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OSCILLATORY CONVECTION IN A POROUS MEDIUM: THE EFFECT OF THROUGHFLOW

by

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SUMMARY

The flow of fluid through a porous medium by natural convection has been shown by Horne and O'Sullivan (1) to be oscillatory under certain conditions in a simplified two-dimensional model of a hydrothermal system. The present investigation extends the range of the model developed in (1) to incorporate the effects of surface recharge and discharge and of fluid sinks inside the region - it is hoped that such a model more realistically describes a geothermal field such as that at Wairakei, New Zealand. It is found that the recharge condition does not prevent the appearance of regular oscillations, in fact their amplitude increases. The addition of a sink into the region increases the volume of fluid flowing in and out of the recharge surface and also raises the usable heat output considerably if the sink is in the correct position.

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1. INTRODUCTION

In Horne and O'Sullivan (1) we developed a simple numerical model of a hydrothermal system which consisted of a closed, two dimensional region filled with an isotropic, fluid saturated permeable material, heated uniformly or non-uniformly from below. With this model we successfully verified the fluctuating solutions to the uniformly heated problem obtained by Caltagirone, Cloupeau and Combarnous (2) and Combarnous and Le Fur (3). Further we discovered that if the lower boundary is heated along only half of its length the unsteady flow is restricted into regular oscillations, generating ascending thermals over the heater and corresponding "antithermals" in the descending flow. The process by which these disturbances are produced is described in (1) and also by Caltagirone et al (2) and Sparrow, Husar and Goldstein (4) - a locally uniform temperature gradient is established close to a horizontal boundary such that the local Rayleigh number exceeds the critical value that allows a convection cell to establish in this region. The disturbances so formed gestate and are conveyed from the boundary by the main circulating convective flow until final dissipation by or loss out of the system. Although this phenomenon is in itself of general interest, clearly the model is too simplified to be applied to a real system such as the Wairakei geothermal field. Therefore we have modified the model so that fluid is allowed to flow freely into or out of the upper boundary and may also be withdrawn (or injected) through point sinks (sources) in the flow region. This provides an analogy to surface water recharge and discharge and to the extraction of geothermally heated water through boreholes. A recharge surface condition has been studied previously by Donaldson (5) but only at relatively low Rayleigh numbers and with a specified surface inflow.

In the following sections we describe briefly the original model and its more recent modifications, then proceed to the results of computations performed with the extended model.

2. THE NUMERICAL ANALOGUE

The convection of fluids through a porous material under the influence of thermal gradients may be described in terms of the stream function ψ and non-dimensional temperature θ, defined respectively by

$$\psi_Y = \lambda \left(\frac{a}{\kappa R} \right) u; \qquad \psi_X = -\lambda \left(\frac{a}{\kappa R} \right) v \tag{2.1}$$

$$\text{and} \quad \theta = \frac{T - T_0}{T_1 - T_0} . \tag{2.2}$$

The governing equations for the flow in non-dimensional region $0 \le X \le 1$, $0 \le Y \le 1$, with a heater at temperature T_1 and surroundings at T_0, are then

$$\nabla^2 \psi = -\theta_X \tag{2.3}$$

$$\text{and} \quad \nabla^2 \theta = \theta_\tau + R \,.\, \delta(\psi, \theta) \tag{2.4}$$

where the Jacobian operator $\delta(\psi, \theta) \equiv \psi_Y \theta_X - \psi_X \theta_Y$.

In these equations R is the Rayleigh number defined by

$$R = \frac{g\,\alpha\,(T_1 - T_0)\;a\,k}{\nu\,\kappa}\,\lambda \tag{2.5}$$

where κ is the thermal diffusivity of the fluid filled medium, k is the permeability, α is the coefficient of thermal expansion of the fluid, ν is the kinematic viscosity of the fluid, λ is the ratio of the volumetric heat capacity of the fluid to that of the saturated medium, a is the width of the region, and τ is non-dimensional time variable defined by

$$\tau = \frac{\kappa}{a^2}\, t . \tag{2.6}$$

The flow regime is dependent on the boundary conditions and on the value of the Rayleigh number. In (1) we determined that, for a lower boundary heated along half its length, the flow is oscillatory when $R > 480$. In this investigation we first consider the effect of the recharge boundary condition on this cutoff value and then examine a typical unsteady solution at $R = 500$ under the influence of a fluid sink. The temperature boundary conditions are the same as those used in (1) namely, $\theta = 0$ on the surface, $\theta = 1$ on half of the lower boundary and both vertical edges insulated. For a closed system the stream function ψ would be zero on all boundaries but for the recharge discharge condition we specify that the pressure over the top surface is constant (i.e. atmospheric); this requires that the velocity must be normal to the upper boundary, which is achieved by specifying ψ_Y zero.

The heat transport equation (2.4) is solved using an explicit fourth order Arakawa (6) finite difference representation of the advection term $\delta(\psi, \theta)$ and a forward difference in time. At each time step the momentum equation (2.3) is solved using the Buneman odd-even reduction algorithm described by Busbee, Golub and Nielsen (7). The isothermal temperature conditions are applied by direct substitution and the insulated condition by symmetry consideration. On the impermeable boundaries the stream function is set to zero and on the recharge boundary reset after every time step such that its vertical derivative is zero.

The original numerical model has been well tested and validated (see (1)) and may now be modified to accommodate internal fluid sinks. The simplest case is of a sink on one of the vertical boundaries, say at a point $(0, Y_0)$. Then, for a sink of strength q, we require

$$\psi(0, Y, \tau) = -q \qquad Y_0 < Y \leq 1 . \tag{2.7}$$

Clearly if ψ is zero on the other vertical boundary such a boundary condition is only sensible if ψ_Y is zero along the top boundary, which is to be expected physically from conservation of mass.

If the sink lies within the boundaries, say at a point (X_0, Y_0), then we introduce a function ψ^* such that

$$\psi^* = -\frac{q}{2\pi} \tan^{-1}\left(\frac{Y - Y_0}{X - X_0}\right) + \psi . \tag{2.8}$$

The governing equations are then,

$$\text{as before,} \qquad \nabla^2\psi = -\theta_X \tag{2.9}$$

$$\text{but now} \qquad \theta_\tau = \nabla^2\theta + R\ \delta(\psi^*, \theta) \tag{2.10}$$

On the zero throughflow boundaries

$$\psi^* = 0,$$

and on the recharge boundary

$$\psi^*_Y = 0.$$

The positioning of the discontinuity in the inverse tangent term in (2.8) requires some care - for convenience this is taken to lie vertically between the sink point and the top boundary. Also it is necessary to require that the point (X_0, Y_0) does not lie on a node of the finite difference mesh as ψ^* is undefined at this point.

3. RESULTS

Solutions for the recharge boundary problem in the absence of any fluid sinks at Rayleigh numbers 250 to 1000 are obtained on 33 x 33 point meshes. The flow is regular oscillatory when the Rayleigh number exceeds a value in the vicinity of 350, which is considerably less than the cutoff value of 480 determined for the closed system in (1). As is seen in figure 1, in this recharge situation the influx of cold fluid through the top boundary nullifies the descending disturbance that was seen in the closed system solution (1) as the temperature gradient is no longer large enough to allow a perturbation to form. Figure 1 is a plot of isotherms (full lines) and streamlines (broken lines) for the flow at R = 500. It should be noted that, for greater clarity,the diagram is constructed of the solution and its mirror image, joined at the centre. The ascending pulses are generated in a similar manner to the closed system but with a higher frequency. Also, as may be seen by comparing the left vertical boundary temperature profiles in figure 2 (these plots are a superposition of the profiles during several successive time intervals) the recharge disturbances have a larger amplitude and move both faster and further than their closed system counterparts.

The solutions at Rayleigh numbers of 500 and 750 are somewhat anomalous in that successive thermals have neither the same magnitude nor frequency but evolve in pairs; for instance when R = 500 a larger pulse is followed by a smaller pulse after a time interval of 0.0037 and a subsequent larger pluse, identical to the first, appears 0.0042 later. These motions are illustrated in figure 3 which is a plot of a) the Nusselt number, b) left boundary middle temperature and c) maximum stream function,throughout two oscillation pairs. A summary of all the solutions is given in table 1.

R	Nu_{min}	Nu_{max}	v^*_{max}	ψ_{max}	τ_ρ
250	2.678	2.678	0.380	0.057	-
375	2.420	3.181	0.400	0.051	0.0068
500/1st pulse	2.294	3.584	0.485	0.051	0.0037
500/2nd pulse	2.194	3.884	0.480	0.051	0.0042
750/1st pulse	2.841	4.784	0.394	0.053	0.0021
750/2nd pulse	2.340	4.564	0.387	0.053	0.0024
1000	2.717	4.820	0.417	0.058	0.0017

Nu_{min} = minimum Nusselt number

Nu_{max} = maximum Nusselt number

v^*_{max} = maximum speed of flow at mid-point of left vertical boundary

ψ_{max} = maximum stream function reached during oscillation

τ_ρ = time period of oscillation.

Table 1 Summary of Recharge Solutions

The solution for the flow at Rayleigh number 500, in which there is a sink on the left boundary (notice that this boundary may also be considered as a line of symmetry) is illustrated in figure 4. In this case the sink is of strength 0.05 and the flow has a Nusselt number varying between 4.4 and 8.7, an oscillation of period 0.0042 and amplitude very close to that of the no sink solution. If the sink is positioned mid-region over the other end of the heater, the flow is as in figure 5, with a greatly reduced Nusselt number varying between 2.60 and 3.56 and an oscillation of period 0.0041 with amplitude approximately 20% of that previously observed. It should be noted that the Nusselt number in this case is the total heat transfer between the heater and the low temperature boundaries plus the amount of heat removed through the sink compared to the amount of conductive heat transfer through the region when fluid motion is disallowed.

4. DISCUSSION

The steadiness of convective flow through a porous medium was shown in (1) to be greatly influenced by the presence of vertical boundaries. With the results generated here the single ascending region of thermals enables closer inspection of the disturbances without the added complication of interacting descending effects - comparison of these solutions with the closed system results of (1) indicates the significance of the coupling between temperature and velocity. This is further highlighted in consideration of the paired oscillations illustrated in figure 3 - as one thermal rises through the porous matrix it creates a velocity perturbation which interrupts the gestation of the thermal which has formed on the heater behind it. This correspondingly smaller thermal is drawn off earlier than its predecessor and thus makes a premature appearance. When the Rayleigh number is sufficiently large, successive disturbances are generated so rapidly that the interaction between them extends further than just adjacent disturbances and it is seen that when R = 1000 the oscillations are regular. On the other hand at lower Rayleigh number the disturbances are widely spaced in both time and position and when R = 375 the oscillation is again regular.

A biperiodic effect was also observed in the closed porous region of (1) where the descending disturbance occurs at exactly half the frequency of the ascending thermals. Also there seems to be a similar effect in the experimental results of Sparrow, Husar and Goldstein (4) (figure 2a, p.797) for a Newtonian fluid heated from below. Thus the appearance of these interactions between the velocity and temperature fields adds weight to the assertion that the advection term in the heat transport equation is of particular importance in this type of problem and justifies the efforts to maintain accuracy in its representation.

The physical implications of these solutions may best be considered by observing the ground level effects. It has already shown in (1) that in the closed boundary model there is a periodic solution at Rayleigh numbers that are below that at Wairakei (an estimated figure being 5000 - 10,000). The comparative diagrams in figure 2 show clearly that the recharge solution produces a much more prominent effect in that the temperature perturbations travel closer to the surface and are much larger in magnitude than those in the closed model. The addition of a sink above the centre of the heater effectively collects these rising thermals and prevents their loss at low temperature through the surface and greatly increases the Nusselt number of the flow. The efficiency of placing the sink in this position is indicated by comparing the maximum Nusselt number of the solution (8.7) to that when the sink is positioned above one end of the heater (3.56). In this latter case the flow of fluid between the surface and the sink chokes the flow through the region and consequently reduces the fluid velocity over the heater, thus reducing the convective heat transfer. Also the sink is drawing fluid of much lower temperature, thus greatly reducing its usefulness.

5. REFERENCES

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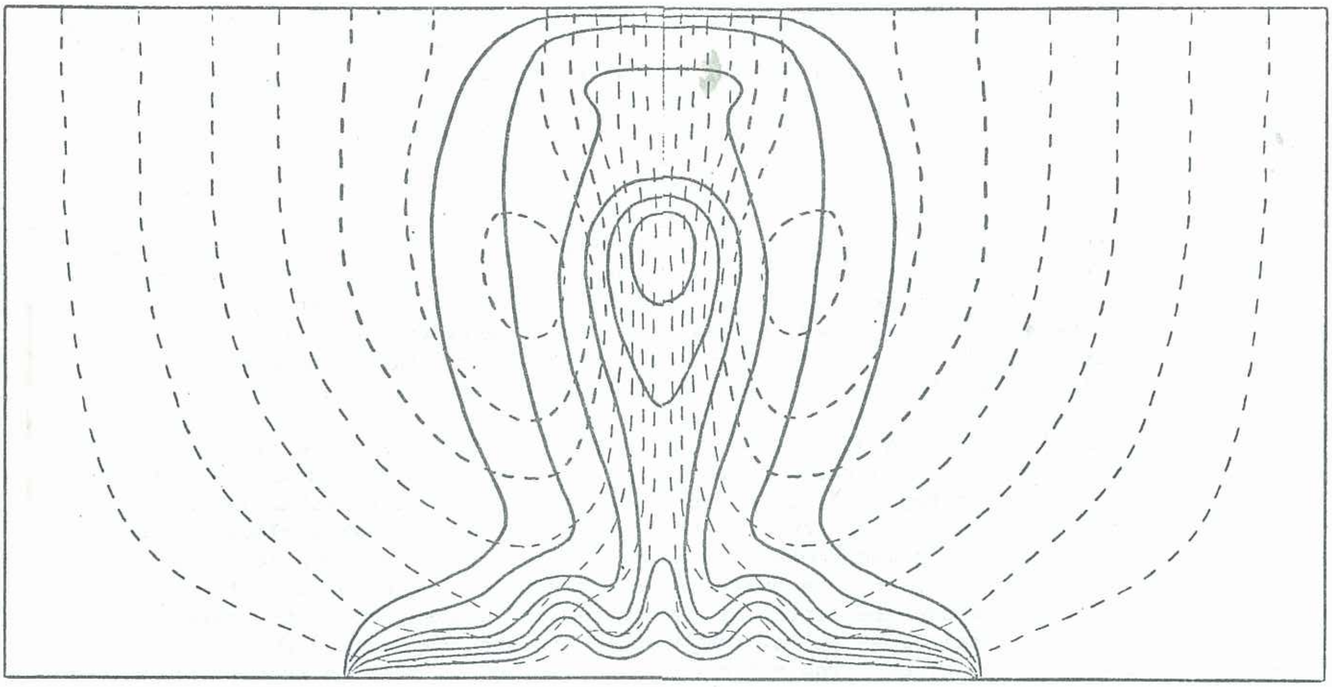

Figure 1: Solution for Rayleigh number of 500, plot of isotherms (——) and streamlines (----) at a particular time.

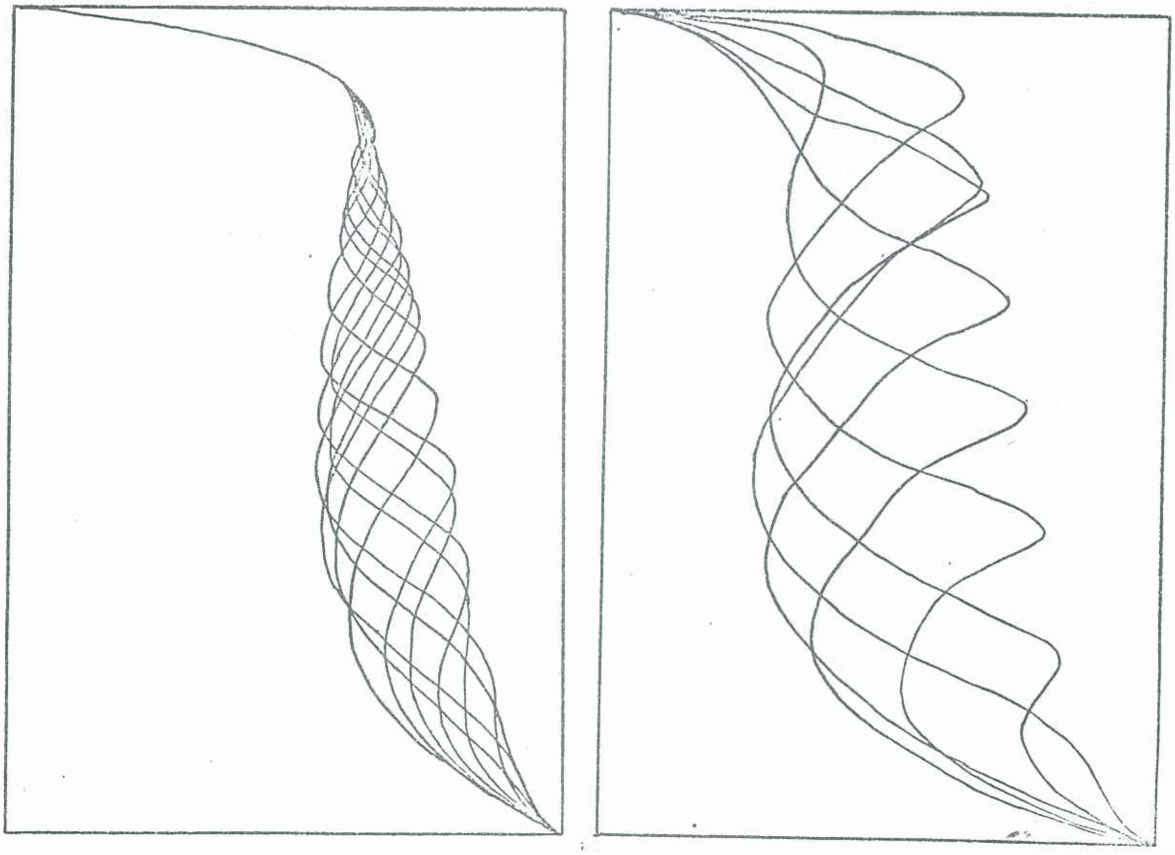

Figure 2: Plots of temperature profiles along left hand vertical boundary at various times for R = 750.
Left: closed system of (1), Right: recharge solution.

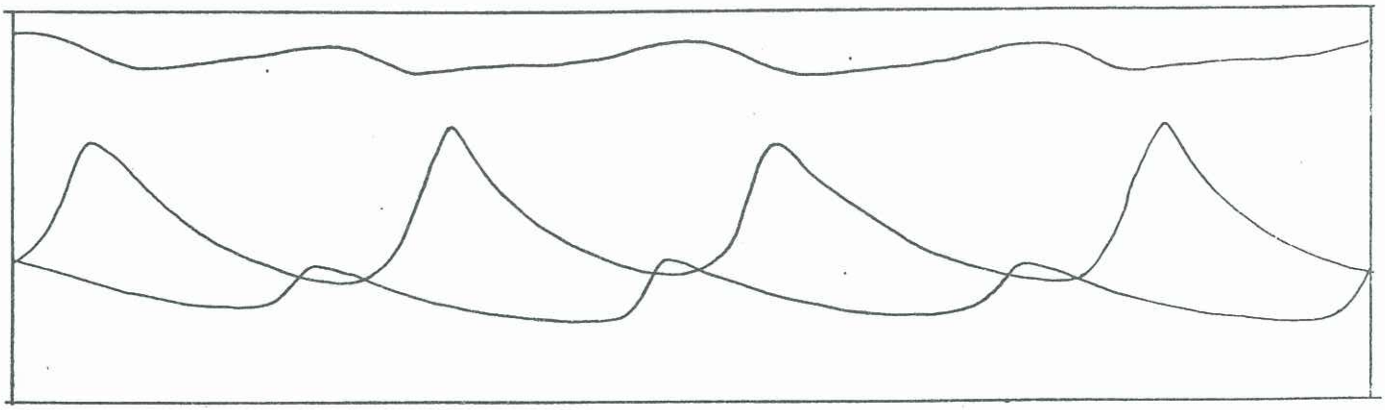

Figure 3: Time variation of temperature (centre plot) at mid-point of left vertical boundary, Nusselt number (lower plot) and maximum stream function (upper plot) for R = 500.

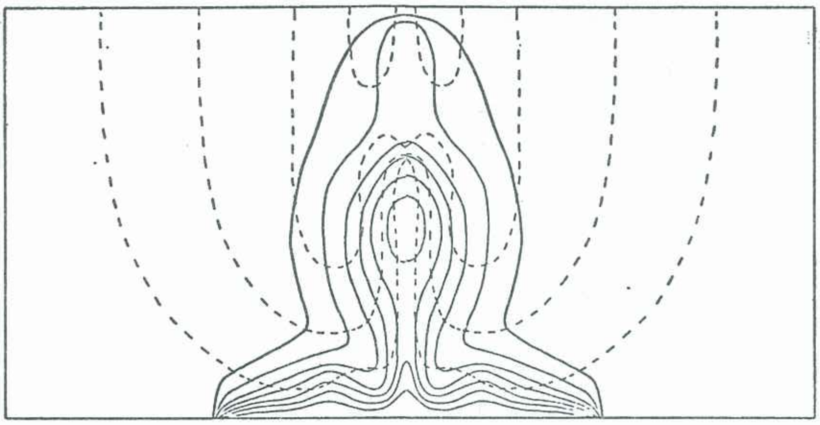

Figure 4: Typical solution for R = 500 with sink on left hand vertical boundary.

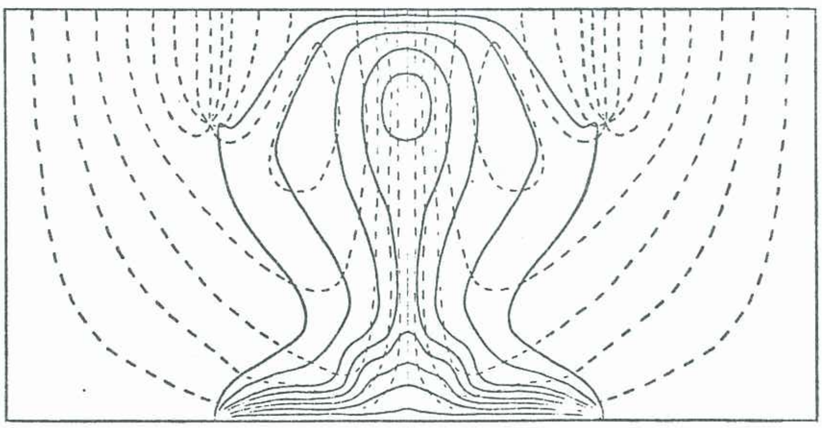

Figure 5: Typical solution for R = 500 with sink in mid-region.