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DESIGN SPEEDS FOR WIND TURBINES

by

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SUMMARY

Predictions of wind turbine performance are required in order to design and cost wind energy conversion systems.

By assuming that the hourly-mean wind-speed frequency can be characterized by the Weibull distribution, an optimum rated wind speed is defined which maximizes the primary energy conversion of an ideal wind turbine. As the overall conversion system design may require a prime operating time greater than that available above the optimum rated wind speed, primary energy conversion performance and prime time are presented as functions of rated wind speed. Secondary energy conversion performance below rated wind speed is presented in a similar manner.

The information given is intended to provide a guide to the rating and performance of wind turbines in given wind regimes.

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INTRODUCTION

Wind is inherently variable. When considering its use as a convenient form of solar energy the variability of wind therefore needs to be characterized and kept in mind when designing a wind energy conversion system. The system will generally be designed to meet a specification which calls for the supply of a given quantity of energy, of given quality, in a given time at a given probability level.

The preferred or prime energy is considered to be that supplied at constant rated power, while the energy supplied below rated power is termed secondary energy. The design time may be a critical irrigation month, an irrigation season or a period during which pumping to storage is required.

In the analysis presented, the design time period has been taken as the whole year and the design year based on the assumption that annual average wind speeds are normally distributed. Within the year and probably for shorter periods, the hourly mean wind speed frequency can be characterized by the Weibull distribution, (Kolodin 1963).[8] This is particularly fortunate for energy conversion performance prediction as the power output of a wind turbine is usually related to hourly mean wind speed. Considering variations within the hour, short period gusts and lulls exert an influence on structural, aerodynamic and control design. Gusts of 1.5 times the mean hourly wind speed are likely and the overall design should be able to survive a gust of about 10 times the annual average wind speed, (Anon 1973, 1974).[1,2]

WIND ENERGY AVAILABLE

The general expression for wind power available is

$$Y = \frac{1}{2}\rho \bar{V}^3 S x^3, \quad y = x^3. \tag{1}$$

The wind energy available in a period of time as a function of atmospheric density, mean wind speed cubed and the energy pattern factor, is given by

$$J = \frac{1}{2}\rho \bar{V}^3 S T E, \tag{2}$$

where

$$\rho = \sigma \rho_0, \tag{3}$$

$$\bar{V} = \bar{V}_{10}\left[\frac{H}{10}\right]^{a}, \tag{4}$$

$$E = \int_0^\infty x^3 P(x)\, dx. \tag{5}$$

The relative density is a function of altitude and latitude. Values for the International Standard Atmosphere are shown in Table 1, and can be used for temperate latitudes. The reduction in density with altitude may well be more than compensated by the increase in the cube of the mean wind speed.

The mean wind speed is a function of ground roughness and height above ground. A power law wind speed profile can be used to project standard 10 metre windspeeds to the height required,

$$J = J_{10}\left[\frac{H}{10}\right]^{3a}. \tag{6}$$

Average values of the power law exponent are shown in Table 2. An exponent value of 0.167 for a typical exposed site would indicate an increase in wind energy available proportional to the square root of the height ratio.

The energy pattern factor is a function of the wind regime, which is uniquely characterized by the exponent of the Weibull distribution. This type of distribution appears well suited to wind turbine design problems: it has been discussed by Goodrich (1926)[7] in relation to plotting rainfall and runoff and more recently by Kolodin (1963)[8] and Davenport (1968)[4] in relation to wind speed frequency data. Appropriately, Yevjevich (1972)[12] and Shigley (1972)[9] refer to the Weibull distribution in hydrology, fatigue and reliability.

The usual form of the Weibull distribution is

$$P(>V) = e^{-\left(\frac{V}{c}\right)^k} . \tag{7}$$

For the present analysis a more convenient form is obtained by using

$$n = \left(\frac{\bar{V}}{c}\right)^k . \tag{8}$$

Then

$$P(>x) = e^{-nx^k} , \tag{9}$$

and

$$P(<x) = 1 - e^{-nx^k} . \tag{10}$$

The non-dimensional wind-speed frequency or probability density is given by differentiating as

$$P(x) = nkx^{k-1} e^{-nx^k} . \tag{11}$$

Taking the first moment about the origin,

$$M_1 = \int_0^\infty nkx^k e^{-nx^k} = 1 . \tag{12}$$

Substituting $v = nx^k$ and using the gamma function,

$$M_1 = \frac{1}{n^{\frac{1}{k}}} \Gamma\left(1+\frac{1}{k}\right) = 1 , \tag{13}$$

or

$$n = \Gamma^k\left(1+\frac{1}{k}\right) . \tag{14}$$

Similarly for the mth moment about the origin,

$$M_m = \int_0^\infty nkx^{k+m-1} e^{-nx^k} dx , \tag{15}$$

giving

$$M_m = \frac{\Gamma\left(1+\frac{m}{k}\right)}{\Gamma^m\left(1+\frac{1}{k}\right)} . \tag{16}$$

In particular the energy pattern factor is given by

$$M_3 = \frac{\Gamma\left(1+\frac{3}{k}\right)}{\Gamma^3\left(1+\frac{1}{k}\right)} = E . \tag{17}$$

The wind speed frequency distribution is therefore described by the coefficients of variation and skew as

$$C_V = \left[M_2 - 1\right]^{\frac{1}{2}} = C_V(k), \tag{18}$$

$$C_S = \frac{M_3 - 3M_2 + 2}{\left[M_2 - 1\right]^{\frac{3}{2}}} = C_S(k) . \tag{19}$$

The mode or most frequent wind speed is given by

$$x^* = \frac{\left[1 - \frac{1}{k}\right]^{\frac{1}{k}}}{\Gamma\left[1 + \frac{1}{k}\right]} . \tag{20}$$

The effect of the Weibull exponent on the shape of the wind-speed frequency distribution is shown in Figure 1 while the characteristics of some typical wind regimes are shown in Table 3. Relating wind to solar energy, Davenport (1968)[4] suggests that a simple model of large scale atmospheric motions causing the wind, is a two-dimensional turbulent motion. If the wind is isotropic: there is no "prevailing" wind, the Weibull exponent is 2 and the wind speed frequency therefore follows the form of the Rayleigh distribution. In temperate latitudes the observed annual exponent is often just less than 2, so that the Rayleigh form is a useful reference regime. The natural motion and exponent are significantly modified by geographic and topographic influences. Considering the spectrum of wind regimes shown in Table 3: at one end of the exponent range wind shadow effects given rise to extreme variation, while at the other end dominant prevailing winds give a more equable type of wind regime with near-normal wind-speed frequency distributions.

In order to take account of variations from year to year the mean annual wind speed used for design will generally be selected on the basis of the student's t, normal or other suitable distribution. Typically the acceptable probability level will be determined from the standard deviation of the annual means and the standard normal distribution curve (Shigley 1972),[9]

$$\frac{J_D}{J} = \left[\frac{\bar{V}_D}{\bar{\bar{V}}}\right]^3 = \left[1 + z\left(\frac{s}{\bar{\bar{V}}}\right)\right]^3 . \tag{21}$$

Assuming as an example that 0.95 probability of exceedance is required and that the per unit standard deviation is 0.125, the design wind speed would be about 0.8 of the longterm mean and the design energy therefore only about half of that obtained using the long term mean value. It is of course essential to obtain reliable means and not be over optimistic when selecting the design wind speed, Golding (1955).[6]

PRIMARY ENERGY CONVERSION

Betz (1926)[3] showed that the wind shaft power developed by an ideal free-running wind turbine rotor with a perfectly matched load is given by

$$W = B\tfrac{1}{2}\rho\bar{V}^3 S x^3 , \quad w = x^3 , \tag{22}$$

where $B = \frac{16}{27}$.

If the ideal wind turbine is reefed or governed to produce constant full-load or rated power when operating above a given rated wind speed, as shown in Figure 2, the rated wind-shaft power is given by

$$W_1 = B\tfrac{1}{2}\rho\bar{V}^3 S x_R^3 , \quad w_1 = x_R^3 . \tag{23}$$

The ideal primary wind-shaft energy, converted above the rated wind speed is then

$$J_1 = B\tfrac{1}{2}\rho\bar{V}^3 S T x_R^3 P(>x_R), \tag{24}$$

$$= B\tfrac{1}{2}\rho\bar{V}^3 S T x_R^3 e^{-nx_R^k}. \tag{25}$$

The primary energy conversion factor is therefore

$$F_1 = \frac{J_1}{B\tfrac{1}{2}\rho\bar{V}^3 S T} = x_R^3 e^{-nx_R^k}, \tag{26}$$

$$= x_R^3 e^{-\Gamma^k(1+\frac{1}{k})x_R^k}. \tag{27}$$

Differentiating to determine the optimum rated wind speed for maximum primary energy conversion (Esman and Mamedzade, 1963)[5]

$$x_{RO} = \frac{1}{\Gamma(1+\frac{1}{k})}\left[\frac{3}{k}\right]^{\frac{1}{k}}. \tag{28}$$

The maximum primary energy conversion factor is then

$$F_{10} = \frac{\left[\frac{3}{k}\right]^{\frac{3}{k}}}{\Gamma^3(1+\frac{1}{k})} e^{-\frac{3}{k}}. \tag{29}$$

The optimum prime time or operating time above optimum rated wind speed is

$$P(>x_{RO}) = e^{-nx_{RO}^k} = e^{-\frac{3}{k}}. \tag{30}$$

Some systems may demand full power operation for a proportion of the total time period greater than the optimum prime time; the general expression for time of full power operation above any required rated wind speed is then

$$P(>x_R) = e^{-nx_R^k} = e^{-\Gamma^k(1+\frac{1}{k})x_R^k}. \tag{31}$$

The ideal primary energy conversion factor and prime time for a Rayleigh wind regime are shown as functions of normalized rated wind speed in Figure 3. The prime time is sensitive to rated wind speed and is reduced by a factor of about 0.5 between the mean and the optimum wind speed, and by a factor of about 0.1 between the mean and twice the mean wind speed. The general expressions for energy conversion factor and time of operation allow wind turbine size and rated power to be determined to meet a given primary energy conversion requirement in a given wind regime.

Some slight loss of prime time and energy will occur when the wind turbine is furled in order to protect it from damage due to excessive gusting.

The time above furling speed is

$$P(>x_F) = P(>fx_R) = e^{-\Gamma^k(1+\frac{1}{k})[fx_R]^k}. \tag{32}$$

To take account of furling the prime time or energy is factored by

$$C_F = 1 - \frac{e^{-n[fx_R]^k}}{e^{-nx_R^k}} = 1 - e^{-nx_R^k[f^k-1]}. \tag{33}$$

For an optimum rated wind speed system

$$C_{FO} = 1 - e^{-\frac{3}{k}[f^k-1]}. \tag{34}$$

Usually the furling speed is about twice the rated speed so that the prime time and energy losses are negligible.

SECONDARY ENERGY CONVERSION

The secondary wind-shaft energy, converted below the rated wind speed, can be regarded as a bonus if the wind turbine is governed at low rated wind speeds or as the main energy supply if the machine has a relatively high rated wind speed. When the wind is sufficiently regular, as in the trade wind regime, governing may be unnecessary and the wind turbine system designed to be completely free-running. This type of design would still need to be furled or otherwise arranged to survive extreme hurricane wind speeds.

If the ideal wind turbine is running free, the secondary wind-shaft energy converted below the rated wind speed is

$$J_2 = B\tfrac{1}{2}\rho\bar{V}^3 ST\int_0^{x_R} x^3 P(x)\,dx. \tag{35}$$

The secondary energy conversion factor is therefore

$$F_2 = \int_0^{x_R} x^3 P(x)\,dx. \tag{36}$$

Substituting $v = nx^k$ and using the incomplete gamma function,

$$F_2 = n^{-\frac{3}{k}}\gamma\left[\left(1+\tfrac{3}{k}\right), x_R\right], \tag{37}$$

$$= E\,x_R^{(1+\frac{3}{k})}\gamma^*\left[\left(1+\tfrac{3}{k}\right), x_R\right]. \tag{38}$$

The trends of the ideal secondary energy conversion factor and the total energy conversion for a Rayleigh wind regime are shown in Figure 3. At the optimum rated wind speed the ideal secondary energy is about equal to the ideal primary energy. Above the optimum rated wind speed the ideal total energy peaks at about twice the mean wind speed. The slight increase in total energy is at the expense of reductions in prime time and energy and a significant increase in rated power.

DISCUSSION

In practice useful secondary energy is only produced above a cut-in wind speed of about half the rated wind speed, so that a typical wind turbine might only have a 4:1 useful operating wind speed range, Figure 2. The trends of total operating time above the cut-in wind speed and loss in ideal secondary energy for a Rayleigh regime are shown in Figure 3.

The near-linear useful power - wind speed relationship between cut-in and rated wind speeds of Figure 2 indicates that significant compound secondary power conversion losses occur in this wind speed range, especially near the cut-in wind speed. In view of this and the increased cost of the rated power, or installed capacity, associated with high rated wind speeds, Vadot[10,11] (1957,58) suggests that the economic optimum rated wind speed will probably be less than twice the mean wind speed. Esman and Mamedzade (1963),[5] who introduced the optimum rated wind speed concept, recommend that the rated speed not exceed the optimum in order to obtain sufficient time of operation; their preferred rated wind speed is again less than twice the mean wind speed. In accord with these views and the present analysis Golding (1955)(6) suggests that at a good site more than half the useful total energy converted is primary energy, in which case the additional secondary energy loss may not be serious.

CONCLUSIONS

The primary and secondary energy conversion factors based on the Weibull wind speed frequency distribution allow ideal wind turbine energy conversion performance to be predicted at various rated wind speeds in various wind regimes. A well balanced wind turbine design will usually be rated at or below the optimum rated wind speed for maximum primary energy conversion in a given wind regime.

The useful energy converted by the system will be significantly less than that converted by an ideal wind turbine. The useful power can be obtained by applying aerodynamic, mechanical, electrical, pump and hydraulic circuit power conversion efficiencies to the ideal wind turbine. The influence of these compound efficiencies on useful energy output performance can be determined for given systems by using appropriately modified primary and secondary energy conversion factors.

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TABLES

Table 1: Variation of Atmospheric Density with Altitude

Altitude (m)	1000	2000	3000	4000	5000
Relative Density	0.9074	0.8215	0.7420	0.6685	0.6007

International standard sea level density 1.2256 kg/ m^3

Table 2: Variation of power-law wind-speed profile exponent with type of terrain

Type of Terrain	Exponent a
Flat open country	0.16
Rough wooded country, city suburbs	0.28
Heavily built-up urban centres	0.40

Source: Davenport (1968)[4]

Table 3: Variation of Weibull exponent with type of wind regime

General Flow Character	Type of Wind Regime	Weibull Exponent k	Variation Coefficient C_V	Skewness Coefficient C_S
Irregular	Polar, wind shadow	1.0	1.0	2.0
	Mediterranean, desert	1.5	0.679	1.070
Isotropic	Temperate maritime and continental	2.00	0.522	0.632
	Roaring fourties	2.5	0.428	0.327
Constant	Trade wind	4.0	0.280	-0.088

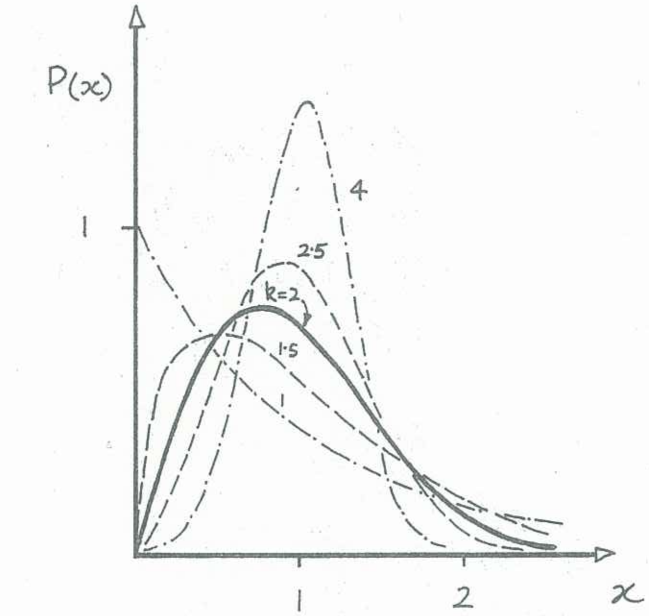

Figure 1. Variation of wind-speed frequency distribution with Weibull exponent.

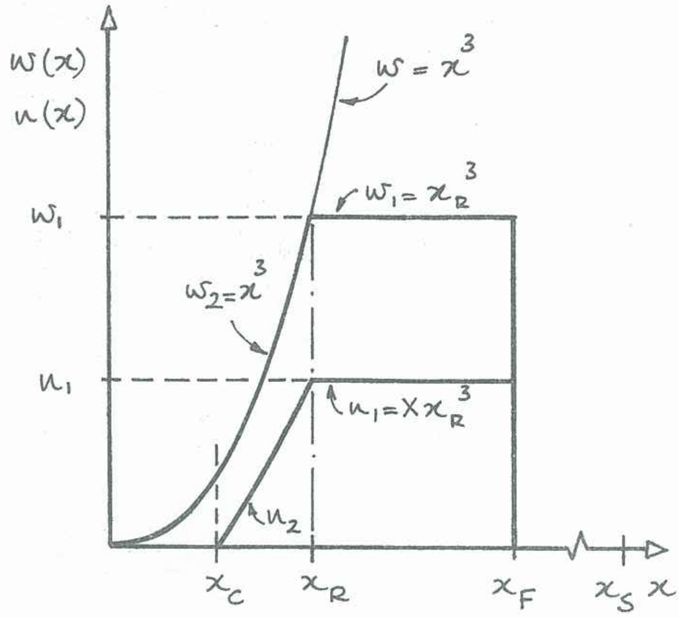

Figure 2. Variation of wind turbine power output with wind speed.

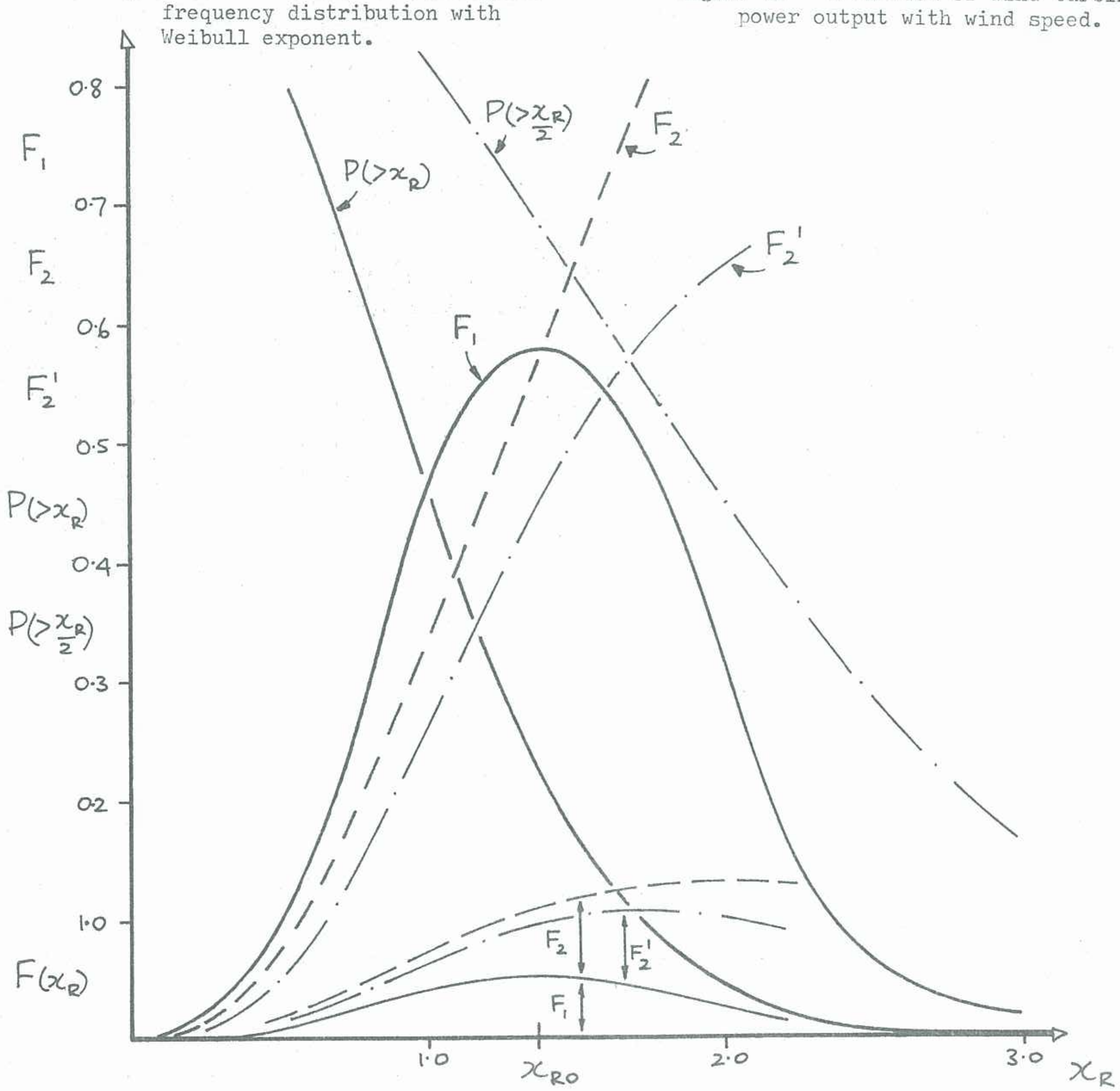

Figure 3. Variation of primary and secondary energy conversion performance and operating times with rated wind speed in a Rayleigh wind regime.