1). The two-dimensional wall profile obtained is then rotated about the x axis to form the axisymmetric duct. The inviscid, incompressible flow through this axisymmetric duct is then analysed numerically with the aid of a computer program written in Algol. The program which was developed by Ashcroft (2) in Atlas Autocode is based on a series solution to the stream function Ψ_s, as given by the Stokes-Beltrami equation:-

$$\frac{\partial^2 \Psi_s}{\partial x^2} + \frac{\partial^2 \Psi_s}{\partial R^2} - \frac{1}{R}\frac{\partial \Psi_s}{\partial R} = 0 \qquad \ldots. (1)$$

the series solution being of the form:-

$$\Psi_s = \sum_{m=1}^{\infty} f_m(x)\, R^{2m} \qquad \ldots. (2)$$

and the functions $f_m(x)$ are evaluated numerically. The method of solution used by Tsien (3), Szczeniowski (4), and Cohen and Ritchie (5) was to adopt a specific axial velocity distribution which gives values to the functions $f_m(x)$. Hence equation (2) can be solved for R at given values of Ψ_s . The method of solution used in this paper differs by evaluating the functions $f_m(x)$ numerically subject to the boundary conditions that $\Psi_s = 0, 1$ on the axis of the duct and on the wall respectively. In addition, the flow is taken to be uniform and parallel to the x axis at both entry and exit. This, of course, leads to an over-specification of the boundary conditions, because at both entry and exit planes and for any chosen value of R the flow has to satisfy two boundary conditions that V_x is equal to the chosen uniform velocity, and V_R is zero. However, when both entry and exit portions of the duct have only gradual changes in wall slope, V_R will be nearly zero, and hence the theoretical results will be in good agreement with experiment.

*Both x and R are non-dimensional

The computer program enables velocity q (the resultant of V_x and V_R), pressure coefficient Cp, and pressure gradient $\frac{2}{\rho q^2}\frac{dp}{dx}$ to be obtained on the boundary streamline, ie. on the wall. In addition, once the functions $f_m(x)$ have been evaluated it is possible to obtain the co-ordinates of any given streamline within the duct, and hence obtain q, Cp, and $\frac{2}{\rho q^2}\frac{dp}{dx}$ on this streamline, by using equation (2) to solve for R. This latter extension to Ashcroft's program is given in reference (6).

In all cases, it was found that only four or five terms were needed in the series solution to produce sufficient accuracy in the results, as the series converges very rapidly with increasing number of terms. The only limit upon the number of terms in the series solution and number of duct intervals chosen, is the storage facility of the KDF9 computer.

The basic requirement of the contractions to be tested experimentally was that the duct boundary should have a step wall velocity distribution, or a velocity distribution which was a good approximation to it*. Also the length of the duct was to be as short as possible. Figs 1 to 4 show the theoretical designs for the contractions used in the experimental programme. Contraction A (see Fig 1) has been designed on the basis of a finite length generating function suggested by Lighthill (1). For this function, the two-dimesional wall slope is given by:-

$$\begin{aligned} \chi &= 0 \text{ for } 0 < \theta < \beta \text{ and } (\pi-\beta) < \theta < \pi \\ &= a\left\{1 - \cos(\theta-\beta)\right\} \text{ for } \beta < \theta < (\pi-\beta) \qquad \ldots. \ (4) \end{aligned}$$

where β , a are constants and β is small.

Contraction A is seen to have adverse wall pressure gradients in the vicinity of the region near to $\frac{x}{D_2} = 0.5$. The inlet and outlet wall slopes for this contraction are both 0.1^o. Contraction A_1 was obtained by curtailing the inlet of contraction A at a wall slope of 3.5^o, as shown in Fig 1. Contraction C was obtained by a curtailing of the outlet of contraction A_1 at a wall slope of 2^o.

Fig 4 shows the theoretical design for contraction B. The two-dimensional generating function for this contraction is:-

$$\begin{aligned} \chi &= a(3\sin\theta - \sin 3\theta) \text{ for } 0 < \theta < 155^o \\ &= 0 \text{ for } 155^o < \theta < \pi \qquad \ldots. \ (5) \end{aligned}$$

Using this generating function the wall shape was obtained by the method of Watkins and King (7). The inlet and outlet wall slopes for contraction B were 0.5^o and 0.1^o respectively. On comparing Figures 2, 3 and 4 it can be seen that the adverse pressure gradients present in contraction B are greater than those in contractions A, A_1 and C.

THE EXPERIMENTAL INVESTIGATION

The object of testing the ducts was two-fold:-

(i) To verify the performance of the ducts predicted by the potential theory.

(ii) To study the effect of adverse pressure gradients occurring on the boundary on the performance of the ducts.

*The ducts were primarily designed for use as diffusers (see reference (5)).

Table 1 gives the basic dimensions of the diffusers tested, the overall contraction ratios being approxmately 4:1. Before testing began velocity profiles were measured at a point 127 mm upstream of the contraction inlet over the range of Reynolds numbers $Re_2 = 1.5 \times 10^5$ to 5.6×10^5. The profiles were found to be fairly uniform (see Table 2) with only slight flow assymmetry, the ratio of the inlet axisymmetric boundary layer displacement thickness to the inlet diameter being about 0.015 for the range of Reynolds numbers tested.

Fig 2 compares the theoretical and experimental static pressure distribution obtained for ducts A and A_1 for Reynolds numbers of 1.5×10^5, 4.4×10^5 and 5.6×10^5. The agreement between the theoretical and experimental values of Cp obtained on the duct axis was also found to be good although the measured decrease in pressure is less than the theoretical decrease prediction. In addition, the actual Cp does not become zero at the duct outlet, as was implied in the assumed boundary conditions for solution of equation (1). However the measurements show that the static pressure can be expected to become uniform across the cross-section in a short distance, say at $\frac{x}{D_2} \approx -0.5$.

Fig 3 shows the corresponding results for duct C. General agreement is similar to that obtained with duct A. Since a high adverse pressure gradient is implied at the outlet end of duct C, which strongly affects the growth of the boundary layer, the dependence of Cp on the Reynolds number, Re_2, is seen to be greater than for duct A. The agreement between the experimental and theoretical values of Cp on the axis at the outlet end of duct C is better than that obtained for duct A. With duct C the static pressure distribution, and hence the velocity profile, was still not uniform at $\frac{x}{D_2} = -0.5$.

Again, general agreement for the results of duct B (see Fig 4) is seen to be similar to that obtained for ducts A and C. Comparing Figs 2, 3 and 4 the dependence of pressure decrease on Reynolds number for duct B is seen to be greater than that for duct A, but less than that for C. For duct B the difference in measured Cp between wall and axis was 0.012 at $x/D_2 = -0.5$ and the total pressure was found to be uniform at $x/D_2 = -0.28$. Thus the velocity profiles were non-uniform up to $x/D_2 = -0.5$, which disagrees with the potential theory assumption of uniform velocity at the duct outlet. Another important observation was that despite the large adverse wall pressure gradient at the outlet end of duct B, $\frac{2}{\rho q_2^2}\frac{dp}{dx}$ being approximately -0.38, no flow separation was discernible.

DISCUSSION OF THE RESULTS

The differences obtained in the measured and theoretically predicted pressure decrease through the contractions, due to boundary layer effects, imply that the effective area ratio of the ducts is less than the geometrical area ratio. The decrease in area ratio appears to depend on the specification of the wall velocity distribution, the inlet and outlet wall slopes, and the inlet flow conditions. Table 3 shows that in the ducts tested the effective area ratio achieved was 16.2% to 23.3% less than the geometrical area ratio. Thus to obtain an effective area ratio equal to the intended area ratio (eg. 3.99 for duct A) the design area ratio should be increased by an amount varying from 19.3% to 30.3%. In practice, wind tunnel contractions are more usually designed with a monotonically increasing velocity along the wall and then the area reduction effect is likely to be even more significant. Thus, if a contraction is designed using a potential theory approach it seems reasonable to make the design duct area ratio 20% to 25% greater than the desired contraction ratio.

The velocity profile at the outlet of the contractions tested was usually non-uniform. The length of parallel pipe required after the outlet to produce a uniform velocity profile appears to depend on two factors, (i) the outlet wall slope, and (ii) the adverse pressure gradients present on the final portion of the boundary. The experimental results given in this paper show that the length requirement increases with increase in both wall slope and magnitude of the adverse pressure gradient, but the latter effect appears to be dominant. In the absence of adverse pressure gradients on the boundary, the results indicate that by terminating the duct at a high outlet wall slope, of the order of 1^o to 2^o, the length requirement is not significantly changed. On the contrary, Table 2 shows that the flow uniformity expected after a given parallel length would be higher with contraction C than with contraction A. In actual practice, if the final portion of the duct boundary has a nearly constant pressure, the boundary layer on it may thicken making the outlet velocity profile more non-uniform. This suggests that a contraction having a monotonically increasing velocity distribution on its boundary should be terminated before the favourable pressure gradients become so weak that the boundary layer starts thickening. Further, the results of ducts A and A_1 show that if the initial portion of the contraction boundary has a nearly constant pressure then the inlet wall slope can be made as high as 3.5^o without affecting the performance of the contraction.

CONCLUSIONS

The following conclusions are drawn from the results:-

(i) There was generally good agreement between measured and theoretical predictions of the pressure/velocity distributions on the wall and axis. But for exact agreement it would have been necessary to include boundary layer effects. An increase of 20% to 25% in the design area ratio would appear to overcome the decrease in effective area ratio produced by the presence of boundary layers.

(ii) A contraction having a monotonic wall velocity distribution and nearly constant pressure on the initial and final portions of the boundary can be terminated at high wall slopes at inlet and outlet, say 2^o and 3.5^o respectively, without inpairing its performance.

(iii) The length of parallel pipe required after the exit of a contraction for the velocity to become uniform depends on the exit wall slope and the amount of adverse pressure gradients in the final portion of the duct, the latter parameter being the more important.

(iv) Adverse pressure gradients up to $\frac{2}{\rho q_2^2}\frac{dp}{dx} = -0.38$ can be permitted on the final portion of the contraction boundary without risking flow separation.

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TABLE 1 - BASIC DIMENSIONS OF CONTRACTIONS TESTED

Contraction	Overall Length (mm)	Inlet Diameter (mm)	Outlet Diameter (mm)	Inlet Wall Slope	Outlet Wall Slope
A	289	103	206	0.11^{o}	0.11^{o}
A_1	193	103	202	0.11^{o}	3.5^{o}
C	150	103	200	2^{o}	3.5^{o}
B	171	103	206	0.1^{o}	0.5^{o}

TABLE 2 - DEGREE OF UNIFORMITY OF VELOCITY PROFILES AT INLET AND OUTLET

Re_2	V_{max}/V_{mean} at 1.25 D_2 Upstream of Inlet	V_{max}/V_{mean} at 0.28 D_2 Downstream of Outlet			
		A	A_1	C	B
1.5×10^5	1.026	1.015	1.014	1.009	1.014
4.4×10^5	1.068	1.013	1.013	1.009	1.012
5.6×10^5	1.061	1.013	1.014	1.007	1.008

TABLE 3 - EFFECTIVE AREA RATIOS ACHIEVED WITH TEST CONTRACTIONS

Contraction	Geometrical Area Ratio	Effective Flow Area Ratio	Percentage Decrease in Design Area Ratio	Suggested Increase in Design Area Ratio
A	3.99	3.34	16.3	19.5
A_1	3.86	3.125	19.0	23.5
C	3.78	2.90	23.3	30.3
B	3.99	3.345	16.2	19.3

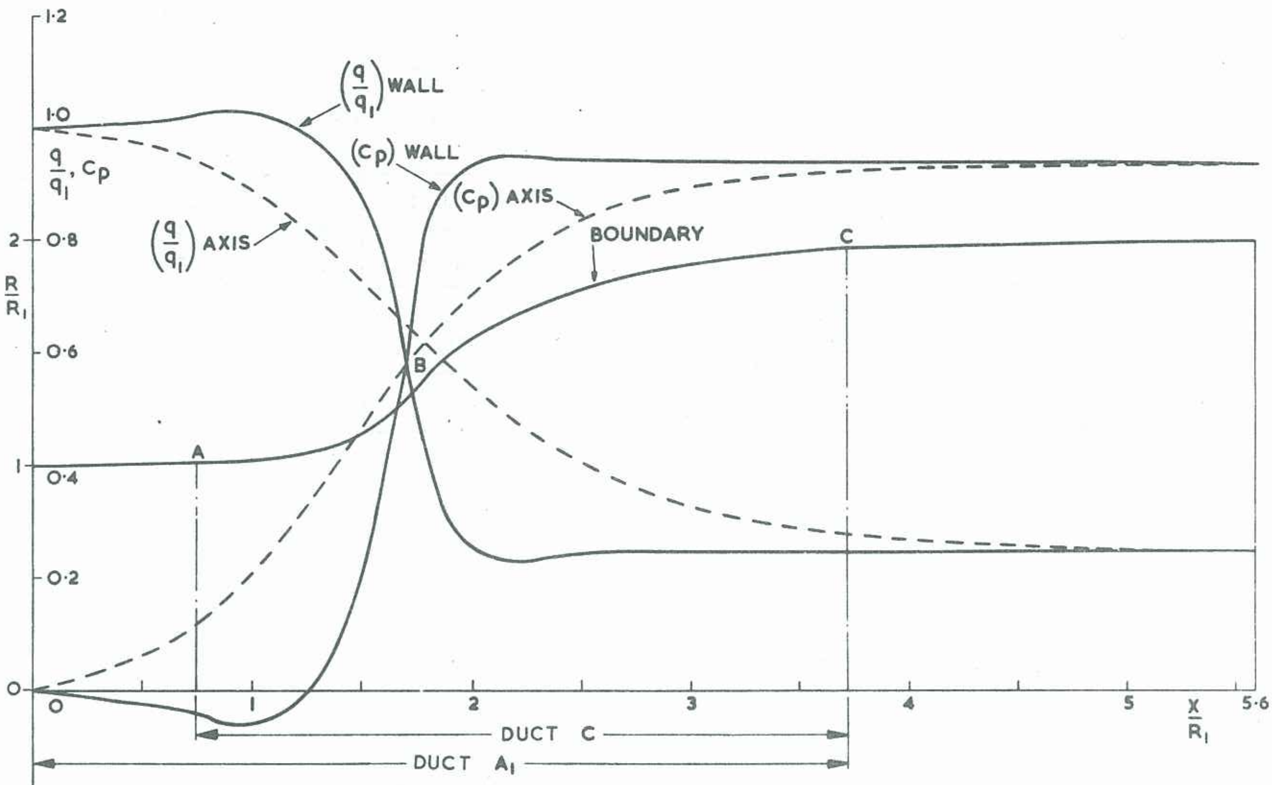

FIG. 1 DESIGN DETAILS OF TEST DUCTS A, A1 AND C.

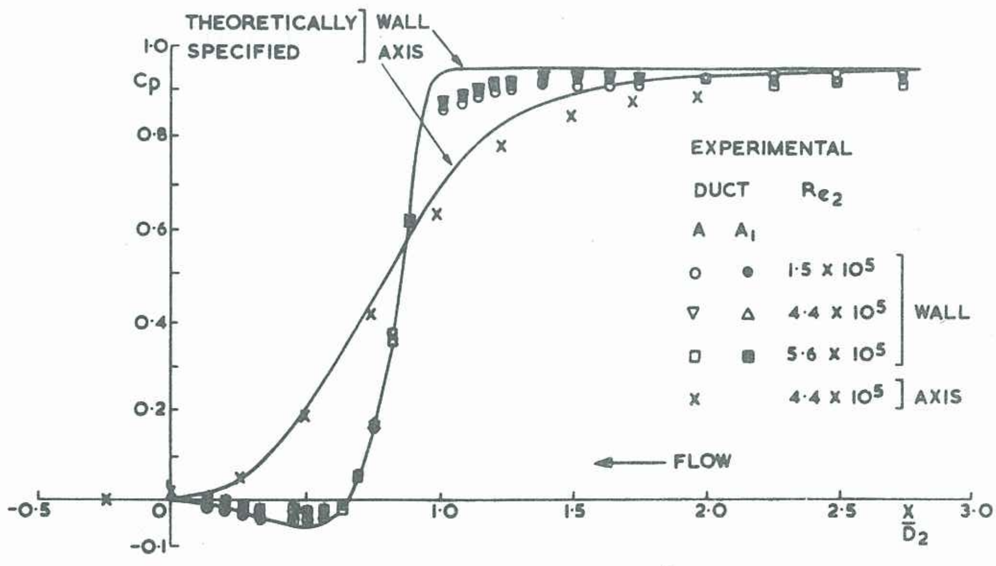

FIG. 2 TEST DUCTS A, A1 - STATIC PRESSURE DISTRIBUTION ALONG THE WALL AND AXIS.

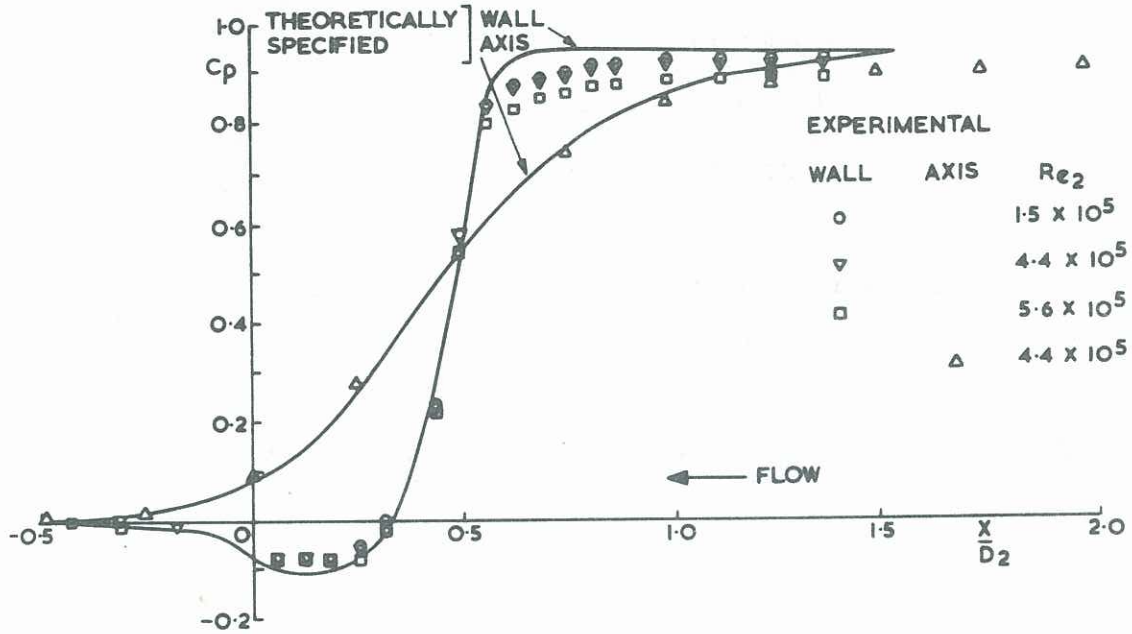

FIG 3 TEST DUCT C - STATIC PRESSURE DISTRIBUTION ALONG THE WALL AND AXIS.

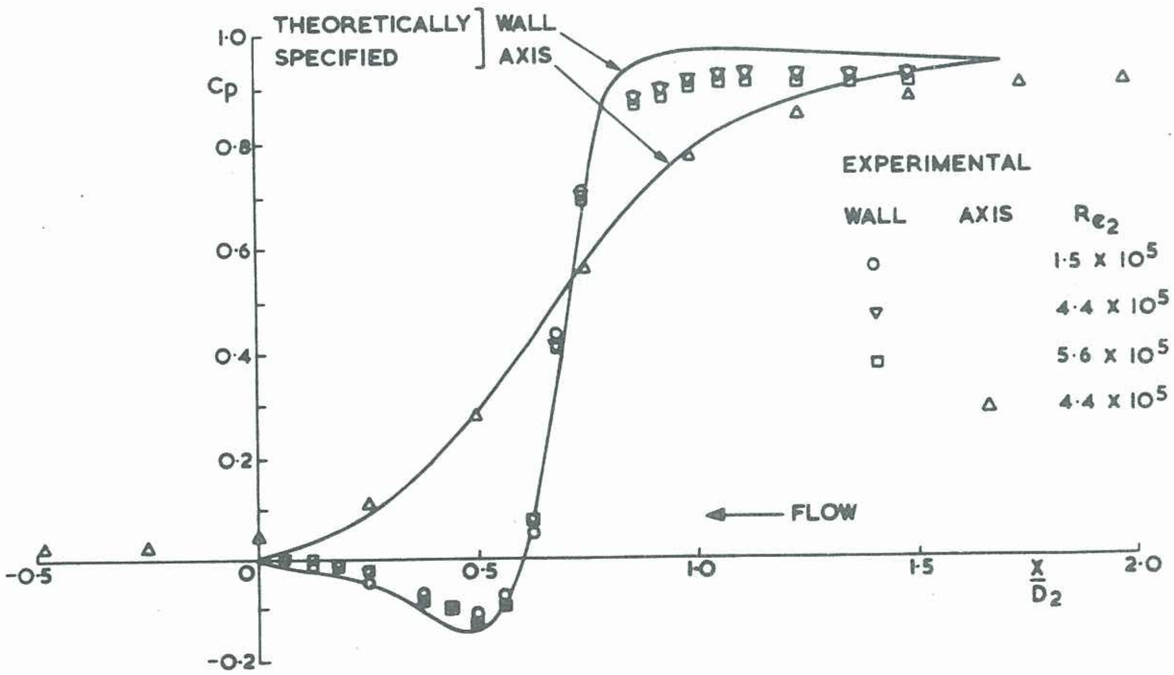

FIG. 4 TEST DUCT B - STATIC PRESSURE DISTRIBUTION ALONG THE WALL AND AXIS.