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FLOW IN A FISSURED POROUS MEDIUM

by

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A fissured porous medium is modelled as an assemblage of porous blocks. The behaviour of a single block is found, as it discharges in a quasi-steady manner; and its diffusivity and relaxation time as functions of water content. Then if the fissures and blocks both behave in a linear and quasi-steady manner, the fissured medium looks just like a homogeneous one.

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GLOSSARY

ε = rock porosity

e = water fraction in rock $1-e$ = steam fraction

X = X_b = block volume λ = block volume/fissure volume

$\rho_{w,m,s}$; $\mathcal{U}_{w,m,s}$ = density, enthalpy of water, rock, steam

$c_{w,m} = \frac{d}{dT}\mathcal{U}_{w,m}$ = specific heat of water, rock

k = permeability κ = diffusivity τ = relaxation time of a block

$\mu_{w,s}$, $\underline{V}_{w,s}$, $Q_{w,s}$, $F_{w,s}(e)$ = viscosity, volume flux density, discharge from a block, permeability reduction factor (1) for water, steam

$\gamma = \mu_w/\mu_s$

T = temperature, P = pressure, $P = P(T)$ saturation pressure

$\mathcal{L}$ = latent heat = $\mathcal{U}_s - \mathcal{U}_w$

ΔT = temperature change of a block.

The following are taken as constant: $c_w, c_m, \rho_w, \rho_s, \rho_m, \mathcal{U}_s$

INTRODUCTION

This paper deals with a fissured porous medium, consisting of an assemblage of porous blocks. The first two parts deal with the quasi-steady behaviour of a single block (which could also be an isolated porous block). As it discharges, there is a simply-obtained relationship between the falling water content of the block, the falling temperature of the block, and the rising enthalpy of its discharge. This is described in section 1. Section 2 calculates the diffusivity (to pressure) of a homogeneous porous block, as a function of its water fraction, and its relaxation time. Section 3 deals with an assemblage of such blocks, and shows that if the flow is quasi-steady and linear, the fissured medium behaves exactly like a homogeneous porous medium of an appropriate permeability.

The fissured porous medium is modelled as an assemblage of porous blocks of low permeability, that discharge into fissures of much higher permeability:

The blocks are storage granules for the larger system of the fissures. Most of the fluid is stored in the porous blocks, but most of the permeability is in the fissures. I assume that changes of pressure and water fraction are sufficiently slow, that the system is quasi-steady.

1. Discharge of a single block.

Some results can be obtained quite easily If a block discharges slowly, it is possible to find the temperature of its output as a function of the water remaining in the block, and this can be found without detailed knowledge of the rate of discharge.

water content of block = $\varepsilon e X$ steam content = $\varepsilon(1-e)X$

$$Q_w : Q_s = F_w/\mu_w : F_s/\mu_s \quad \therefore (Q_w, Q_s) = (F_w, \gamma F_s).(Q_w + Q_s)/(F_w + \gamma F_s) \qquad (1.1)$$

Mass is conserved in the block:

$$X\frac{d}{dt}\{\varepsilon e\rho_w + \varepsilon(1-e)\rho_s\} = X\varepsilon(\rho_w - \rho_s)\frac{de}{dt} = -\rho_w Q_w - \rho_s Q_s \qquad (1.2)$$

and energy: $$X\frac{d}{dt}\{\varepsilon e\rho_w U_w + \varepsilon(1-e)\rho_s U_s + \rho_m U_m\}$$

$$= X\{\varepsilon(\rho_w U_w - \rho_s U_s)\frac{de}{dt} + (\varepsilon e\rho_w C_w + \rho_m C_m)\frac{dT}{dt}\} = -(P + \rho_w U_w)Q_w - (P + \rho_s U_s)Q_s \qquad (1.3)$$

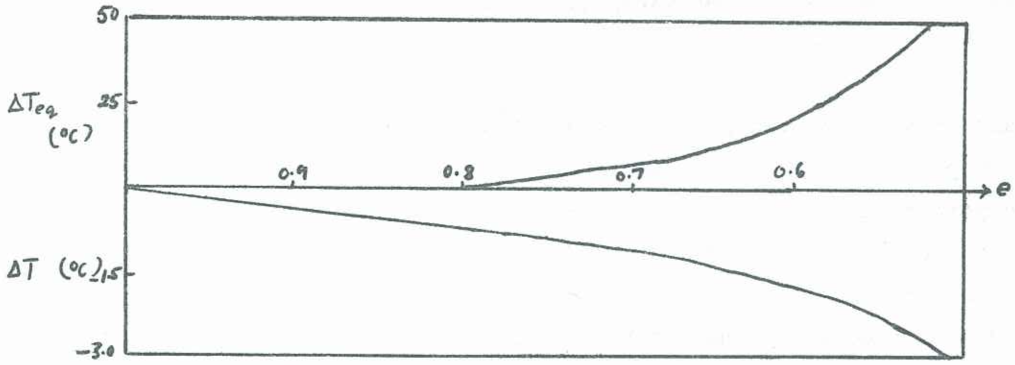
Between these three equations:

$$\boxed{\frac{dT}{de} = \frac{P(\rho_w - \rho_s) + \mathcal{L}\rho_w\rho_s}{\varepsilon e\rho_w C_w + \rho_m C_m} \cdot \frac{\varepsilon(F_w + \gamma F_s)}{\rho_w F_w + \gamma\rho_s F_s}} \qquad (1.4)$$

As water is expelled by steam, the temperature drops. (1.4) gives the relation between the falling temperature and the falling water fraction. As the water fraction in the block falls, the permeability to steam increases (as $F_s(e)$), and so the output enthalpy rises. I express this in terms of the equivalent temperature rise, ΔT_{eq}, that would be obtained if the steam in the discharge were condensed:

$$\boxed{\Delta T_{eq} = \frac{\rho_w F_w \Delta T + \gamma\rho_s F_s \mathcal{L}/c_w}{\rho_w F_w + \gamma\rho_s F_s}} \qquad (1.5)$$

Fig. 1 shows ΔT and ΔT_{eq} vs e, from an initial temperature of 235°C:



At 235°C, a fall of 1°C implies a drop of 8p.s.i., and so fig.1 relates the output enthalpy to falling pressure.

2. Time-dependence of the block discharge.

I now consider the full fissure-block system. I ignore gravity, in this analysis, since its effect over a block of 100m, is quite small. It is assumed that pressures change sufficiently slowly with distance, that the fissure pressure outside any block, can be regarded as a function of time alone. Each block's pressure will then depend only parametrically on fissure coordinates. Let the subscript $_b$ refer to the block, and $_f$ to the fissure. $P_f(\alpha t)$ is the fissure pressure outside a block. α is small, and is inserted to emphasise that P_f varies slowly compared to the relaxation time of the block. Inside the block, pressure and water fraction consist of a basic term, independent of $\underline{x}_f$, and slowly varying in time, plus a small perturbation:

$$P_b(t, \underline{x}_b) = P_b^{(0)}(\alpha t) + \alpha P_b^{(1)}(\alpha t, \underline{x}_b) + O(\alpha^2) \qquad P_b^{(0)} = P_f \qquad (2.1)$$

$$\underline{V}_{b\,s,w}(t, \underline{x}_b) = \alpha V^{(1)}_{b\,s,w}(\alpha t, \underline{x}_b) + \ldots$$

Ignoring gravity, the equations of motion are: (2)

$$\frac{\partial}{\partial t}\{\varepsilon e\rho_w + (1-e)\varepsilon\rho_s\} + \nabla.\{\rho_w V_w + \rho_s V_s\} = 0 \qquad (2.2)$$

$$\frac{\partial}{\partial t}\{\varepsilon e\rho_w U_w + \varepsilon(1-e)\rho_s U_s + \rho_m U_m\} + \nabla.\{(P + \rho_s U_s)\underline{V}_s + (P + \rho_w U_w)\underline{V}_w\} = 0 \qquad (2.3)$$

$$\underline{V}_{w,s} = -\frac{k}{\mu_{w,s}} F_{w,s}(e)\nabla P \qquad (2.4)$$

Inserting these into (2.1) and expanding in powers of α:

$$\alpha^2\varepsilon(\rho_w - \rho_s)\frac{\partial e_b^{(1)}}{\partial t} - \alpha k_b\{\frac{F_s}{\mu_s}\rho_s + \frac{F_w}{\mu_w}\rho_w\}\nabla_b^2 P_b^{(1)} = -\alpha\frac{d}{dt}\{\varepsilon e_b^{(0)}\rho_w + \varepsilon(1-e_b^{(0)})\rho_s\} \qquad (2.5)$$

$$\alpha^2 \cdot \varepsilon(\rho_w U_w - \rho_s U_s)\frac{\partial e_b^{(1)}}{\partial t} + \alpha^2 \cdot (\varepsilon e \rho_w C_w + \rho_m C_m)\frac{\partial T_b^{(0)}}{\partial t} - \alpha k_b \left\{ \frac{F_s}{\mu_s}(P + \rho_s U_s) + \frac{F_w}{\mu_w}(P + \rho_w U_w) \right\} \nabla_b^2 P_b^{(1)} = S_2 = -\alpha \frac{d}{dt}\left\{ \varepsilon e_b^{(0)} \rho_w U_w + \varepsilon(1 - e_b^{(0)}) \rho_s U_s + \rho_m U_m \right\} \quad (2.6)$$

with $F_{w,s} = F_{w,s}(e_b^{(0)})$. At highest order in α (2.5,6) are of the form $\nabla_b^2 P_b^{(1)} = S_{1,2}$ and equating S_1 and S_2 gives eqn. (1.4).

From these equations comes the relaxation time. Set $\alpha = 1$, since we now want O(1) time dependence. Eliminating $e_b^{(1)}$ between (2.5) and (2.6) yields:

$$\left\{ A\frac{\partial}{\partial t} + B\nabla_b^2 \right\} P_b^{(1)} = C \quad (2.7)$$

where

$$A = -(\rho_w - \rho_s)\{\varepsilon e \rho_w C_w + \rho_m C_m\}\frac{dT}{dP}\ ;\quad B = k_b\left\{\frac{F_s}{\mu_s} + \frac{F_w}{\mu_w}\right\}\{P(\rho_w - \rho_s) + \mathcal{L}\rho_w\rho_s\};\quad C = (\rho_w - \rho_s)\{\varepsilon e \rho_w C_w + \rho_m C_m\}\frac{dT_b^{(0)}}{dt} \quad (2.8)$$

and hence the diffusivity

$$\kappa = -B/A = k\frac{dP}{dT}\left(\frac{F_s}{\mu_s} + \frac{F_w}{\mu_w}\right)\frac{P + \mathcal{L}\rho_w\rho_s/(\rho_w - \rho_s)}{\varepsilon e \rho_w C_w + \rho_m C_m}$$

For a block of dimension a, the relaxation time is $\tau = a^2/\kappa$ At 235°C, with $k = 3\times10^{-10}\,cm^2$

$$\boxed{\kappa = \frac{5(F_w + 2\tfrac{1}{2}F_s)}{e + 2\tfrac{1}{2}}} \quad (2.9)$$

If a = 100m. in wet rock $(e \sim 1)$ κ = 1½ cm /sec, τ = 2yr (2.10)

in steamy rock $(e \sim 0)$ κ = 5 cm /sec, τ = yr

The quasi-steady condition means that e must vary little in one relaxation time:

$$1 \gg \tau\frac{de}{dt} = \tau\frac{de}{dT}\frac{dT}{dt} = \tau\frac{de}{dT}\frac{dT}{dP}\frac{dP}{dt} \quad (2.11)$$

If A is the block size in units of 100m, these limits, for wet rock, are:

$$\frac{dP}{dt} \ll \frac{11}{A^2}\ \text{p.s.i./yr} \qquad \frac{dT}{dt} \ll \frac{1\tfrac{1}{2}}{A^2}\ {}^{\circ}\text{C/yr} \quad (2.12)$$

For steamy rock, the limits are much larger.

3. Interaction with the Fissures.

Integrating over block volume

$$k_b\left(\frac{\rho_w F_w}{\mu_w} + \frac{\rho_s F_s}{\mu_s}\right)\int \frac{\partial P_b^{(1)}}{\partial n}dS = \varepsilon(\rho_w - \rho_s)\frac{de_b^{(0)}}{dt}\cdot X_b \quad (3.1)$$

$= \mathcal{J}_1$ = mass flux out of block.

Likewise, $\mathcal{J}_2$ = energy flux = $X_b \frac{d}{dt}\left\{ \varepsilon e_b^{(0)}\rho_w U_w + \varepsilon(1 - e_b^{(0)})\rho_s U_s + \rho_m U_m \right\}$ (3.3)

I now drop the $^{(0)}$ superscripts.

$$\mathcal{J}_1 = \varepsilon(\rho_w - \rho_s)\frac{de_b}{dT}\cdot\frac{dT}{dP}\frac{\partial P_f}{\partial t}\cdot X_b = X_b\sigma_1\frac{\partial P_f}{\partial t}$$

$$\sigma_1 = (\rho_w - \rho_s)\frac{dT}{dP}\cdot\frac{(\rho_w F_w + \rho_s F_s)(\varepsilon e \rho_w C_w + \rho_m C_m)}{(F_w + \gamma F_s)(P(\rho_w - \rho_s) + \mathcal{L}\rho_w\rho_s)} \quad (3.4)$$

$$\mathcal{J}_2 = X_b\sigma_2\frac{\partial P_f}{\partial t}\ ;\qquad \sigma_2 = (\rho_w - \rho_s)\frac{dT}{dP}\cdot\frac{F_w(P + \rho_w U_w) + \gamma F_s(P + \rho_s U_s)}{F_w + \gamma F_s}\cdot\frac{\varepsilon e \rho_w C_w + \rho_m C_m}{P(\rho_w - \rho_s) + \mathcal{L}\rho_w\rho_s} \quad (3.5)$$

I now use partial derivatives, $\frac{\partial}{\partial t}$ on $P_b = P_b^{(0)} = P_f$ to emphasize that we now have space-dependence in the fissures as well. The fluxes $\mathcal{J}_1$, $\mathcal{J}_2$ appear as source terms in the fissures, and so the fissure equations are:

mass: $$\frac{\partial}{\partial t}\{e\rho_w + (1 - e)\rho_s\} + \nabla\cdot\{\rho_s \underline{V}_s + \rho_w \underline{V}_w\} = \lambda\sigma_1\frac{\partial P_f}{\partial t} \quad (3.6)$$

where λ = block volume/fissure volume. It is large, and the first term (storage in the fissure) is then small. So:

$$\nabla.\ (\rho_s \underline{V}_s + \rho_w \underline{V}_w) = \lambda\sigma_1 \frac{\partial P_f}{\partial t} \tag{3.7}$$

and for energy:

$$\nabla.\ ((P+\rho_s U_s)\underline{V}_s + (P+\rho_w U_w)\underline{V}_w) = \lambda\sigma_2 \frac{\partial P_f}{\partial t} \tag{3.8}$$

Now I further assume that the fissures can be regarded as a porous medium, of permeability k_f:

$$\underline{V}_{s,w} = -\frac{k_f}{\mu_{s,w}} F_{s,w}(e_f) . \nabla P_f$$

and, that the flow can be linearised about a basic state $e_f = e_f(t)$ - that the fissure flow also is quasi-static. Then (3.7,8) are now:

$$-k_f \left(\frac{\rho_s}{\mu_s} F_s(e_f) + \frac{\rho_w}{\mu_w} F_w(e_f) \right) \nabla^2 P_f = \lambda\sigma_1 \frac{\partial P_f}{\partial t} \tag{3.9}$$

$$-k_f \left(\frac{\rho_s U_s}{\mu_s} F_s(e_f) + \frac{\rho_w U_w}{\mu_w} F_w(e_f) \right) \nabla^2 P_f = \lambda\sigma_2 \frac{\partial P_f}{\partial t} \tag{3.10}$$

Because of the form of σ_1, σ_2 which have the ratio

$$\sigma_1 : \sigma_2 = \left(\frac{\rho_s}{\mu_s} F_s(e_b) + \frac{\rho_w}{\mu_w} F_w(e_b) \right) : \left(\frac{\rho_s U_s}{\mu_s} F_s(e_b) + \frac{\rho_w U_w}{\mu_w} F_w(e_b) \right)$$

we have

$$F_w(e_f) : F_s(e_f) = F_w(e_b) : F_s(e_b) \quad \text{so} \quad \boxed{e_f = e_b} \tag{3.11}$$

This came about because the terms involving $\frac{\partial e_f}{\partial t}$ were small, and dropped from the fissure equations. If e_f deviated from e_b the fluxes into the fissure, with their prescribed ratio $\sigma_1 : \sigma_2$ would correct e_f in a short time. It is noteworthy that this conclusion, $e_f = e_b$ depended upon the fissure and block having the same law of motion. If, for some reason, the fissure had different F_w and F_s functions, this conclusion would not hold.

The equation for P_f is:

$$\left(\frac{\partial}{\partial t} + \kappa_f \nabla^2 \right) P_f = 0$$

$$\kappa_f = \frac{k_f}{\lambda} \cdot \frac{dP}{dT} \cdot \frac{F_w + \gamma F_s}{\rho_w - \rho_s} \cdot \frac{P(\rho_w - \rho_s) + \mathcal{L}\rho_w \rho_s}{\varepsilon e \rho_w c_w + \rho_m c_m} \tag{3.12}$$

This is just the diffusion equation, as if the entire fissured medium were a homogeneous porous medium, of permeability k_f/λ.

This conclusion required that:

(i) the systems evolve in a quasi-steady manner

(ii) the fissures and blocks behave like linearised porous media.

The assumption most likely to be violated, is that the fissures obey a linearised (or even linear) flow law. Since the flow is confined into them, velocities will be much higher, and nonlinearities correspondingly stronger.

REFERENCES

(1) Permeability reduction factors for two-phase flow are discussed in 'The Physics of Flow through Porous Media', Scheidegger, University of Toronto Press 1957.

(2) A. McNabb, Geothermal Circular.