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REAERATION COEFFICIENT IN STREAMS AND RIVERS

by

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SUMMARY

Number of formulae have been proposed to estimate the value of reaeration coefficient in streams and rivers. One such equation is the Thackston-Krenkel formula. Although based on the fluid flow characteristics, these equations are unsatisfactory for practical use as they presume two dimensional flow conditions and disregard the width of the stream as a variable. Another deficiency of the equations is that they do not consider the surface turbulence due to waves. A modified equation is proposed in this paper which is felt to be easier to use and more significant.

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REAERATION COEFFICIENT IN STREAMS & RIVERS

Glossary of Terms

k_2 = Reaeration coefficient (sec^{-1}) at 20°C base 10.

h = average depth of flow = $\frac{\text{area of flow}}{\text{width of flow}}$

$\bar{u}$ = mean velocity

Dm, D_L = mixing coefficients, $\left(\frac{L^2}{T}\right)$

S = average slope of energy line

F = Froude Number

u_* = shear velocity $= \sqrt{gRS} \simeq \sqrt{ghS}$

(R is hydraulic radius)

Q = discharge

C, n = constants

Introduction

The capacity of the natural streams to assimilate liquid waste within safe limits depends upon its reaeration characteristics. It is, therefore, hardly surprising that considerable amount of environmental research is directed to assess the reaeration capacity of the streams. O'Connor and Dobbins[1] were the first to attempt a correlation of the reaeration coefficient, as defined by the Streeter-Phelps equation, with the measurable hydraulic parameters of streams. Their attempt was based on a mathematical model of a mass transfer mechanism. In the subsequent years, a number of formulae were developed along similar lines.

One object of this paper is to review the different formulae and to understand their limitations. Another object is to propose still another formula which is relatively easier to use in practice and to offer suggestions for its practical use.

Review of Formulae

Some of the formulae, which have been proposed during the last twentyfive years are listed below. The list is by no means very comprehensive although it is fairly representative.

In all, ten formulae together with the names of the researchers are given here, as far as possible in chronological order.

O'Conner Dobbins[1] ..... $k_2 = \frac{\sqrt{Dm\,\bar{u}}}{2.3\ h^{3/2}}$ ...(1)

Krenkel-Orlob[2] ..... $k_2 = 4.3 \times 10^{-5}\ D_L^{-1.15} h^{-1.92}$ ...(2)

Churchill-Elmore-Buckingham[3] ..... $k_2 = \frac{5\ \bar{u}}{h^{5/3}}$ ...(3)

(k_2 in $days^{-1}$; FPS units)

Thackston-Krenkel[4] ..... $k_2 = 0.000125\ (1+\sqrt{F})\frac{u_*}{h}$ ...(4)

Thackston-Krenkel-Parker[5] ..... $k_2 = CQ^n$ ...(5)

Parkhurst-Pomeroy[6] ..... $k_2 \propto \frac{(S\bar{u})^{3/8}}{h}$ ...(6)

Owen *et al*[7] ..... $k_2 = 0.09\ \bar{u}^{\,0.73}\ h^{-0.75}$ ...(7)

(k_2 in $hours^{-1}$, FPS units)

Isaacs and Gaudy[8] ..... $k_2 = \frac{0.29\,\bar{u}}{\sqrt{h}}$ .... (8)

Shastry et al[9] ..... $k_2 = 0.00026\,F^{1.3}\left(\frac{D_L}{uh}\right)^{1.14}$ .... (9)
(indices rounded by author)

Lam-Lau[10] ..... $k_2 = 0.0126\,\frac{\bar{u}}{h}\left(\frac{u_*}{u}\right)^3$ ....(10)

Comments

i) All of these equations are based on either laboratory data or the actual stream measurements. In some cases, (e.g. Owen et al) the data is subsequently found to be biassed due to various reasons.

ii) The formulae are arrived at after statistical curve-fitting. The degree of scatter is considerable.

iii) Equations 2, 3, 5, 6, 7 and 8 are dimensionally incorrect, thus bringing in a severe limitation on their general use.

iv) Use of mixing coefficients Dm or D_L are as a result of Fluid Dynamics approach. The equations 1, 2 and 9 may perhaps be intellectually more satisfying but may not necessarily be more accurate. Dm and D_L are also not readily measurable quantities.

v) Thackston-Krenkel have suggested that eq. 5 could be used for an approximate assessment. Available data does not support this contention.

vi) Eq. 4 by Thackston-Krenkel has received considerable notice since it was first published and deserves, therefore, particular attention. The equation has resulted out of a conviction that reaeration coefficient is linearly proportional to turbulent diffusivity which in turn is also linearly related to average shear velocity. This arguement, propounded for two-dimensional flow, is not valid for three dimensional flow with secondary currents. The linear relationship may, therefore, be limited to two-dimensional flows only.

u_* is not readily measurable, as in actual streams it would depend upon the hydraulic radius, thus making the equation more cumbersome for use.

The Froude Number of flow is introduced in the equation to account for the effective area of the ruffled surface. The ruffled surface of a stream is either due to wind waves or due to the chopped surface prevailing in super-critical flows when Vedernikov Number is more than one. Both these factors are not considered in the development of the formula. Hence the inclusion of Froude Number cannot be explained adequately. The normal surface disturbance prevailing in turbulent flow is already separately accounted for by turbulent diffusivity or the shear velocity in the expression.

The mean velocity of flow, $\bar{u}$, does not explicitly appear in Thackston-Krenkel formula and this may be called a weak point of the formula. However, little rewriting of the proposed equation results in -

$$\frac{k_2 h}{\bar{u}} \propto \frac{u_*}{\bar{u}} \qquad \text{....(11)}$$

This appears to be a more suitable form to express the formula, as the ratio $\frac{\bar{u}}{u_*}$ has specific significance is open channel flow.

However eq. 11 bears comparison with eq. 10 by Lam-Lau. Both are as a result of almost the same data. The anamoly in the two equations is puzzling.

Another curious aspect of Thackston-Krenkel equation is that it applies for both hydraulically smooth and hydraulically rough surfaces. Their laboratory runs in the open channel correspond to S series (smooth bed) and B and G series (rough surfaces) while A series corresponds to an intermediate position. It seems illogical that the controlling equation in all the cases will have the same form. It is more reasonable to assume that the equation is valid only for rough turbulent flow, which is usually the case for natural streams. For hydraulically smooth flows, viscosity will be an important variable, featuring in the end equation in the form of Reynolds Number.

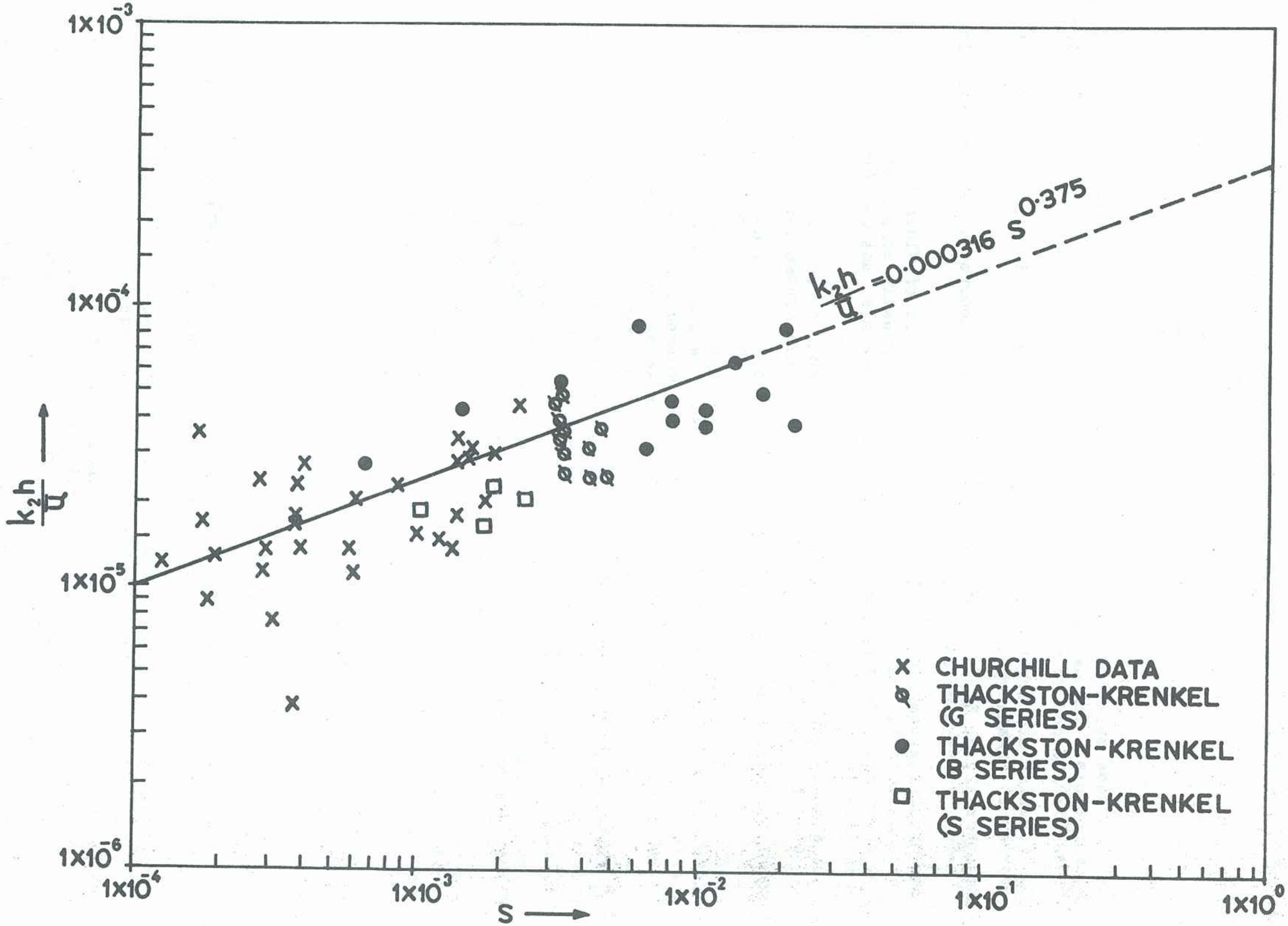
1x10^-3
1x10^-4
1x10^-5
1x10^-6
k2h/u
1x10^-4
1x10^-3
1x10^-2
1x10^-1
1x10^0
S
k2h/u = 0·000316 S^0·375
× CHURCHILL DATA
ø THACKSTON-KRENKEL (G SERIES)
● THACKSTON-KRENKEL (B SERIES)
□ THACKSTON-KRENKEL (S SERIES)

FIG. 1.

vii) Equation 7 by Churchill et al and eq. 8 by Isaacs & Gaudy are much simpler to use. Their dimensional non-homogeniety is their essential drawback. Besides this, the equations do not include the energy slope, an important variable. No doubt, S is to some extent correlated with $\bar{u}$ & h; but it is not exclusively dependent upon these two factors only. The energy slope is controlled by both the grain roughness and form roughness of the bed. It also depends upon secondary flow in the streams and the meandering pattern. For these reasons, S has to be explicitly included in any formula for reaeration coefficient.

viii) Parkhurst-Pomeroy's eq. 6 is an improvement in this respect. However, the equation, besides being nonhomogenous, is also mainly for sewers rather than for natural streams.

From the above discussions, two points emerge. Firstly, the Fluid Dynamics approach, although desirable, involves many simplifying but questionable assumptions. The resulting equation from such an approach, may, therefore, be little better than a purely empirical equation.

Secondly, the role of wind velocity is not considered by any of the researchers. Juliano[n] states that observed reaeration coefficients have been found many times greater (in tidal and windy reaches) as compared to those calculated by formulae. The value of k_2, with the use of the above formulae and neglecting wind effects may be on the oversafe side. The otherwise excellent work by Churchill et al does not mentionethe wind velocities.

A New Formula

In view of the above discussion, it can be presumed that in rough turbulent flow the basic and easily measurable variables on which the reaeration coefficient depends are h, $\bar{u}$ and S. Then the following dimensionally homogeneous form of equation results:

$$\frac{k_2 h}{\bar{u}} = C\,S^n \qquad \text{....(12)}$$

where C and n are constants. In order to determine the values of C and n, Churchill data and Thackston-Krenkel data (corresponding to roughened bed) was used resulting in a plot shown in Fig. 1. The equation of the mean line (curve-fitting by eye only) is as follows:

$$\frac{k_2 h}{\bar{u}} = 0.000316\ S^{0.375} \qquad \text{....(13)}$$

The equation is nondimensional, easier to use and connects only easily measurable quantities. The curve-fitting by eye only was done as further refinement of the equation is not desirable in view of the considerable scatter.

It may be noted that Thackston-Krenkel S series corresponding to smooth bed is also plotted in Fig.1 and all the points lie lower than the prediction equation line as anticipated. This fact supports the statement that the equation is to be used only for rough turbulent flow.

Field-Prediction

The value of k_2 as arrived by eq. 13 has to be adjusted for three effects, namely: a) temperature effects b) pollution load effects and c) wind effects.

Very little information is available on the wind and pollution load effects,while temperature effect can be reliable predicted with the available information.

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