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A SPECIALIZED COMPUTER TECHNIQUE FOR SIMULATION OF FLOW OSCILLATION IN SUDDEN AND GRADUAL EXPANSION

by

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S U M M A R Y

The history of motion and the development of oscillatory flow has been studied by means of mathematical model. The initiation of asymmetry occurs at early stages and at a relatively low Reynold's Number. The vorticity time-dependant equation was solved using a finite differences technique. The computational procedure allowed the flow pattern to be obtained at any time step. The effect of side flow on the symmetrical flow in a sudden expansion was also simulated.

Geometry of the expansion was shown to affect the structure of the vortices. It was also possible to make a tentative comparison between mathematical model results with visual flow pattern of a physical model. The comparison was not necessary at the same Reynolds Number. However it confirms the existance of asymetry and periodicity of flow in sudden and gradual expansion.

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INTRODUCTION

Flow of a jet of fluid into a sudden or divergent conduits or open channel is often associated with assymetry and periodicity. It has been observed that flow in wide rivers and estuaries also oscillate as a result of the same mechanism. Experimental methods do not show in details, the consequences which lead to the oscillation and the bistable conditions which may occur in sudden and gradual expansion(4). The development seems to occur fast at early stages of shear flow where the fluid starts to accelerate from rest.

Macagno and Hung(3),(7) have studied the laminar eddies in two-dimensional expansion, using a computational technique for the solution of the full navier-stokes equations. They have considered symmetrical model and flow Reynolds number of 200.

Since it is the object of the study described here to demonstrate the initiation of assymetry observed in laboratory models, it was decided to consider the full section and impose no restriction on the centre-line to allow for any assymetry to develope. Also, considering a higher Reynolds Number = 1,000 based on the prescribed velocity and width of the inlet.

A gradual expansion was also studied using the same techniqué which proved that the method could be used to conduct expansion with different geometry.

Finite-differences scheme was used to describe the full equations of motion. Flow pattern was directly obtained by an X-Y plotter attached to the computer. Only through a visulization technique by photographing the surface flow pattern was it possible to make a satisfactory comparison with the mathematical model results for the sudden and gradual expansion.

EQUATIONS GOVERNING THE FLOW

The main equations governing the flow are:
The vorticity equation as derived from the basic Navier-Stokes equations of motion

$$\frac{\partial \omega}{\partial t} + \frac{\partial u\omega}{\partial x} + \frac{\partial v\omega}{\partial y} = \nu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \qquad (1)$$

and Poisson's equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \qquad (2)$$

where u, v are the velocities in the x and y direction, ν is the kinematic viscosity, ψ is the stream function and ω is the vorticity and is equal to

$$\omega = -\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad (3)$$

also

$$u = \frac{\partial \psi}{\partial y} \; ; \quad v = -\frac{\partial \psi}{\partial x} \; . \qquad (4)$$

In equation number (1), the term $\left(\frac{\partial u\omega}{\partial x} + \frac{\partial v\omega}{\partial y}\right)$ represents the transport term, $\nu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$ represents the diffusion term and $\frac{\partial \omega}{\partial t}$ relates the change of vorticity with time.

The fluid and its environment in both sudden and gradual expansion represented by finite cells of dimensions δx and δy: every mesh point was defined by indices i and

j in the x and y direction respectively as in Figs. (1) and (2).

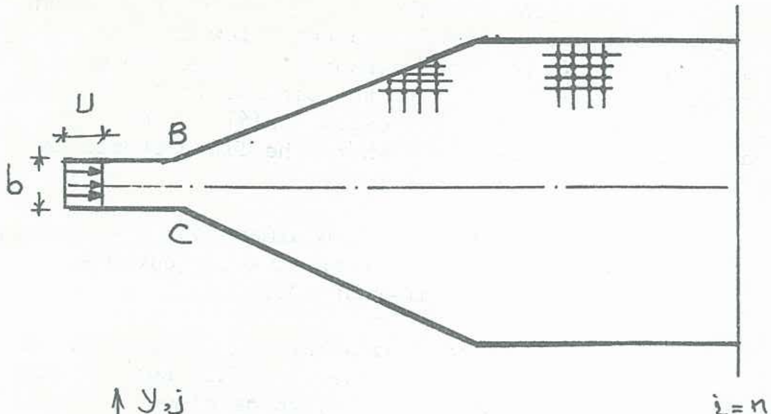
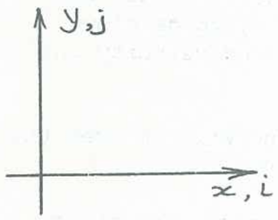
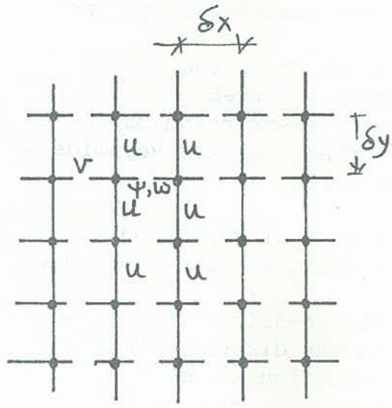


Fig. 1





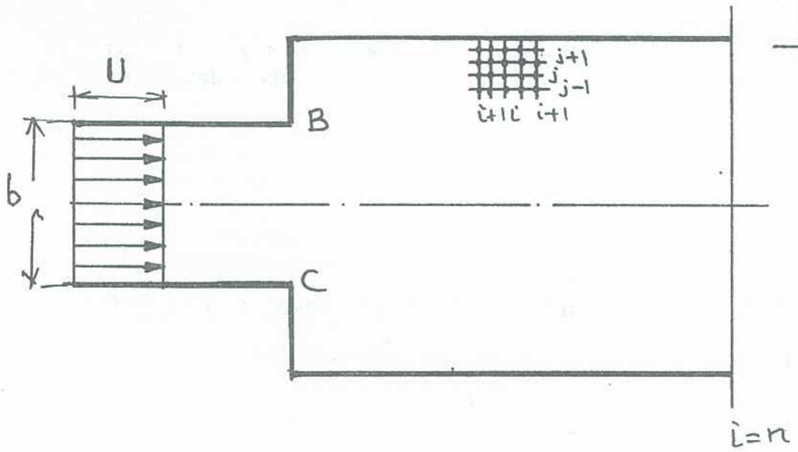


Fig. 2

FINITE DIFFERENCE APPROXIMATIONS

Assuming that the entire set of values of the variable u, v, ψ and ω were known at sometime t, a finite difference approximation for equation (1) was used to obtain a new value for the vorticity at a time $t + \delta t$. Equation (1) can be expressed in different finite difference forms. The stability analysis of the vorticity equation by Fromm(1) showed that time-centred differences are unconditionally stable for small perturbations. Different forms of expressions for Equation (1) have been used by many investigators for the solution of transient flow problems.

In this investigation, different finite differences schemes for equation (1) were tested in the case of sudden and gradual expansion. One of the favoured form which proved to be satisfactory is

$$\omega_{i,j} = \left(\frac{1}{1 + \frac{4\nu\delta t}{\delta y^2}}\right)\Big\{\omega^{t-1}_{i,j} + t\left(\frac{u\omega^{t}_{i-1,j} - u\omega^{t}_{i+1,j}}{\delta x} + \frac{v\omega^{t}_{i,j-1} - v\omega^{t}_{i,j+1}}{\delta y}\right)$$

$$+ 2\nu\delta t\left(\frac{\omega^{t}_{i+1,j} + \omega^{t}_{i-1,j} - 2\omega^{t-1}_{i,j}}{\delta x^2} + \frac{\omega^{t}_{i,j+1} + \omega^{t}_{i,j-1}}{\delta y^2}\right)$$

Also Poisson's equation (2) was expressed in finite difference form as follows:

$$\frac{1}{\delta x^2}(\psi_{i+1,j} + \psi_{i-1,j} - 2\psi_{i,j}) + \frac{1}{\delta y^2}(\psi_{i,j+1} + \psi_{i,j-1} - 2\psi_{i,j}) + \omega_{i,j} = 0$$

To obtain a new value of ψ an iterative process was applied in which a new value of the vorticity ω at a time $t + \delta t$ was used. In this process an accelerating factor was used to speed up the convergence to an acceptable criterion. The criterion of convergence was similar to that reported by Macagno and Hung[2].

INITIAL AND BOUNDARY CONDITIONS

The computation scheme was carried out over the whole domain of the conduit expansion and hence the points on the centreline were treated as any other interior points in the field. Any oscillation in the streamline was then allowed to develop. It is clear that the distribution of velocity assumed at the inlet could affect the eddy zone downstream of the expansion. A uniform distribution was imposed at the inlet. The effect of boundary layer growth was appreciable at the start of the expansion.

An initial solution in terms of velocities and vorticities was established before commencing the time advancement. The streamfunction was defined at the two boundaries by fixing Reynolds Number = $\frac{Ub}{\nu}$, where U is the mean velocity at the inlet and b = width of the inlet. Initial values for velocities and vorticities were obtained by means of Equations (3) and (4). At the beginning vorticity occured only at the solid walls and was zero everywhere else.

The non-slip condition was imposed on the wall, $u = v = 0$; thus the effect of wall friction was always included. The vorticity at the interior points was calculated by means of Equation (3). At the solid wall, the vorticity is due to the relative motion between the fluid and the wall. The non-slip condition would present an infinite vorticity value at the wall[7], but it was found practical by Macagno and Hung to use a value related to the velocities over a layer of half mesh size thickness. Any change in the velocity field can affect the vorticity values in the field and along the solid boundary. Equation (3) was used for calculating vorticity at interior points and at points on the solid boundary, but in the latter velocities at ficticious points outside the boundary were considered zero. At corner points B and C Fig. (1) and (2), vorticity was calculated taking into account flow component normal to the axis and hence eddies could develop close to the corner. Macagno and Hung preferred to assume flow from the corner to be parallel to the upstream wall, but they also assumed axial symmetry. The assumption adopted by the Author was chosen to allow full freedom for vorticities to develop across the centreline. All the outlet a continuation boundary conditions was assumed for both ψ and ω. When the eddies extended to the outlet boundary, this assumption broke down, as shown in Fig. (3) at T = 0.12. For points at a concave corner, the vorticity was calculated by Equation (3), and it was zero since the non-slip conditions of $u = v = 0$ still prevailed. Each time cycle was divided into different phases:

1. The cycle began with initial values for the variables ψ, ω, u and v given for all mesh points and the boundaries.

2. A new value for the vorticity was obtained at a time $t + \delta t$ using the vorticity equation (5).

3. The ψ field was then revised for the new time by solving Poisson's equation using an iterative method. The iteration process was considered to be satisfactory when

   $\frac{\psi^{K+L} - \psi^{K}}{\psi_o} < 0.0002$, where K is the order of the iteration and ψ_o is the streamfunction value at the boundary.

4. Equation (12) was used to obtain a value for ψ at the outlet boundary.

5. The velocity and vorticity values were deduced at all mesh points.

6. The vorticity values were calculated at the solid boundary and outlet boundary.

7. At any cycle for which the output was required, the coordinates x, y of the points of equal ψ values were calculated and fed to an x-y plotter machine which provided the flow pattern at any particular time.

RESULTS AND DISCUSSIONS

1. Sudden expansion; area ration 2 : 1

The flow in a sudden expansion of area ratio 2 : 1 and at Reynolds Number = 1,000 was first investigated. No restrictions were imposed on the centreline and any asymmetry was allowed to develop.

Flow patterns for this case at different time steps as obtained directly from the plotter are shown in Fig. (3).
Fig. (3), T = 0 represents the potential flow solution. The other figures illustrate the progressive growth of symmetrical vortices up to time T = 0.48. At first, vorticity was zero at all points except at the solid boundary. It was then transferred to the main body of the flow by viscous action. Separation occured at the convex corners and symmetrical eddies formed on each side at T = 0.32, Fig. (3). The eddy growth was more rapid than that formed by Macagno and Hung[7] who used a Reynold Numberof only 200. When the eddies became fully developed they begun to migrate downstream and there were indications that a new eddy would form in their place, but before this could happen, the primary eddies began to affect the downstream boundary and the computation was terminated.

Two models were tested; in one, flow at the convex corner was assumed parallel to the x-axis (after Macagno and Hung) while in the other, normal components were also permitted. The only difference between the two cases was tha, in the former the strength of the eddies was marginally greater and the rate of migration slightly higher, but these differences were very small.

Flow remained symmetrical about the axis in both cases, until breakdown of the down-stream condition had clearly occured.

2. Sudden expansion; Addition of side flow

The photographs shown in Fig. (4) are of flow made visible by means of aluminium powder on the surface. They are for a sudden expansion of 2 : 1 area ratio; however, they do not correspond to the same Reynolds Number as the numerical experiments reported here but they showed clearly that the flow usually starts symmetrical and then may develop into oscillatory flow or into a bistable flow.

As reported by Macagno and Hung, it was found difficult to obtain a symmetrical flow in a sudden expansion.

In photograph (1), Fig. (4) symmetrical vortices were formed on both sides. In the next photograph (2), the vortices started to elongate in a manner similar to the split eddy formed in Hung and Macagno's mathematical model for the sudden expansion. Very soon after that, one vortex on one side gained more momentum and caused the main stream to re-attach to the wall again, as shown in photograph (3). The stream was skewed and one vortex had grown bigger than the other. This was found to happen even at relatively low Reynolds Numbers in an experimental model.

This bistable flow in the experimental model could be sewitched from one wall to the other by means of a very small temporary flow from a side stream.

A mathematical model similar to the previous one for the sudden expansion case was set up to show the effect of the side flow on the normal separated flow in a sudden expansion.

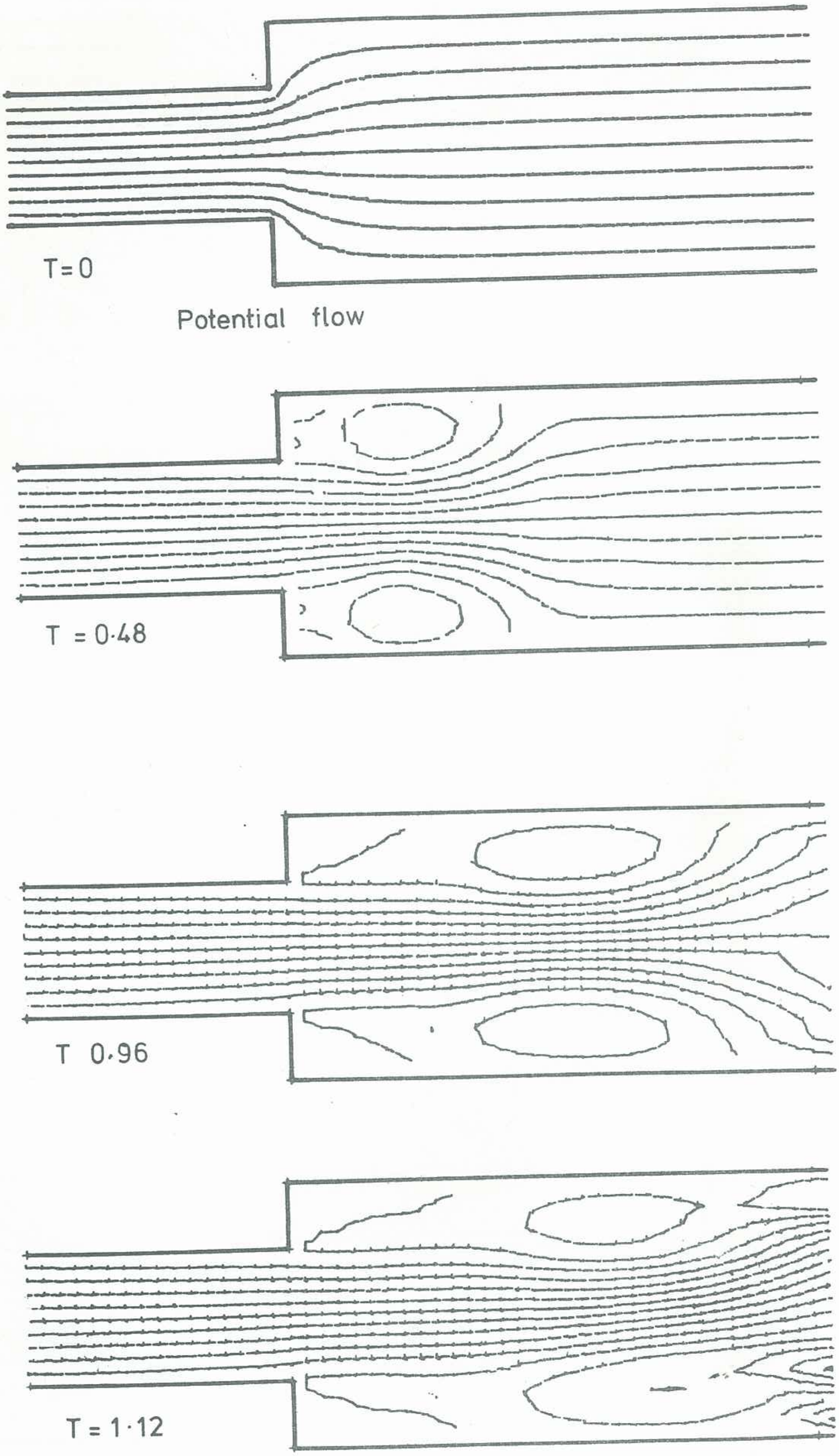


Fig. 3 SUDDEN EXPANSION.

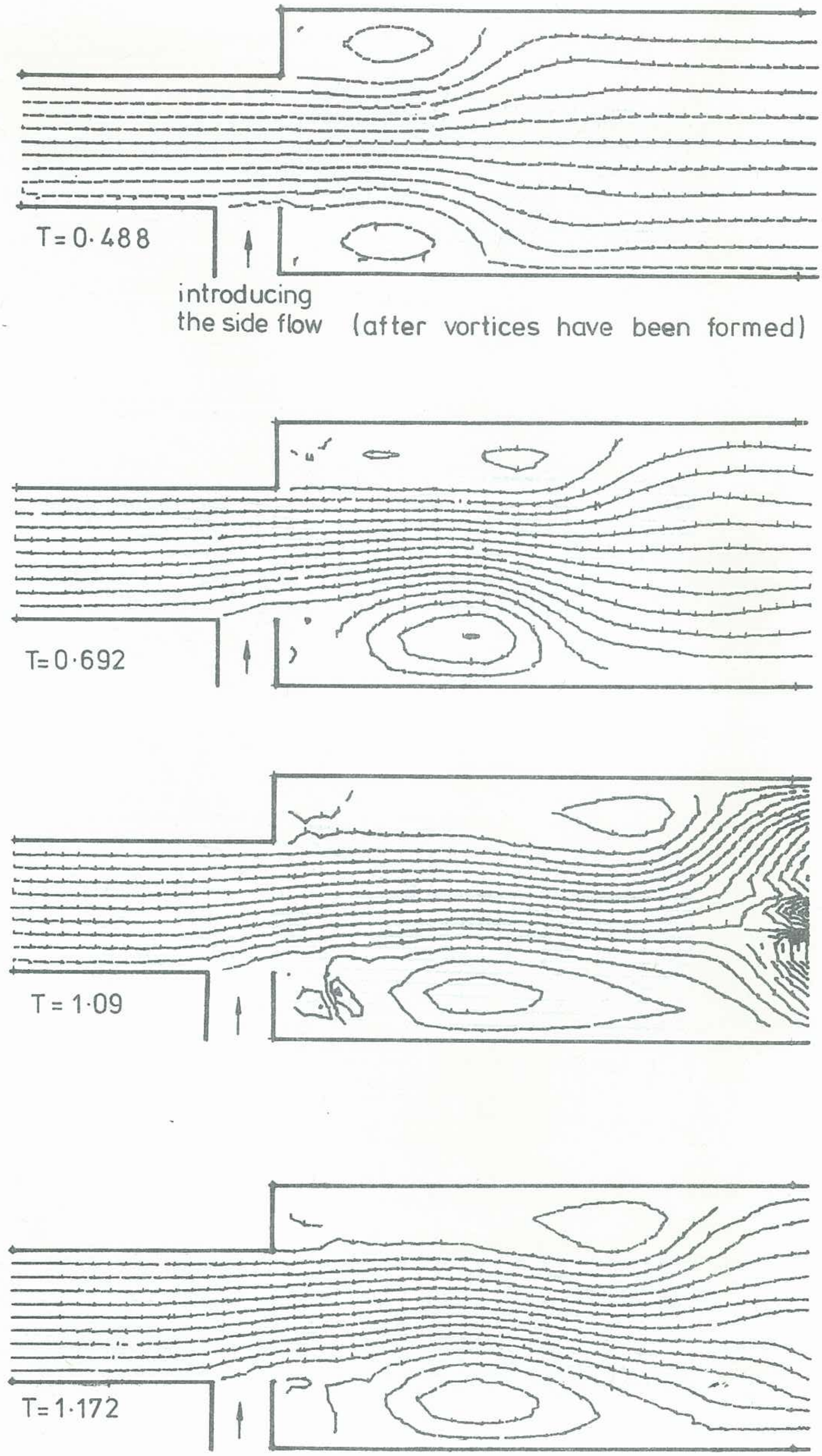


Fig. 6 EFFECT OF SIDE FLOW.

STABLE
ASYMMETRICAL FLOW
ESTABLISHED

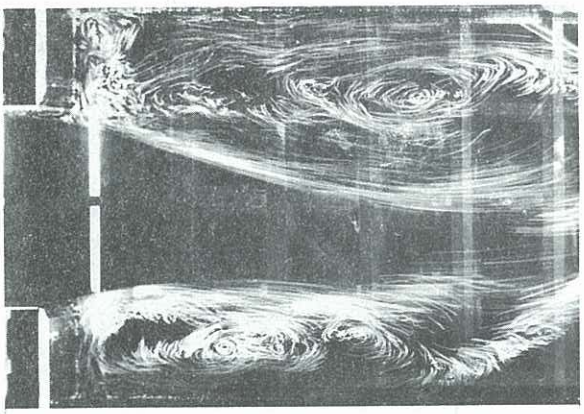


FIG. (4·3)

VORTICES
ENLARGING.

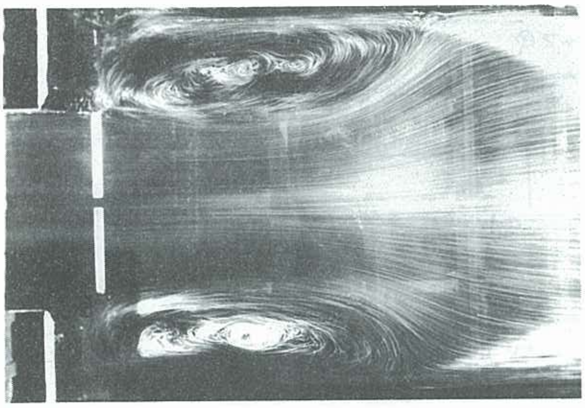


FIG. (4·2)

SYMMETRICAL
VORTICES
DEVELOPING.

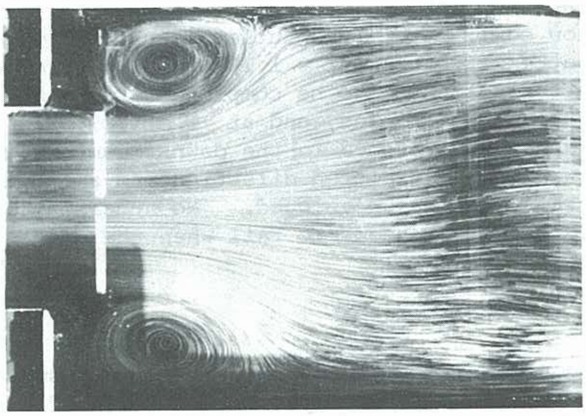


FIG. (4·1)

DEVELOPMENT OF FLOW PATTERN IN SUDDEN ENLARGEMENT

The development of the vortices started symmetrically as before in Fig. 3 and ψ values were kept constant at the solid boundaries. Reynolds Number based on the prescribed discharge and width was 1,000.

When the vortices had grown to a reasonable size but were still symmetrical, the side flow was introduced at T = 0.488 Fig. (6). This was done by changing the values at the lower boundary and consequently changing the outlet discharge from 2Q into 2.1Q as shown in Fig. (5)

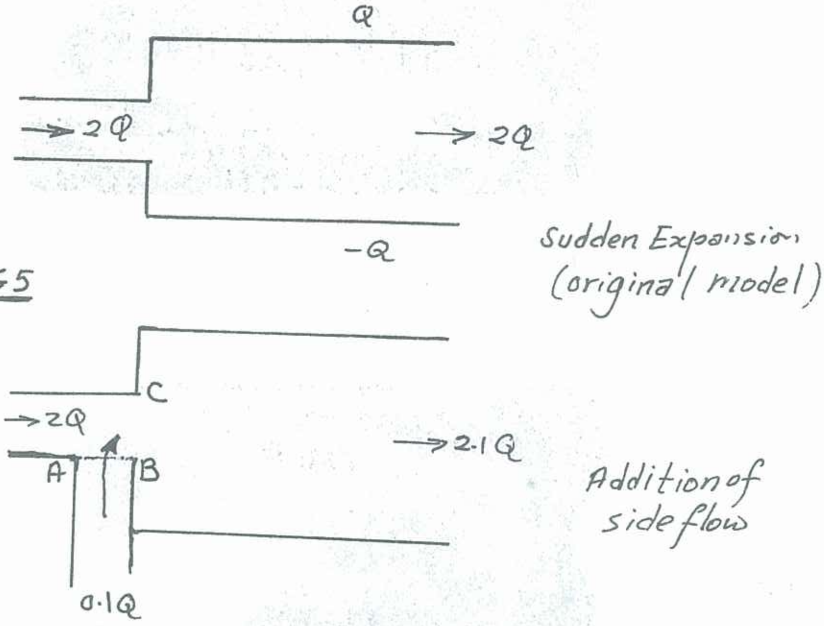


FIG5

A component of velocity normal to the flow was assumed to act across the flow at grid points on the mouth of the side channel AB from time T = 0.488 onwards.

Fig. (6), up to T = 0.48 show the development of vortices in the same order as before. At the beginning it was noticed that the momentum of the side stream had negligable effect; it showed only an extra streamline in the flow pattern Fig. (6), T = 0.488. However, through diffusion in the y-direction the momentum of the side flow had been transferred into the main stream. At T = 0.692 Fig. (6) the effect of the side flow momentum started to become significant and the stream was steered towards the other wall expecially in line with the centre of the main vortex. Before full development of asymmetrical flow could occur, the outlet conditions were invalidated by the rapidly-changing flow pattern Fig. (6), T = 1.172. A much longer outlet conduit would have been needed to allow this development to be completed. At T = 0.692, the eddy on the upper wall had split into two small ones, and at T = 1.09, Fig. (6), the small one was decayed and the other has been stretched in the downstream direction. Also at T = 1.09 the main stream was skewed in the expected direction and a similar flow pattern to that in photograph (3), Fig. (4) was developed. No numerical instability was arised and only the outlet boundary condition seems to fail when the eddy stretched to the downstream end.

GRADUAL EXPANSION

Flow in gradual expansion is very similar to that of a sudden expansion. Symmetrical flow was very difficult to attain in an experimental model of a gradual expansion. The flow is usually associated either with periodicity and oscillations or with a bi-stable state. Using the computational techniques described before, it was possible to study the accelerated flow in a divergent conduit at Reynolds Number = 1,000 and with a bigger area ratio 7 : 1. At the same time investigation on an experimental model was carried out. It was very difficult in the case of bistable flow to show by visualization technique the development of the vortices as shown in the sudden expansion model. However, the Author observed an oscillatory flow in a divergent expansion with the same area ratio, but only at a very low depth of flow as shown in photograph (7).

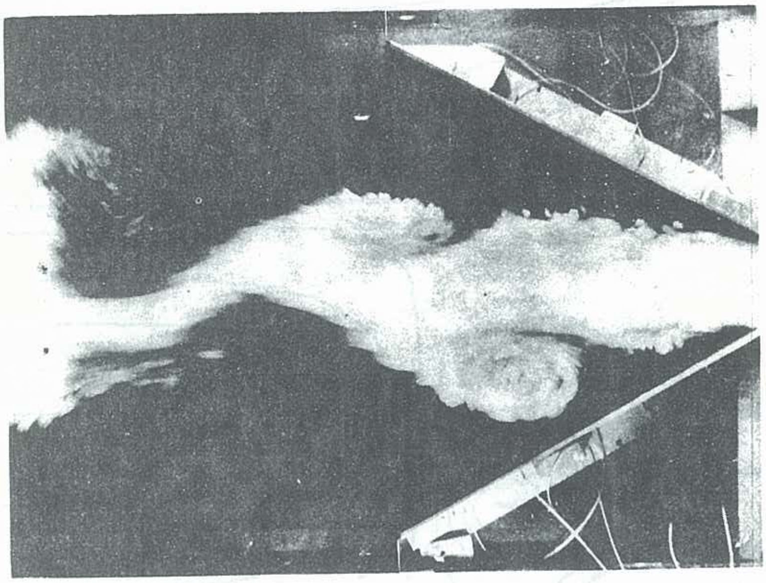


Fig. 7

It is very similar to jet flow discharging into a stagnant fluid.

In the mathematical model the same computation steps were carried out as before. A uniform flow was assumed at the inlet. The whole model was considered and no restrictions were imposed on the centreline. Starting from a potential flow solution as in Fig. (8), after advancement in time, separation occurred at the sharp corner and symmetrical vortices formed on each side. It was noticed in Fig. (8) T = 0.08 that the first two vortices had moved downstream and new vortices had formed. The two newly formed vortices grew bigger in size and intensity equally on each side.

The length of the side walls in this model was larger than that used in the sudden expansion model. As the vortices moved downstream, some asymmetry of the main stream began to develop (see Fig. (8) for period T = 0.16) and, subsequently, the main stream became oscillatory. From T = 0.20 onwards, however, the downstream boundary conditions had become invalid. Further work on the effect of downstream conditions is clearly required.

CONCLUSIONS

Laminar accelerated flow with higher Reynolds Number was studied in a sudden and gradual expansion - no restriction was imposed on the centreline. The whole model was considered and any asymmetry was allowed to develop. Flow visualization in sudden and gradual expansion showed great tendency to the flow to oscillate and sometimes became bistable.

Geometry of the expansions affect the structures of the formed vortices as has been shown in the case of sudden and gradual expansion.

It was shown in detail here the simulation of the effect of side flow on the symmetrical flow in a sudden expansion. A remarkable increase in the intensity of the eddy on one side was observed and the eddy on the other side was split into two. One grew in size and stretched downstream and the other decayed. It would be interesting in future

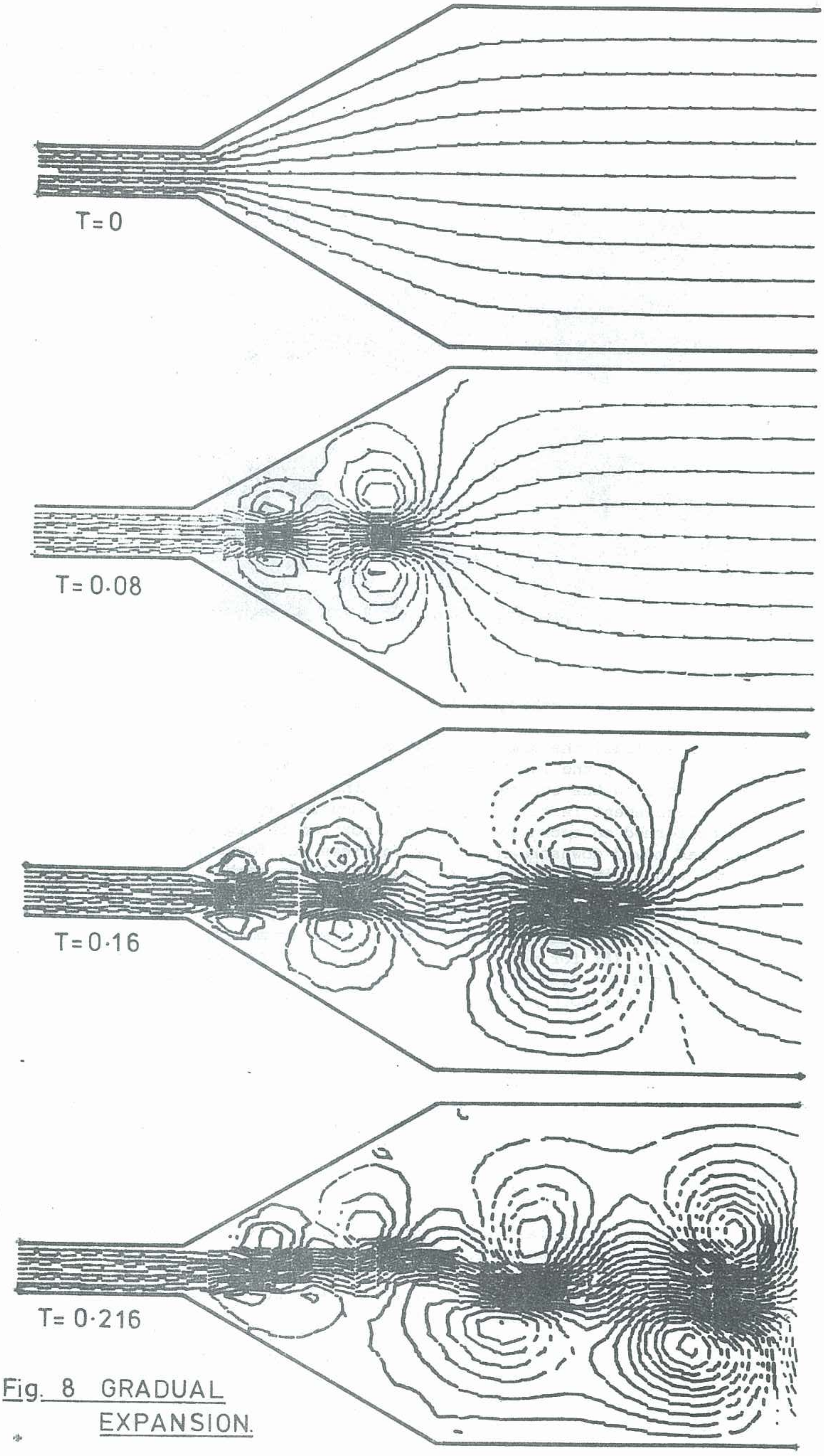


Fig. 8 GRADUAL EXPANSION.

investigations to consider different ratios of side flow.

In a gradual expansion of 7 : 1 are ratio, the formation of vortices on both sides was similar to jet flow from a finite slot into a stagnant fluid. A symmetrical row of vortices were in a stagard position and the main stream was in natural oscillation.

At higher Reynolds Number in laminar flow the eddies grow faster in size and intensity.

The continuative boundary conditions assumed by simple linear interpolation method at the outlet may only be successful for uniform flow conditions at the outlet and seems to fail whenever reversed flow was established.

It was possible to compare methematical model results with visual pattern of a physical model, even not necessarily at the same Reynolds Number. However, it confirms the existance of assymetry and periodicity of the flow in sudden and gradual expansion.

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