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DYNAMIC MODELLING TECHNIQUES FOR FLUID POWER CONTROL SYSTEMS

by

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SUMMARY

Fluid power systems, whether pneumatic or hydraulic, are used in control and automation situations where dynamic response is an important factor. To investigate dynamic response at the system design stage, mathematical models are required, the solutions of which provide the predicted response characteristics. The type of model which can be utilized is directly dependent on the facilities available to solve or simulate the model. The advent of computers has allowed the utilization of complex, potentially more accurate, models. Further, it has encouraged the development of new modelling procedures much more amenable to utilization by the control system designer.

The paper reviews the conventional modelling procedures, linear and non-linear, and the modern computer-simulation oriented power flow modelling procedures. The review and comparison is carried out on the basis of the requirements of the system designer who wishes to ensure a predetermined optimum dynamic response of his proposed system. Examples are included.

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1. INTRODUCTION

The term Fluid Power Control Systems encompasses hydraulic powered systems, pneumatic powered systems, and pneumatic logic systems (conventional and fluidic). The present paper is concerned only with powered control systems, both hydraulic and pneumatic, which can be classified as servomechanisms or power transmissions. Such systems are used :-

- where substantial forces have to be developed, such as in clamping, presses, or brake systems;
- where substantial forces are required to position substantial resisting loads. This is the servomechanism situation inferring both power amplification and positioning capability;
- where flexible powered rotary drives are required. This is the power transmission situation embracing pneumatic motor drives and hydraulic hydrostatic transmissions.

Hydraulic control systems are used where large forces, or fast stiff responses, or high power-to-weight ratios, or very precise positioning, are dominant requirements. These aspects of performance are attainable because of the high pressure used (to 30 MPa) and the high bulk modulus of the fluids used. Typical applications are aircraft controls, mobile equipment for digging and loading, and machine tool drives. Pneumatic systems, usually operated by air at around 1 MPa, are softer and slower in their power transmission actions due to the low bulk modulus of air. However, they are also much less expensive and are quite adequate in many situations of clamping, transfering, positioning, hoisting, etc.. Hydraulic and pneumatic systems have their own well defined areas of dominance; there are also areas in which either can be used, and those in which they experience rivalry from alternative electrical, mechanical, and hybrid systems.

Fluid power systems are often used where controlled motion is intended and where forces must be developed to induce the required motion in the face of resisting forces. Thus, dynamic performance and response is often a factor to be considered when specifying, testing, or designing such systems. Response should be stable, adequately but not excessively fast, smooth with zero or acceptable oscillation and overshoot. That is, concern exists not only with the fact that the driven load will go from state A to state B if the appropriate command (automatic or manual) is given, but also with the manner of the change of state. The quality of dynamic performance required of the system varies with the application. It may be:-

- vitally important, such as in the control of high speed aircraft manoeuvring surfaces;
- reasonably important, such as with manually controlled mobile digging and handling equipment;
- of lesser importance, such as in hydraulic press applications or in pneumatic clamping on production lines.

It follows that the need to predict dynamic performance at the system design stage, and the accuracy to which it needs to be predicted, range from essential and accurate to secondary and approximate. This is an important engineering consideration because it takes a lot more time, effort, and expertise to work at the high end of this scale than at the low end. There are many situations where a relatively crude but quick analysis is adequate.

Dynamic performance prediction is either:-

- analytical in nature, being carried out after the event, probably with the intention to eliminate or minimize performance deficiencies which the completed system revealed;
- synthetical in nature, being carried out before the proposed system is built, and having the intention of ensuring that the system will have the desired dynamic characteristics.

Fluid power control systems can usually be regarded as of "small scale", and so are amenable to the latter approach. Pre-construction dynamic analyses of high cost systems as used in aircraft, military, and some machine tool applications, is a routine matter. It is not so for the majority of industrial applications, where past experience coupled with afterassembly experimentation and adjustment is more common. It is proposed that the designer-analyst of fluid power systems would always be advantaged if he could ensure, as an adjunct to design, that the dynamic performance of a proposed system and application approached a predetermined quality of response. To fully achieve this, four steps need to be carried out:-

- specification of the desired response. This should be a routine engineering consideration dictated by the task rather than by the system chosen to perform the task;

- description of the dynamic potential of the proposed system by a numerical (rather than algebraic) mathematical model;

- solution of the model to provide predicted dynamic response;

- variations to system model parameters to converge its response towards the desired response - i.e. optimization of response.

To be useful, the model formed must be quite dependent on the solution procedures available. If only manual methods of solution are available, the model must be simple and probably linear. If computers are available the model can be complex and non-linear. Widespread availability of computers, and availability of general digital computer simulation and solution procedures in particular, relegates the solution and subsequent optimization of system models to a minor position. This leaves the formation of a mathematical model and the specification of its parameter values as the major obstacles to be overcome by the predictive designer-analyst.

It is on this basis, that the present paper considers conventional and newly developing mathematical model forms and procedures suitable for fluid power systems.

2. CONVENTIONAL MODELLING PROCEDURES

"Conventional" is used here only to separate the relatively recent development of power flow modelling procedures from the longer established techniques to be discussed in this Section. Power flow procedures will be introduced and discussed in Section 3.

Conventional modelling of hydraulic control systems can be of a number of forms:-

a. A set of simultaneous equations, linear and non-linear, algebraic and differential, describing the relationships between the system variables and parameters.

b. A transfer function, which is a linear differential equation relating, in time, a chosen response variable to a designated input command. All equations used to form a transfer function must be linear.

c. A vector-matrix model, which relates a chosen set of system variables to a designated set of inputs to the system, through inherent system parameters. The useful vector-matrix model is essentially a simultaneous set of transfer functions.

d. A block diagram, or other form of signal flow diagram, which is a diagrammatic representation of the system's dynamic equations. If all terms on the diagram are linear, it can be reduced to provide a transfer function or vector-matrix model. However, computer solutions (simulations) of models are readily made from (unreduced) signal flow diagrams.

All approaches to modelling require insight of the physical laws and relationships which govern the behaviour of the various devices used in the system components, together with an appreciation of the interaction of the components and the structure of the system. The successful formation of dynamic models is an art as much as a science, and experience and experiment are important assets.

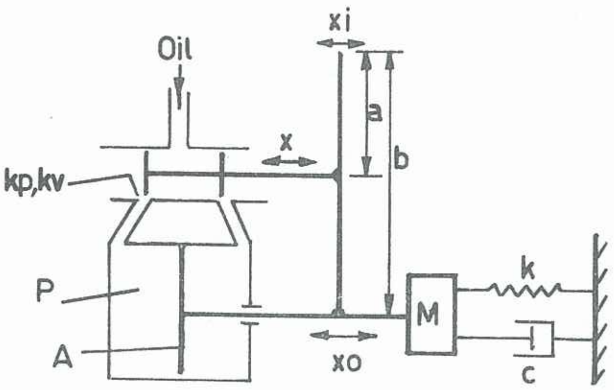
The procedure for developing a model can be summarised:-

1. Obtain a diagrammatic representation of the system to be modelled, showing clearly how the system is intended to function.

2. Select that response variable (or set of variables), study of which will provide clearest appreciation of system response.

3. Designate an input (or set of inputs) which will cause the system to respond in its real operational mode. It is not necessary to specify the form of the input at this stage, merely its physical variable and location.

4. List assumptions which are valid in the particular circumstances, and which will render modelling easier but still adequately valid. Typical assumptions which might be valid include: input drive speed constant; pressure relief valve does not lift; all driven inertias can be lumped in with the designated load inertia; all friction is viscous; pressure in the return lines is negligibly small (zero); motor-load coupling is rigid; the model is for small scale response predictions only; etc. This is an area requiring considerable thought for each modelling attempt. However, there is a lot of confirmed past experience available in the literature.



(a) Schematic of System

$$x = (b - a)/b.xi - a/b.xo$$
$$q = kv.x - kp.P$$
$$q = A.D.xo + V/\beta.D.P$$
$$M.D^2.xo = P.A - c.D.xo - k.xo$$

(b) Equation Set.

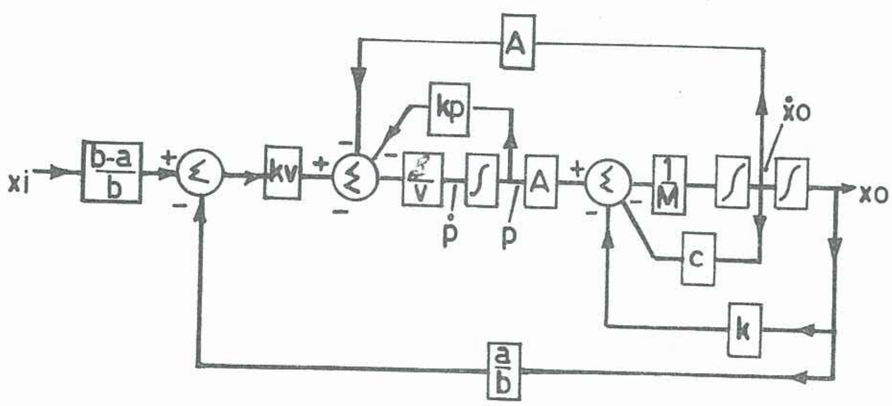
$$\frac{xo}{xi} = \frac{K}{K1.D^3 + K2.D^2 + K3.D + 1}$$

where the K's are combinations of the system coefficient.

(c) Transfer Function.

$$\begin{bmatrix} \dot{xo} \\ \ddot{xo} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k/M & -c/M & A/M \\ -a/b.\beta.kv/V & -\beta.A/V & -\beta.kp/V \end{bmatrix} \begin{bmatrix} xo \\ \dot{xo} \\ P \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ (b-a)/b.\beta.kv/V \end{bmatrix} \begin{bmatrix} xi \end{bmatrix}$$

(d) Vector Matrix Model.



e. Block Diagram

FIG 1. CONVENTIONAL MODELS

5. Imagine the system to be in equilibrium. This can be at zero value state, or at some preferred steady-state operational state. Note all initial values of system variables, if other than zero.

6. Imagine the designated input to be applied at the instant designated t = 0.

7. Form equations describing the subsequent behaviour or actions of the various physical components which comprise the system.

The relationships so formed can be expressed as a set of equations or as a signal flow diagram, as previously discussed.

It is necessary to ensure:-

a. that correct relationships are used;

b. that all relevant relationships are included.

The equations should be algebraic rather than numerical. The substitution of numbers for algebraic representation of parameters is left until the model has been developed and is ready for solution.

Fig. 1 shows the schematic of a loaded hydraulic control system, the model of the system as a set of equations, as a transfer function, as a vector matrix model, and as a block diagram. The models could have been more complex, or even simpler, according to the proposers requirements. Non-linear terms could be included in the equation set or in the block diagram.

Variations to the model forms discussed exist.

3. POWER FLOW MODELLING

Power flow modelling procedures are based on the concept that the dynamics of a system comprised of a set of interconnected sub-systems or components is dictated by the power being interchanged between the components. Power being the product of two simultaneous variables (a potential variable x a flow rate variable), this implies a dual variable approach to modelling, a concept quite distinct from conventional approaches. As far as fluid power systems are concerned, power flow modelling is developing along two lines

- the power port approach
- the power bond graph approach

which differ in terminology, symbology, and in preparation of the model for solution (usually digital simulation).

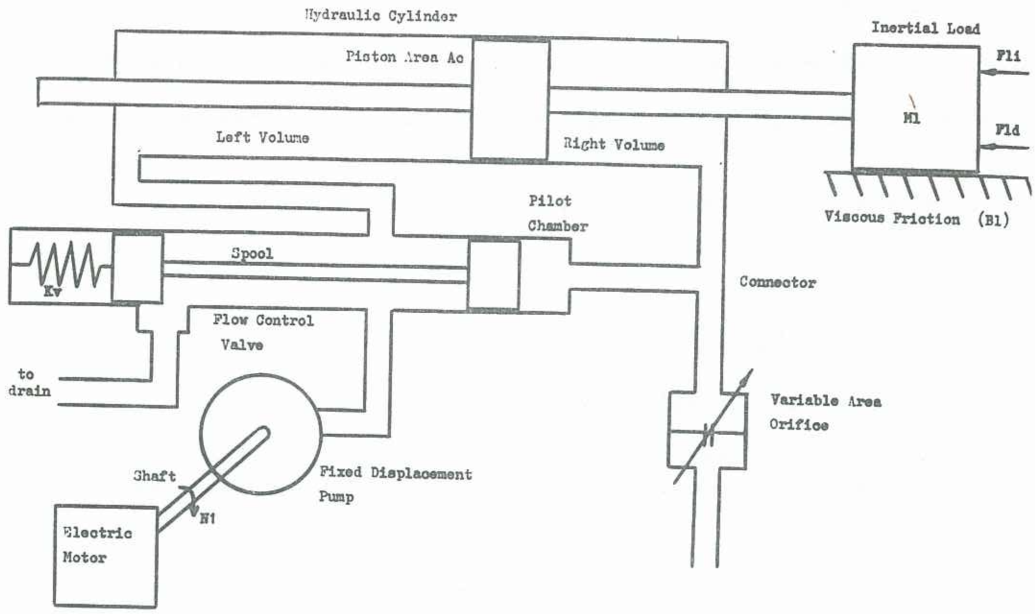
3.1 Power Port Procedure

A power port model is regarded as a set of independently formulated component models linked by paths representing power flows. The end of each power path is regarded as a port, through which power can flow into or from the component in a dynamic mode. A particular hardware component, and hence its model, can have 1 or more power ports. Each power path "contains" the appropriate dual variables of power (potential x flow rate), and thus "contains" simultaneously two system variables.

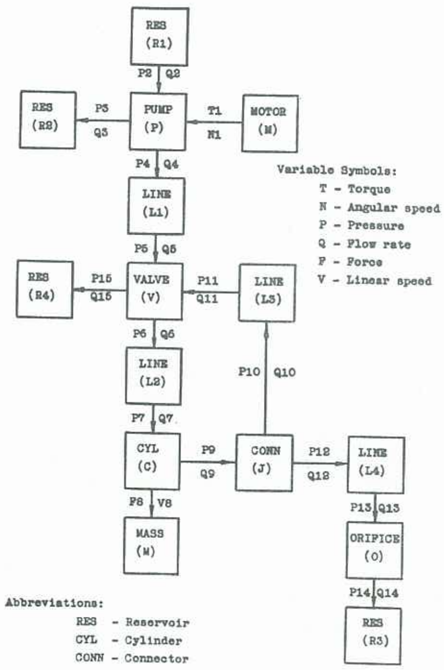
Fig. 2 illustrates a hydraulic velocity control system consisting of a hydraulic cylinder driven by a positive displacement pump via a flow control valve. This valve attempts to maintain constant pressure upstream of the control orifice placed in the discharge line, thus ensuring constant flow through the orifice and constant speed of the load. Excess flow from the pump is discharged to drain by the flow control valve. The area of the control orifice can be varied to change the desired speed of the driven load. The load driven by the system experiences a steady external resisting force Fli plus a variable force Fld.

Fig. 2(b) shows a power port schematic of the system. It can be seen that this schematic model is readily formed via consideration of how the physical system components are connected. Each physical component which affects system dynamics is represented by a block having the n ports through which it is judged that significant power flows can take place. Each such port is connected to a port in another component by a power line or path. Thus, the structure of the proposed dynamic model is simply decided, bearing a direct similarity to the structure of the proposed physical system.

Power flow directions are indicated by arrows to help in appreciating how the system



a. System Schematic



b. Power Port Schematic

$$
\begin{aligned}
\dot{N1} &= (Te-Tls-Tpt-Tvl-Tcl-Tc)/(Jm+Jp)\\
\dot{P1} &= (Bm1/V1t).(Q2-Q3-Q15-Ac.V8)\\
\dot{Pr} &= (Br/Vrt).(Ac.V8-Av.\dot{X}-Q13)\\
\ddot{X} &= (Pr.Av-Bv.\dot{X}-Kv.(X+Xs)-Cv.W.Xv.P1)/Mv\\
\ddot{Z} &= (F8-B1.\dot{Z}-Fli-Fld)/M1\\
Q2 &= Dp.N1-Cs.Dp.P1/\mu\\
Q3 &= Ld.P1/(2.0)\\
Q15 &= Cq.W.Xv\sqrt{P1}\\
F8 &= (P1-Pr).Ac\\
V8 &= \dot{Z}\\
Q13 &= Q14\\
Q14 &= Cq.At.\sqrt{Pr}\\
Te &= Kt.(No-N1)\\
Tls &= Tcm+Dm.N1\\
Tpt &= Dp.P1\\
Tvl &= Cdv.Dp.U.N1\\
Tcl &= Cf.Dp.P1\\
V1t &= Vp+V11+V12+V1c\\
V1c &= V1i+Ac.Z\\
Vrt &= Vc+Vrc+V13+V14\\
Vc &= Vci+Av.X\\
Vrc &= Vri-Ac.Z\\
N1 &= \int \dot{N1}.dt\\
P1 &= \int \dot{P1}.dt\\
Pr &= \int \dot{Pr}.dt\\
\dot{X} &= \int \ddot{X}.dt\\
X &= \int \dot{X}.dt\\
\dot{Z} &= \int \ddot{Z}.dt\\
Z &= \int \dot{Z}.dt\\
Xv &= 0 \quad , X < 0\\
&= X \quad , Xmax \geq X \geq 0\\
&= Xmax , X > Xmax
\end{aligned}
$$

c. Equation Set

FIG 2. POWER PORT MODEL

functions. The dual variables associated with each power bond can be recognized and placed on the schematic. On Fig. 2(b) P denotes pressure, Q flowrate, T torque, N speed, F force, and V linear velocity.

It remains now to complete the model by expressing the equations describing the various system variables in terms of each other, and system coefficients, and external modulating variables if they exist. Most of the relationships employed for hydraulic systems will be simple and well known. Some will be non-linear. Completion of the equation set and organization of it for solution, using an approach formalized by Unruh (1) and Young (2), provides the equations of Fig. 2(c). Basically, an equation is required for each of the power bond variables, plus an integral equation for each state variable, plus an equation for each modulating input (if any). However, simplifying assumptions usually allow some variables to be equal, reducing the number of equations. Fig. 2(c) is included only to indicate the nature of a set of power port model equations. Reference 2 contains details of the nomenclature and of the procedures used to produce Fig. 2(c) from Fig. 2(b).

3.2 Power Bond Graphs

The power bond graph technique of modelling has been developed and brought to prominence largely by Karnop (3). It is a refinement of power flow modelling, and involves its own symbology and terminology. As far as fluid power systems are concerned, the principle ideas and symbols are illustrated in Fig. 3. The symbols of column 2 are used to form the structure of the model. The structure is formed much as for the power port schematic, except that the various phenomena affecting dynamic performance are recognized as resistive (R), capacitive (C) or inertial (I) in nature. The structured power bond graph is equivalent to a dual signal (3-dimensional) block diagram. Because of the duality of variables, two summing junction symbols are required; one indicates summing of flow variables at constant potential, the other the summation of potential variables at constant flow rate. The transforming and gyrating symbols indicate transformation of power from one form to another - typically from hydraulic to mechanical. A source indicates that one of the variables of power at that point is constant, the other can vary.

The third column of Fig. 3 concerns the assigning of power flow direction and of causality to the model. This process is carried out after the model has been completed and the various relationships to be used are decided, and it prepares the model for solution. Dransfield and Barnard (4) gives more detailed consideration of power bond graphs applied to hydraulic control system modelling.

Fig. 4 shows a pump sub-system, with the power bond graph model structure with and without causality. The modular construction of the model is apparent, and is analogous with the modular construction of the system itself. One of the very real advantages of power flow modelling is that component models can be joined to form system models much as the hardware components are joined to form the system. The structure (Fig. 4(b)) is first formed; the power flow direction arrows and causal bars (Fig. 4(c)) are added later; and finally the various relationships relating the R, C, and I elements to the system power variables is decided. The relationships may be simple or complex, linear or non-linear, according to the analyst's judgement and experience. Fig. 4(c) plus the set of relevant relationships is the model in a form ready for solution, preferably by digital simulation, or ready for connection to the models of other system components.

4. CONCLUSION

Power flow modelling procedures are particularly suited to the modelling of fluid power control systems, because;

- they are modular in nature, as is the system itself;
- they allow ready coupling of component models to form system models;
- model formation is reasonably formal;
- the model can be developed in a formal way to a stage where it is ready for solution;
- they allow non-linear relationships to be included;
- they allow re-use of component models in other systems;

to a degree well beyond the capability of conventional modelling procedures.

Power bond graph techniques extend the formality of model development to a level which should bring prediction of dynamic response within the capability of increasing numbers of fluid power control system designers.

5. REFERENCES

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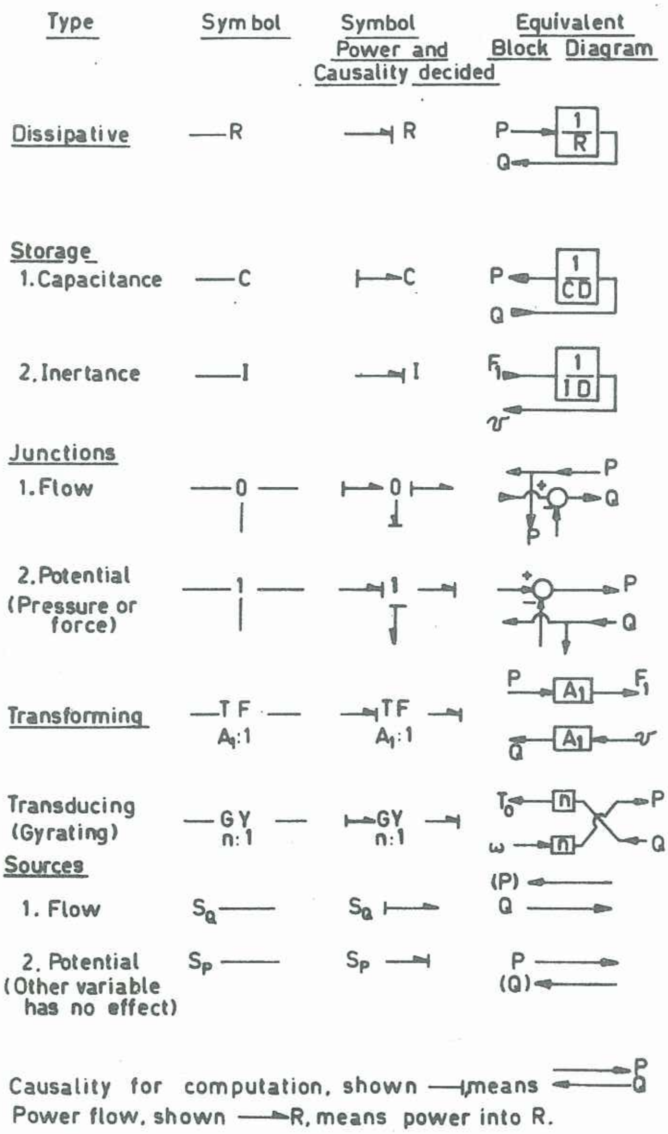
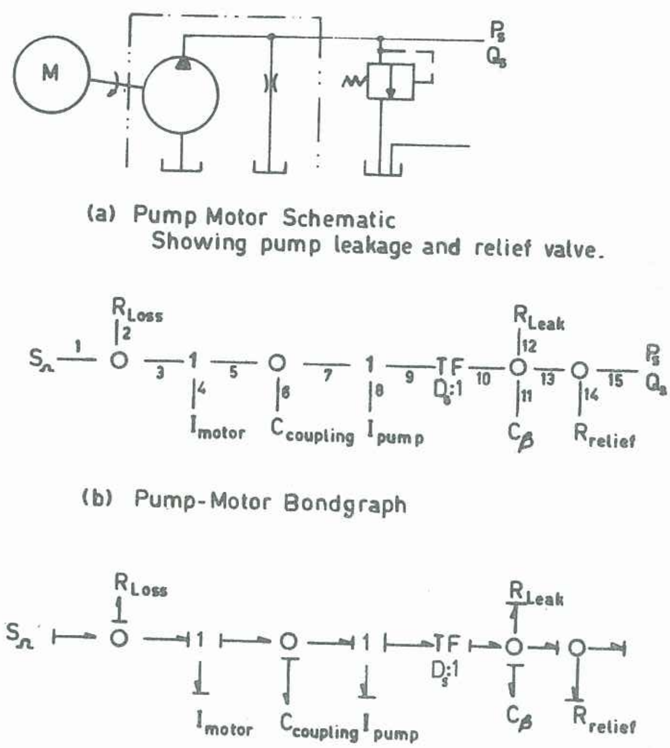


FIG 3. POWER BOND GRAPH SYMBOLS



(a) Pump Motor Schematic Showing pump leakage and relief valve.

(b) Pump-Motor Bondgraph

(c) Bondgraph with causality added.

FIG 4. PUMP SYSTEM