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CONVECTIVE MOTIONS AND RESULTING ENTRAINMENT IN A TWO LAYERED FLUID SYSTEM HEATED FROM BELOW

by

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SUMMARY

An interface with a stable temperature gradient is stirred by large Rayleigh Number convection and the entrainment at the interface is measured. It is found that for a fixed heat flux (Q) causing convection and a given temperature difference across the interface (ΔT_i) then the rate of entrainment (V_e) is almost independent of the distance of the interface from the source of heat and indeed is given by

$$V_e = k \frac{Q}{\Delta T_i}$$

It is shown that this is consistent with the results of the stirred grid experiment if there is no change in the properties of the thermal turbulence after leaving the boundary layer at the source of heat.

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INTRODUCTION

A great deal of work has been done on the erosion of a stable sharp interface by turbulence induced by an oscillating grid by Turner (7). In this case the sharp interface moves away from the grid but remains sharp and the rate at which the non turbulent fluid is entrained into the turbulent fluid is defined as the entrainment velocity V_e. Turner (7) has shown that for an extremely wide range of V_e and for a stable interface produced by a temperature difference that

$$\frac{V_e}{V_s} \propto \left[\frac{\alpha\, g\, \Delta T_i\, Z_s}{V_s^2}\right] \tag{1}$$

where V_s is the r.m.s. value of the horizontal component of the turbulent velocity
Z_s is the integral length scale
ρ is the density of the fluid
α coefficient of volumetric expansion
$\Delta\rho$ is the density difference across the interface
ΔT_i interface temperature difference
and g is the acceleration of gravity.

In the case of grid stirred turbulence, Z_s and V_s are functions of the distance from the grid. For the case of thermal turbulence we would expect a similar relationship if the properties of this type of turbulence (V_s and Z_s) were known at the interface and at this stage it is worth discussing the generation of thermal turbulence. The best experimental evidence suggests that thermal turbulence is caused by thermals or plumes which leave the horizontal heated surface and move in the ambient fluid above this surface. The generation of thermals is thought to be the result of an instability in the conduction layer adjacent to the heated surface. A phenomenalogical theory which explains the generation of thermals has been produced by Howard (4) and the experimental evidence produced by Sparrow et al (5) suggests that the theory is reasonable. According to this model, thermals are produced by a periodic process, each period of which consists of a conductive phase followed by a break-off and mixing phase. At the beginning of the conductive phase, the fluid adjacent to the plate is envisioned as having a uniform temperature which is different from that of the plate. As a result, a temperature front moves away from the plate into the fluid. When the thickness of the conduction layer contained between the moving front and the plate surface is such that the corresponding Rayleigh number exceeds a critical value, the layer becomes unstable and breaks up, thereby producing a thermal. The mixing and agitation associated with the break-up of the conduction layer restores the fluid adjacent to the plate to a uniform temperature, and the entire process begins again.

It is apparent therefore that the r.m.s. velocity scale and the integral length scale of the turbulence would be expected to be functions of the properties of the layer next to the boundary through which the thermals are being generated and the distance from this boundary. Townsend (6) shows that the characteristic velocity and length scale appropriate to the unstable boundary layer are

$$V_s = (\alpha g K Q)^{\frac{1}{4}}, \quad Z_s = \left[\frac{K^3}{\alpha g Q}\right]^{\frac{1}{4}}$$

where K = thermometric diffusivity
Q = heat flux through the plate

Further Deardorff and Willis (2) showed that once away from the boundary the r.m.s. of the velocity fluctuations is independent of the depth. If it is also assumed that the length scale remains independent of the depth then substituting for V_s and Z_s we get for Equation 1

$$V_e\, \Delta T_i = k\, Q\,. \tag{2}$$

It is important to note that this equation would only be applicable if the conditions were slowly varying and if unsteady effects were negligible. Now following Betts (1) the implications of this equation can be explored. We have

Heat in system at time t

$$= h.T_a - \Delta T_i.d$$

where h is the total depth of the fluid
T_a is ambient hot layer temperature
T_c is mixed cold layer temperature
ΔT_i is interface temperature difference $= T_a - T_c$
d is height of the interface

at time $t + \Delta t$ the heat in the system is

$$= h.T_a - (\Delta T_i.d + \frac{\partial}{\partial t}(\Delta T_i.d)\,\Delta t)$$

∴ Heat gain

$$Q.\Delta t = -\frac{\partial}{\partial t}(\Delta T_i.d)\,\Delta t$$

$$\therefore \quad Q = -d.\partial\left(\frac{\Delta T_i}{\partial t}\right) - \Delta T_i \cdot \frac{\partial d}{\partial t}$$

Now $\frac{\partial d}{\partial t} = V_e$ and we have $V_e.\Delta T_i = k.Q$

∴ eliminating Q we have

$$\frac{V_e.\Delta T_i}{k} = -d.\partial\left(\frac{\Delta T_i}{\partial t}\right) - \Delta T_i.V_e$$

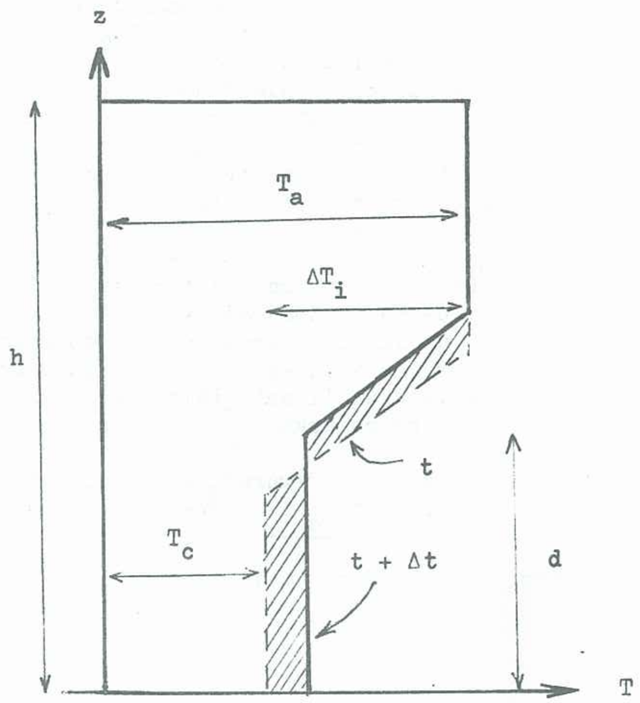
Hence rearranging

$$V_e.\Delta T_i.\frac{(1+k)}{k} = -d.\partial\left(\frac{\Delta T_i}{\partial t}\right)$$

$$\therefore \quad \frac{1}{d} \cdot \frac{\partial d}{\partial t} = \frac{-k}{1+k} \cdot \frac{1}{\Delta T_i} \cdot \frac{\partial(\Delta T_i)}{\partial t}$$

Integrating

$$\log_e \frac{d}{d_o} = \frac{-k}{1+k} \cdot \log_e \frac{\Delta T_i}{\Delta T_o}$$



Thus if in an experiment we plot d versus t on log scales the value of k may be deduced.

EXPERIMENTS

The experimental apparatus illustrated in Figure 1 consists of a tank 30cm square with a height of 60cm. A 0.635cm brass plate at the bottom of the tank was heated with circulating hot water. The heated water controlled by a thermistor based circuit was pumped from a tank into 5 perforated inlet pipes and by this means an even temperature distribution was obtained beneath the plate. Two reference thermometers were set at 8 and 41cm above the plate and another was set in a heating chamber below the brass plate. For the main temperature recording however chrome-alumel thermocouples were used. These were set in the heated chamber in the brass plate and one on the end of a probe which would be lowered or raised vertically. Early tests showed that variations of the mean temperature in the horizontal direction were insignificant and thus a single probe gave sufficiently accurate vertical temperature profiles. To start an experiment the tank was filled with hot ambient fluid and the colder water was allowed to flow under this hot water from a gate at one side of the tank when the tank was filled with ambient hot water and the lower cold water, heating of the bottom plate started. Measurements were made of temperature at 1cm intervals through the tank and a plot of temperature versus time for given depths was obtained. From this graph the vertical temperature profiles at fixed times were read off and plotted (Figure 2). The constant shape of the depth contours on the temperature versus time graph made this possible with minimum errors.

Some preliminary measurements of temperature versus time were also made with fixed thermocouples at various levels in the tank and examples of these are shown in Figure 3. Figure 3(d) shows the trace from a thermocouple just next to the plate and this trace may be interpreted as showing the slow build up in temperature followed by a rapid break up of the sublayer. Figure 3(c) shows the trace of a thermocouple in the convection layer and shows the temperature change caused by the occasional passage of a thermal. Figure 3(b) is the trace of a thermocouple at the interface level and the rapid drop in temperature caused by a cold dome lifted above the interface by the presence of a thermal and then receding. Finally the wave like motion caused by the localized fluctuations of the diffusion zone above the interface can be noted on the thermocouple placed just above the interface.

It remains at this stage to define the interface in a more specific manner. A simple definition is the point of inflexion on the temperature profile (see Figure 4) and this point is more easily determined if the profiles are plotted with the temperature in a non dimensional form. Once the interface value has been determined then the depth of the interface may be plotted against interface temperature on log-log paper giving us the value for each experiment of $\frac{k}{1+k}$. Before discussing this however the similarity of the temperature profiles through the interface should be noted. Indeed the experimental evidence suggests that the thickness of the interface is almost a constant throughout the experiment and all that changes during the experiment as the interface moves upwards is the temperature difference across the interface (Figure 5). Thus the thickness of the interface is constant and does not depend on the distance from the plate or on the heat flux from the plate. It again must be emphasised that the above statements are appropriate only after an initial period. i.e. When the layers are initially set up then there is a period before the heating commences when molecular diffusion is the dominant mechanism of change and it is only after heating for some time that the temperature distribution set up by this diffusion is changed by entrainment to the distribution illustrated in Figure 4. At this stage the run is considered to have commenced. In some runs the cold water became sufficiently warm that the interface ceased to entrain and at this stage the temperature beneath the plate was increased until entrainment again started. After a sufficient time for the profile to readjust the remarks re the interface thickness would again apply.

The values of k from Figure 6 are listed in Table (1) below and values deduced directly from the relationship $k = \frac{V_e \Delta T}{Q}$ are also included. The consistency of the values deduced from the two different methods is satisfactory.

Run	log d_i v log ΔT_i graph		$k = \frac{V_e \Delta T_i}{Q}$	
	k_L (d_i<18.8cm)	k_u (d_i>18.8cm)	k_L (d_i<18.8cm)	k_u (d_i>18.8 cm)
A	0.46	0.29	0.44	0.29
B	0.44	0.26	0.40	0.27
C	0.38	0.27		
D	*	0.34		
E	0.31	*		
F	0.46	*		
G	0.37	*	0.42	
H	0.31	*		
I	0.45	0.36	0.43	
J	0.43	*		
K	*	0.30		
L	*	0.32		
mean k	0.40	0.31		
standard deviation	0.06	0.03		

Table 1 - Values of the entrainment constant $k = \frac{V_e \Delta T_i}{Q}$ for two layered system.

CONCLUSION

For an interface stirred by thermal turbulence it appears that over certain depth ranges $V_e \Delta T_i = k Q$. This is consistent with the stirred grid experiments and the work on convection above heated grids. For the particular experiments carried out k had values of 0.40 for depths from the heated plate to the interface of less than 19cms and 0.30 for depths greater than 19cms. Further research is needed to explain this apparent transition. The most important feature of this work is, however, that the transport of heat and energy is independent of depth. It is suspected in the present experiment that the total motion consists of evenly distributed columns of rising thermals which rise to the top of the layer without losing an appreciable amount of heat. The heat is distributed throughout the fluid by the downswards flow between the rising columns. Further work will be carried out in the near future to check the validity of this hypothesis.

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6. Townsend, A.A. (1959) "Temperature fluctuations over a heated horizontal surface" J. Fluid Mech. 5.

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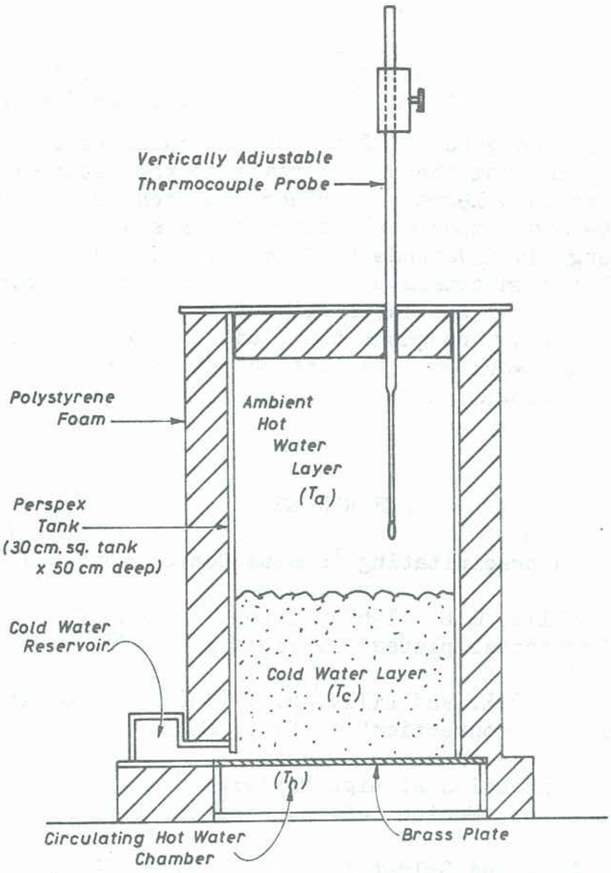

Fig.1: EXPERIMENTAL APPARATUS

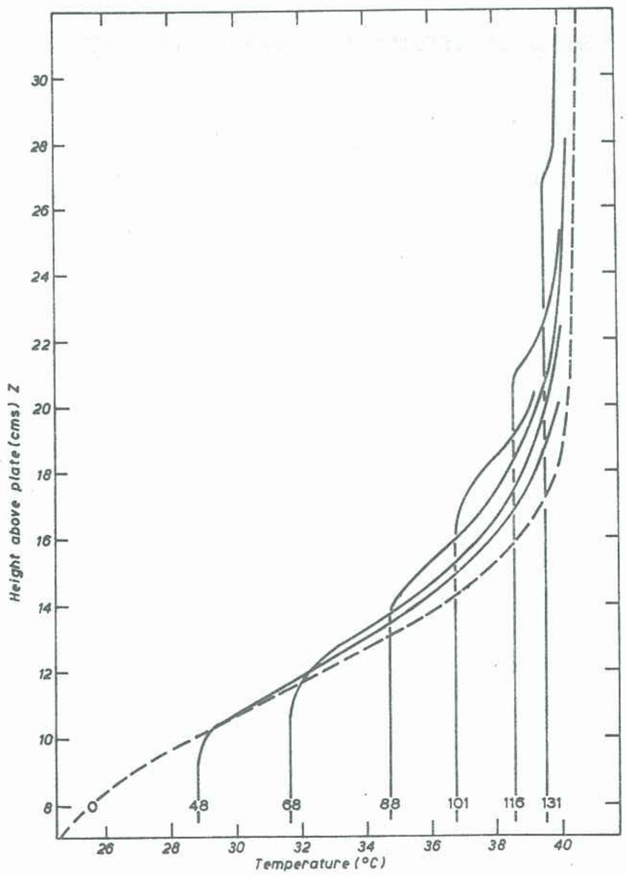

Fig. 2: VERTICAL TEMPERATURE PROFILES—RUN A (Profile labels give time in minutes)

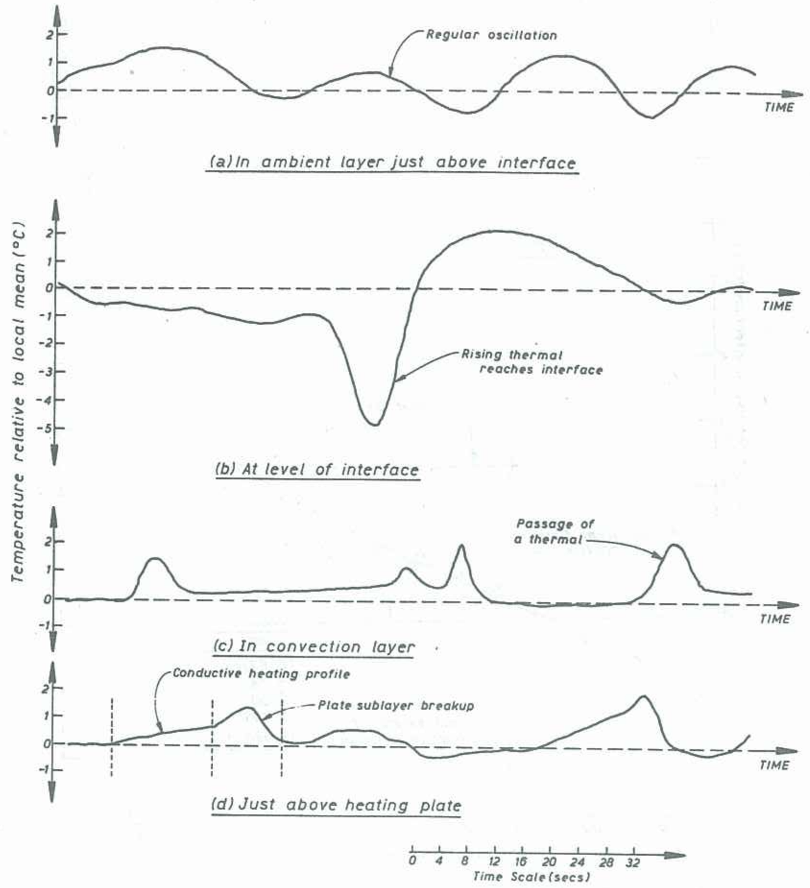

Fig.3: EXAMPLES OF TEMPERATURE PROBE TRACES FOR VARIOUS LEVELS

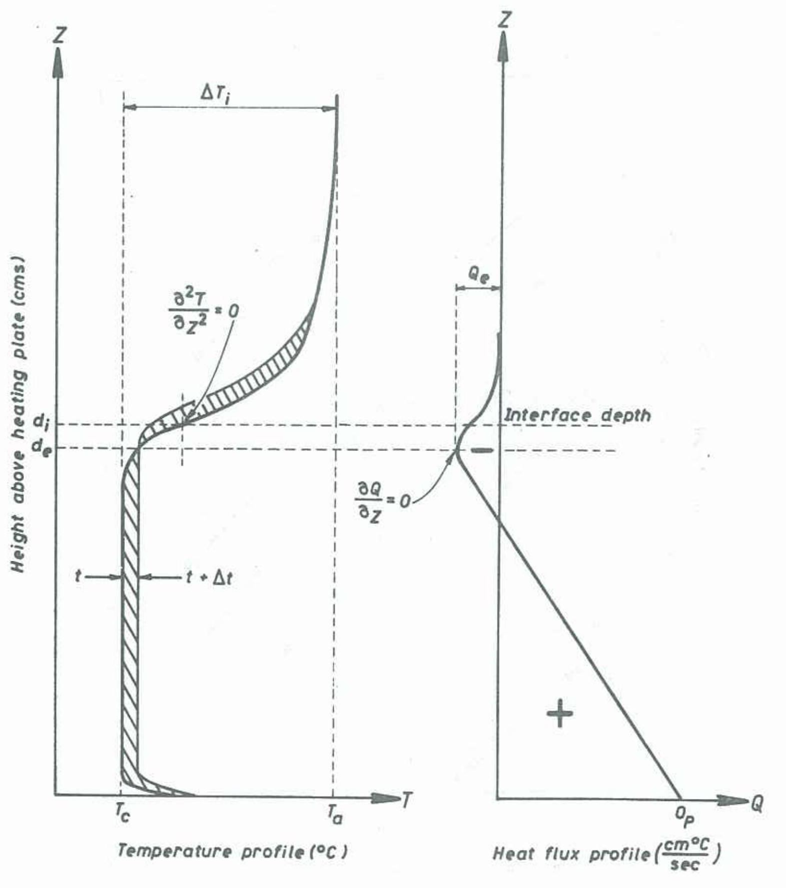

Fig.4: VERTICAL DISTRIBUTIONS OF TEMPERATURE & HEAT FLUX FOR THE CONVECTIVE ENTRAINMENT SYSTEM

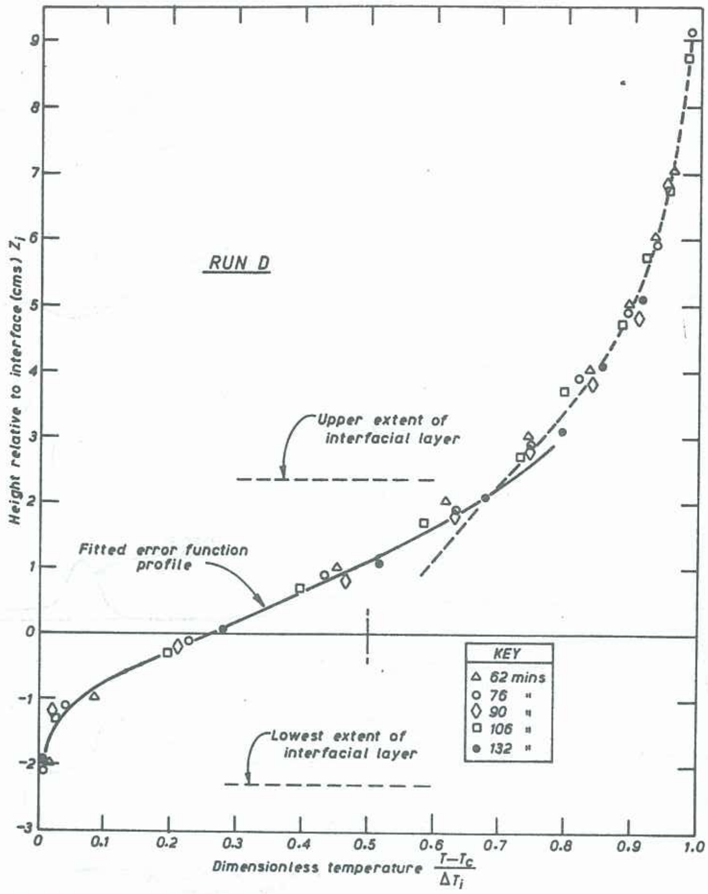

Fig.5: DIMENSIONLESS TEMPERATURE PROFILES PLOTTED WITH THE ORIGIN AT THE INTERFACE

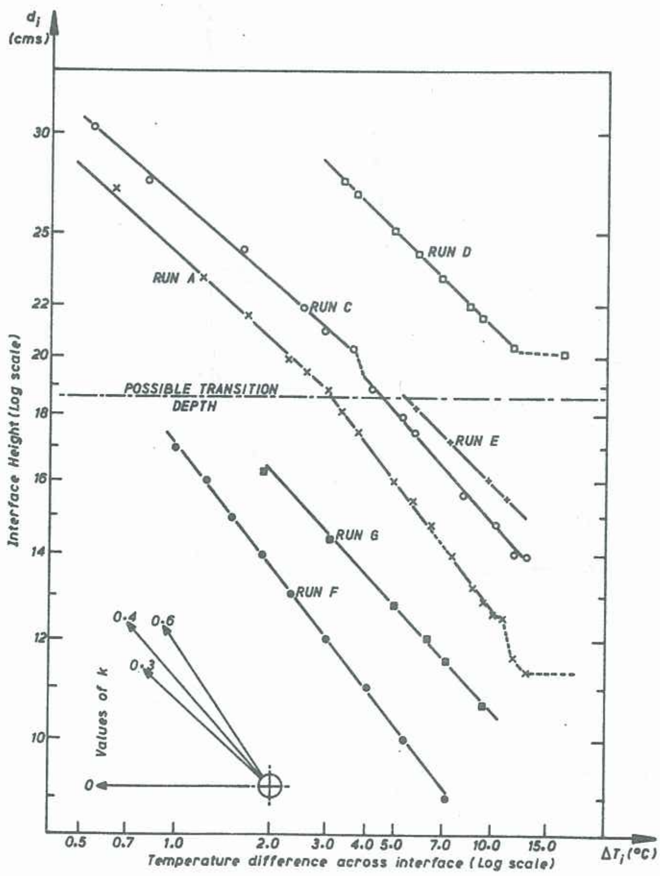

Fig. 6: GRAPH SHOWING RELATIONSHIP BETWEEN INTERFACE DEPTH & INTERFACE TEMPERATURE DIFFERENCE