FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand

1974 December 9 to December 13

THE RATIOS OF EDDY DIFFUSIVITIES OF HEAT AND MOMENTUM FOR LIQUID METALS

by

A. Feingold*, A.C. Gupta**, and K.G. Gupta**

SUMMARY

The evaluation of ratios of eddy diffusivities of heat (ε_H) and momentum (ε_M) for liquid metals flowing turbulently in a cylindrical pipe has important practical applications, but unfortunately presents considerable difficulties in the choice of a suitable mathematical model. No definitive analysis of this problem can be found in the literature to date. Azer and Chao in their paper "A Mechanism of Turbulent Heat Transfer in Liquid Metals" (Int. J. Heat Mass Transfer, Vol. 1, pp. 121-138, 1960) attempted to obtain a solution based on the modified Prandtl mixing length theory. They assumed the eddy to be at a constant average temperature during its flight time and allowed the temperature of the surrounding fluid to vary. The alternative model adopted in the present paper presupposes the fluid temperature to be constant while the eddy temperature varies. The paper contains a detailed theoretical analysis as well as a graphic presentation of results for representative values of Prandtl and Reynolds numbers, and distance from the pipe wall. These results are then compared with published theoretical and experimental values.

* Professor, ** Research Assistant
Department of Mechanical Engineering
University of Ottawa, Ottawa, Canada

Introduction

The pioneering work of Martinelli [1] in 1947 on the heat transfer in liquid metals was further developed by Lyon [2], who suggested the following relation for calculating heat transfer coefficient:

$$Nu = 7 + 0.025\,(\psi\,Pe)^{0.8},$$

where ψ is the ratio of the eddy diffusivity of heat to the eddy diffusivity of momentum and $Pe = Re\,Pr$ is the Peclet number. The value of Nu, the Nusselt number, determined from this relation was found to be considerably higher than that obtained experimentally [3]. The discrepancy was suspected to be at least partly due to the choice of unity as the value for ψ by Martinelli and Lyon. Subsequent investigations [4,5,6,7,8,9,10], both experimental and theoretical tend to support this view and it is now generally accepted that ψ is not constant but varies with the distance from the pipe wall, Reynolds number, and Prandtl number. There is, however, no general agreement on the values of ψ to be adopted in relation to these variables.

Much of the work done during the past several years centered around various proposed modifications to the well-known Prandtl mixing length theory [4,5,6,7], getting away from the rather simplistic proposition that neither heat nor momentum is transferred from or to the eddy during its flight. In particular, the model adopted by Azer and Chao suggested that an eddy exchanges heat with the surrounding fluid during its flight and, specifically, that for the purpose of calculating this heat exchange the temperature of the eddy be set at an arithmetic average between its initial and its final values, while the temperature of the surrounding fluid is made to vary.

The model chosen in the present paper proposes the opposite (and in our view more logical) premise — the fluid temperature is maintained constant at its average value, while the temperature of the eddy varies.

The eddy is assumed to be spherical as was done by many other investigators, e.g. Lykoudis and Touloukian [7]. With the following nomenclature (see Fig. 1):

t_1' = mean temperature of the eddy at the instance it reaches layer 2

t_1 = temperature of the eddy as well as of the fluid at layer 1

t_2 = temperature of the fluid at layer 2,

the eddy is initially at a uniform temperature $T_o = \frac{1}{2}(t_1 + t_1')$ while the average fluid temperature is $T_f = \frac{1}{2}(t_1' + t_2)$.

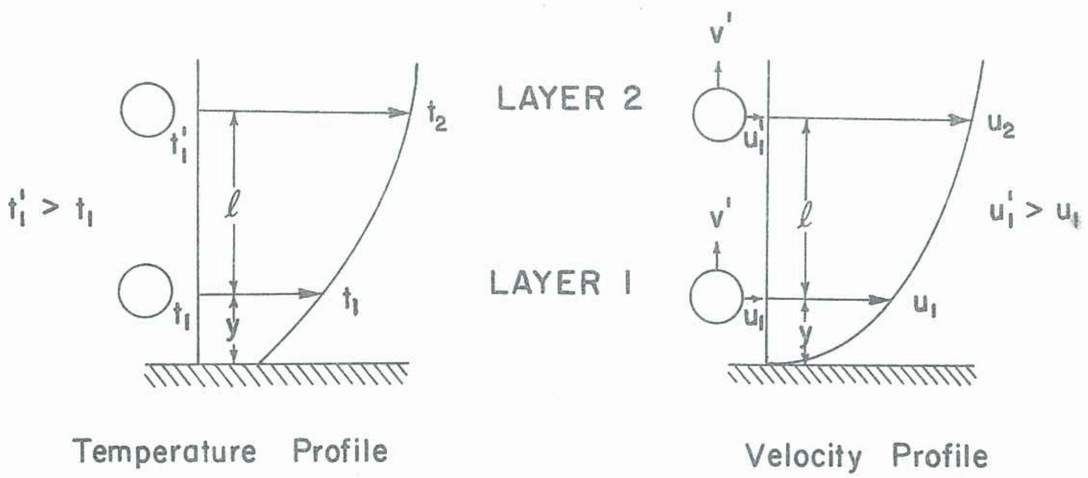


FIGURE I

Temperature distribution within the eddy during its flight

Let h be the constant uniform convective heat transfer coefficient at the surface of the eddy. The solution for temperature within a spherical eddy of radius a as a function of time θ and of a variable distance from the center R must satisfy the equation

$$\frac{\partial^2 T}{\partial R^2} + \frac{2}{R}\frac{\partial T}{\partial R} = \frac{1}{\alpha}\frac{\partial T}{\partial \theta}, \quad 0 \leqslant R \leqslant a \tag{1}$$

The initial and boundary conditions are:

$T = T(\theta, R) = T_o \quad$ at $\quad \theta = 0$

$\frac{\partial T(\theta,R)}{\partial R} = -\frac{h}{k}\left(T(\theta,R) - T_f\right) \quad$ at $\quad R = a$

$T(\theta,R)$ tends to a finite value at $R = 0$.

With these conditions the solution of Eq. (1) is

$$\frac{T-T_o}{T_f-T_o} = \frac{2a(H+1)}{R}\sum_{n=1}^{\infty}\frac{\sin(R\lambda_n)\, e^{-\lambda_n \alpha\theta}}{(H^2 + a^2\lambda_n^2 + H)\sin(a\lambda_n)}, \tag{2}$$

where H is a dimensionless number

$$H = ah/k - 1 \tag{3}$$

and $a\lambda_n$ are the consecutive roots of

$$\tan(a\lambda_n) = -\, a\lambda_n/H. \tag{4}$$

Mean temperature of the eddy at the end of its flight

The quantity of heat Q flowing into the eddy during its flight time ℓ/v' (ℓ is the mixing length, v' is the fluctuating velocity component normal to the general direction of flow) is

$$Q = \int_0^{\ell/v'} k(4\pi a^2)\left(\frac{\partial T(\theta,R)}{\partial R}\right)_{R=a} d\theta$$

$$= 8\pi ak(T_f - T_o)(H+1)^2\sum_{n=1}^{\infty}\frac{1 - e^{-\lambda_n^2\alpha\ell/v'}}{(H^2 + a^2\lambda_n^2 + H)\lambda_n^2\,\alpha}. \tag{5}$$

This quantity of heat is equal to the change in enthalpy of the eddy in time ℓ/v' i.e. during its temperature change from t_1 to t_1', thus

$$Q = \frac{4}{3}\pi a^3 \rho\, c_p\,(t_1' - t_1). \tag{6}$$

Equating equations (5) and (6), substituting $(T_f - T_o) = \frac{1}{2}(t_1 + t_2) - \frac{1}{2}(t_1 + t_1') = \frac{1}{2}(t_2 - t_1')$, and solving, we get

$$\frac{t_1' - t_1}{t_2 - t_1'} = 3(H+1)^2\sum_{n=1}^{\infty}\frac{1 - e^{-\lambda_n^2\alpha\ell/v'}}{a^2\lambda_n^2(H^2 + a^2\lambda_n^2 + H)}. \tag{7}$$

Calculation of diffusivity ratio

Equation (7) contains h(implicit in H), ℓ, a, and v'. None of these is known at this point. Certain transformations and assumptions will therefore be needed.

We shall first of all introduce the customary dimensionless numbers. For this purpose, a characteristic length has to be chosen with respect to the eddy. In order to be able to compare our results with those of Azer and Chao, we shall adopt their choice of half-circumference πa, though evidently there is no sound physical basis for this. With this assumption, Nusselt number of the eddy is $Nu' = \pi ah/k$.

For Reynolds number of the eddy a velocity is needed, and the logical choice is the mean between the relative velocities of fluid and eddy at layers 1 and 2. Thus, $Re' = \pi a(u_2 - u_1')/2\nu$.

Assuming the Prandtl number of the eddy to be the same as that of the surrounding fluid, we have the Stanton number of the eddy $St' = Nu'/Re'\ Pr$.

Diffusivity ratio can be shown [11] to be

$$\psi = \frac{\varepsilon_H}{\varepsilon_M} = \frac{1 + (u_1' - u_1)/(u_2 - u_1')}{1 + (t_1' - t_1)/(t_2 - t_1')} . \tag{8}$$

In this expression appears the quantity $(u_2 - u_1')$ which in our case is equal to the fluctuating velocity component in the general direction of the flow. This component can be related to the previously mentioned component v' following experimental work by Laufer [12] correlated in a simple formula by Azer and Chao [6]

$$\frac{u_2 - u_1'}{v'} = 1.81 \frac{1 - 0.64\ y/r_w}{1 - 0.36\ y/r_w} , \tag{9}$$

where r_w is the radius of the pipe.

On the other hand, the ratio of mixing length to the pipe radius was shown by Nikuradze [13] to be representable by the equation

$$\frac{\ell}{r_w} = 0.14 - 0.08\ (1 - \frac{y}{r_w})^2 - 0.06\ (1 - \frac{y}{r_w})^4 . \tag{10}$$

There seems to be no conclusive evidence pointing to the actual values for the mixing length and for the radius of the eddy. It is thought [6,14] that a value of $\ell/a = 2$ may be a reasonable choice for the ratio of these two quantities.

Using Eq. (9), Azer and Chao expressed the Reynolds number of the eddy in terms of pipe Reynolds number and Darcy friction factor f:

$$Re' = 0.766\ Re \sqrt{\frac{f}{8}}\ (1 - 0.64 \frac{y}{r_w})\left[0.14 - 0.08(1 - \frac{y}{r_w})^2 - 0.06(1 - \frac{y}{r_w})^4\right]. \tag{11}$$

The factor f could be approximated [15] by

$$f = 0.0032 + 0.221/Re^{0.237} .$$

Azer [16] has obtained for liquid metals the following theoretical relation

$$St'Pr^{1/2} = 1.06/Re'^{1/2} . \tag{12}$$

The quantities H and $\alpha\ell/v'$ can now be written:

$$H = 0.337\ Pe'^{1/2} - 1 \tag{13}$$

$$\alpha \frac{\ell}{v'} = \frac{5.69\ a^2(1 - 0.64\ y/r_w)}{Pe'\ (1 - 0.36\ y/r_w)} . \tag{14}$$

Equation (7) now becomes

$$\frac{t_1' - t_1}{t_2 - t_1'} = 0.341\ Pe' \sum_{n=1}^{\infty} \frac{1 - \exp \dfrac{- 5.69\ a^2\lambda_n^2(1 - 0.64\ y/r_w)}{Pe'(1 - 0.36\ y/r_w)}}{a^2\lambda_n^2(0.1136\ Pe' - 0.337\ Pe'^{1/2} + a^2\lambda_n^2)} . \tag{15}$$

This is the temperature function appearing in the denominator in Eq. (8). We shall now evaluate the velocity function appearing in the numerator in the same equation.

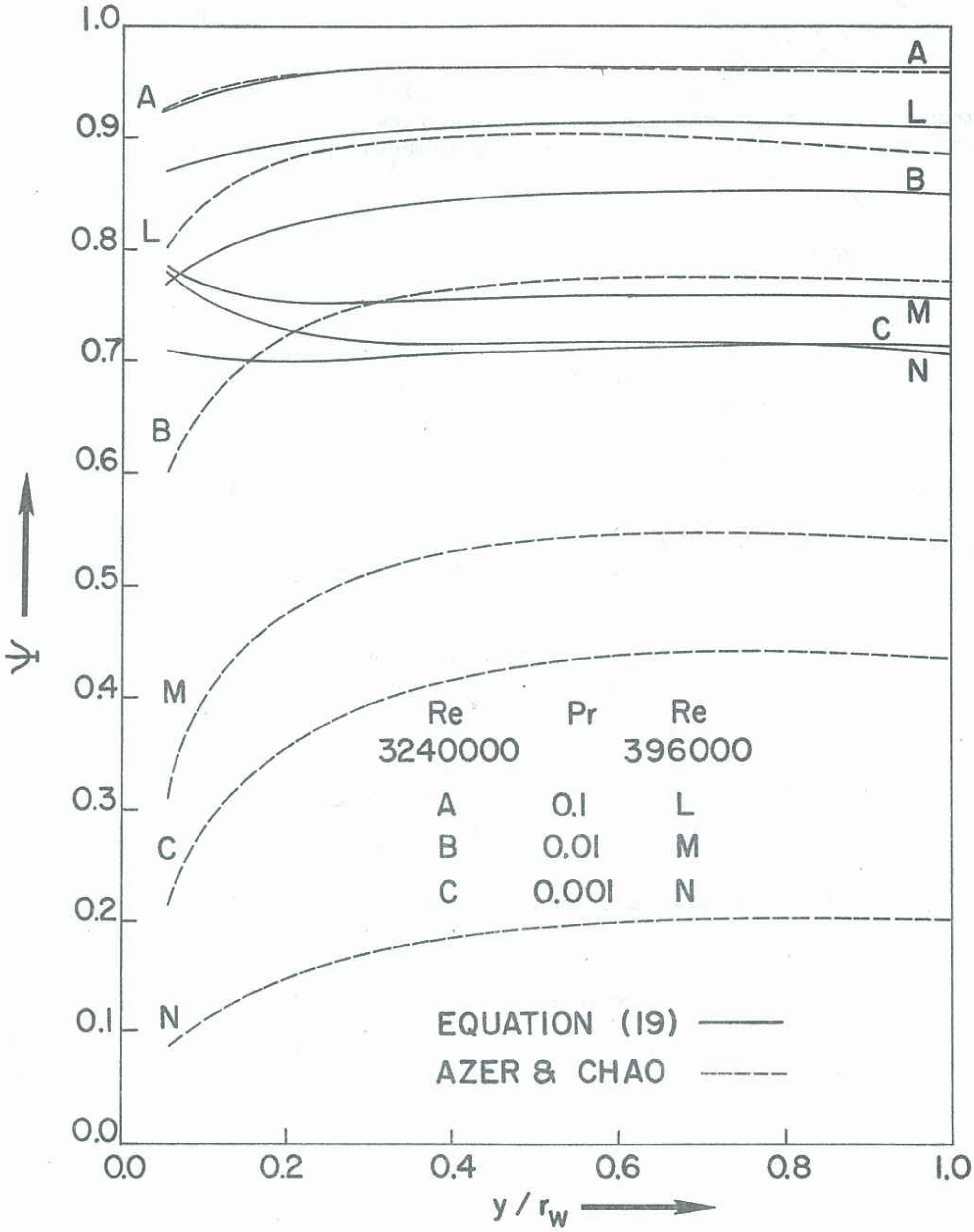
From the momentum principle

$$\rho V \frac{(u_1' - u_1)}{\ell/v'} = g_c S \tau_e , \tag{16}$$

where $S = 4\pi a^2$, $V = \frac{4}{3}\pi a^3$, and τ_e, the shear stress at the surface of the eddy is obtained from

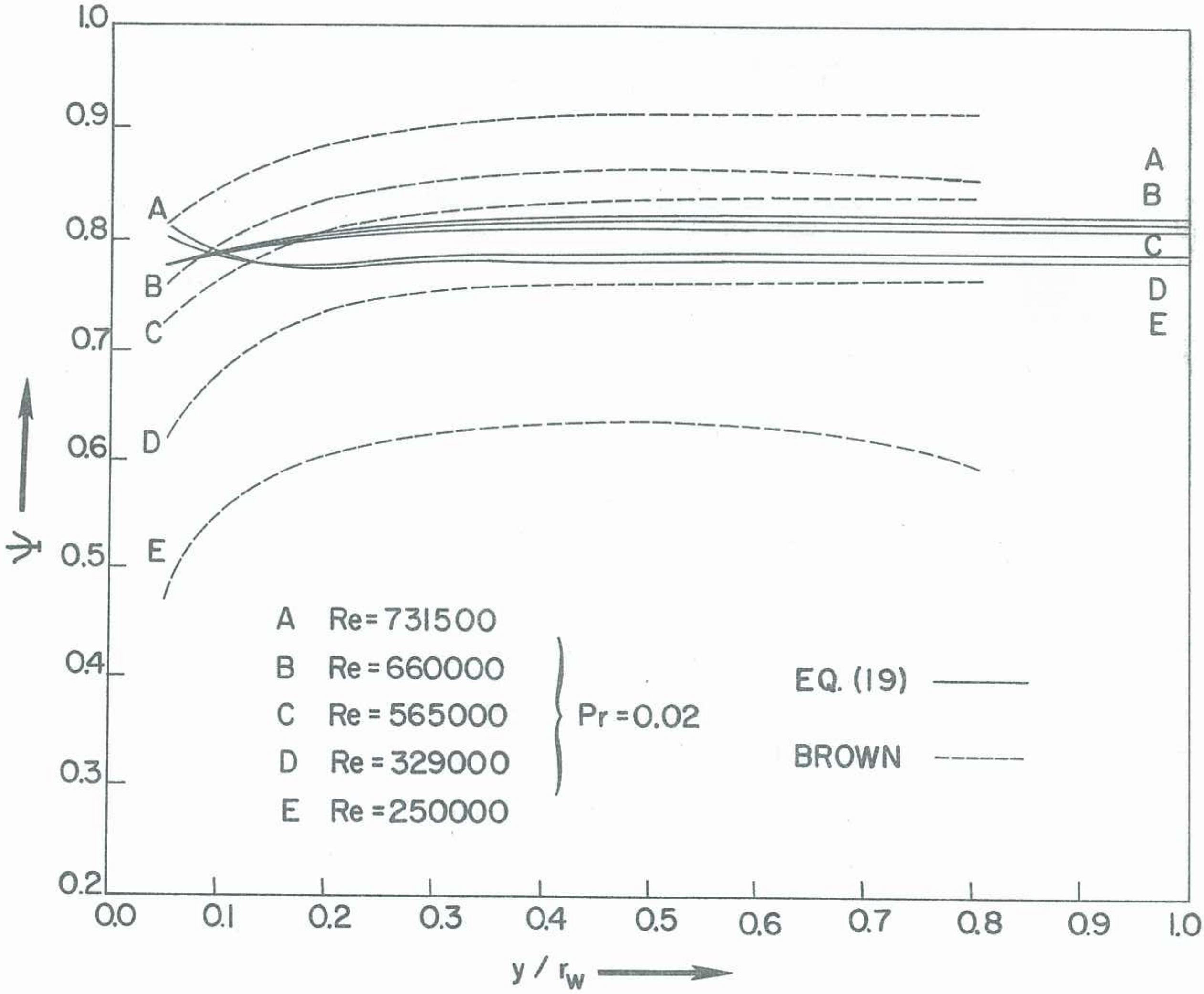
$$\frac{(0.664)8}{Re'^{1/2}} = \frac{4\tau_e g_c}{[(u_2 + u_1)/2 - (u_1 + u_1')/2]^2\ \rho/2} , \tag{17}$$

$(u_2 + u_1)/2$ and $(u_1 + u_1')/2$ being the average axial velocities of the fluid and the eddy, respectively. Substitution of these values into Eq. (16) yields



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
ψ̄
0.0
0.2
0.4
0.6
0.8
1.0
y / r_w
A
L
B
C
M
N
Re | Pr | Re
3240000 | | 396000
A | 0.1 | L
B | 0.01 | M
C | 0.001 | N
EQUATION (19)
AZER & CHAO

FIGURE 2



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
ψ
A
B
C
D
E
A Re=731500
B Re=660000
C Re=565000
D Re=329000
E Re=250000
Pr=0.02
EQ. (19)
BROWN
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
y / r_w

FIGURE 3

$$\frac{u_1' - u_1}{u_2 - u_1'} = \frac{1.803\ (1-0.64\ y/r_w)}{Re'^{1/2}\ (1-0.36\ y/r_w)} \ . \tag{18}$$

The substitution of Eqs. (15) and (18) into Eq. (8) produces

$$\psi = \frac{\varepsilon_H}{\varepsilon_M} = \frac{1 + \dfrac{1.803(1 - 0.64\ y/r_w)}{Re'^{1/2}\ (1 - 0.36\ y/r_w)}}{1+0.341\ Pe' \displaystyle\sum_{n=1}^{\infty} \frac{1-\exp \dfrac{-\ 5.69\ a^2\lambda_n^2\ (1 - 0.64\ y/r_w)}{Pe'\ (1 - 0.36\ y/r_w)}}{a^2\lambda_n^2(0.1136\ Pe' - 0.337\ Pe'^{1/2} + a^2\lambda_n^2)}} \ . \tag{19}$$

Conclusion

Figures (2) and (3) present the comparisons between values of ψ calculated from Eq. (19) and corresponding values found by Azer and Chao [6] and by Brown [8] respectively.

Our values are higher than those of Azer and Chao which accords well with the observation of Carr and Balzhiser [17] that the values of Azer and Chao are very much lower than the experimental values.

There seems to be no major disagreement between our values and those of Brown.

References

1. Martinelli, R.C., "Heat Transfer to Molten Metals", A.S.M.E., 69, 947, 1947.
2. Lyon, R.N., "Liquid Metal Heat Transfer Coefficient", Chem. Engg. Progr., 47, 75, 1951.
3. Lubarsky, B. and Kaufman, S.J., "Review of Experimental Investigations of Liquid Metal Heat Transfer", N.A.C.A. Tech. Note No. 3336, 1955.
4. Deissler, D.G., "Analysis of Fully Developed Heat Transfer at Low Peclet Number in Smooth Tubes with Application to Liquid Metals", N.A.C.A., RM ES2F05, 1952.
5. Jenkins, R., a paper in Proc. Heat Transf. Fluid Mech. Inst., pp. 147-158, Stanford University Press, Stanford, 1951.
6. Azer, N.Z. and Chao, B.T., "A Mechanism of Turbulent Heat Transfer in Liquid Metals", Int. J. Heat Mass Transfer, 1, 121, 1960.
7. Lykoudis, P.S. and Touloukian, Y.S., "Heat Transfer in Liquid Metals", Trans. A.S.M.E., 653, 1968.
8. Brown, H.E., Amstead, B.H., and Short, B.E., "Temperature and Velocity Distribution and Transfer of Heat in Liquid Metals", Trans. A.S.M.E., 79, 279, 1957.
9. Isakoff, S.E. and Drew, T.B., "General Discussion on Heat Transfer", Inst. of Mech. Engineers, London, 405, 1951.
10. Sesonske, A., Schrock, S.L., and Buyco, E.H., "Eddy Diffusivities Ratios for Mercury Flowing in a Tube", A.I. Ch.E. Chem. Engg. Progr. Symposium Series, 57, 61, 101, 1965.
11. Gupta, A.C., "The Ratio of Eddy Diffusivities of Heat and Momentum for Liquid Metals", M.Eng. Thesis, Dept. of Mech. Engg., Univ. of Ottawa, Canada, January, 1972.
12. Laufer, J., "The Structure of Turbulence in Fully Developed Pipe Flow", N.A.C.A. Tech. Note No. 2954, 1953.
13. Nikuradse, J., V.D.I. Forschungsh., vol. 356, 1932.
14. Tien, C.L., "On Jenkins Model of Eddy Diffusivities for Momentum and Heat", J. of Heat Transfer, 389, 1961.
15. Vennard, J.K., Elementary Fluid Mechanics, 3rd Edition, John Wiley and Sons, Inc., New York, Chapman and Hall Ltd., London, 201, 1954.
16. Azer, N.Z., "On the Mechanism of Turbulent Heat Transfer of Liquid Metals Flowing in Circular Conduits", Ph.D. Thesis, Dept. of Mech. Engg., Univ. of Illinois, Urbana, Illinois, 1959.
17. Carr, A.D. and Balzhiser, R.E., "Temperature Profiles and Eddy Diffusivities in Liquid Metals", Jr. Chem. Engg., 12, 53, 1967.