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A NUMERICAL MODEL OF THERMALLY BUOYANT PLUMES

by

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SUMMARY

A simple numerical model is used to compute the vertical velocity and temperature profiles of air ascending in dry and moist convection. The model allows for entrainment of ambient air in an arbitrary manner with height and an arbitrary ambient temperature profile can be used. One application is to thermally buoyant pollution where dry convection is used with air being entrained to produce a conical plume, assuming that vertical accelerations are small. These results are compared with the analytic solutions of Morton, Taylor and Turner (1956). The arbitrary ambient temperature profile allows for a study of the penetration of ground based inversions and inversions aloft.

A second application is to moist convection within cumulus clouds. An adiabatic model of moist convection can be used without entrainment to predict the maximum storm top with considerable success, but the vertical velocities and parcel (in cloud) temperatures are higher than those observed. The addition of entrainment at the rate of about 10% km^{-1} corrects this to a large extent without significantly effecting the storm top. The total potential buoyant energy is related to the observed storm severity and precipitation activity. The model has been valuable in forecasting and occurrence and severity of hailstorms in Alberta in the Alberta Hail Studies Project, and elsewhere in the world.

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Glossary of terms

α	the proportional entrainment ratio,
c	the spreading coefficient of the plume $(c = \frac{dR}{dz})$,
c_p	the specific heat of air at constant pressure,
F_B	the buoyancy force per unit mass,
g	the acceleration due to gravity,
Γ	the dry adiabatic lapse rate (9.76 ^{o}K km^{-1}),
m	the mass of the parcel,
m_l	the water loading per unit mass,
n	the ratio of the vertical temperature gradient to Γ,
Q	the source strength of the plume,
R	the radius of the plume,
ρ	the ambient air density at the source level,
T_o	the ambient air temperature at the source level,
T	the virtual temperature of the parcel,
T'	the ambient virtual temperature,
$\bar{T}'$	the mean ambient virtual temperature across the layer,
w	the parcel vertical velocity,
w_o	the initial parcel vertical velocity, at height z_o,
z	the vertical coordinate, the height above the plume source,
dz	the layer thickness.

1. INTRODUCTION

Convection is an important mechanism by which the atmosphere, through vertical mixing, attains a more stable state. In the boundary layer the air close to the ground is warmed when the surface of the earth is heated by solar radiation. This heat is initially propogated upwards by conduction but as the heating increases it becomes more efficient for heat to be transported by bulk motion of the air and it is carried upward by buoyant convective plumes or thermals. When a significant wind is present there is also vertical mixing in the boundary layer by turbulent eddies. Buoyant plumes are also formed over heat sources such and chimneys and fires.

Higher in the air vertical instability can arise when a cold dry air stream flows over a warm moist air stream. In this case convective, or cumulus, clouds are formed in the thermally buoyant plumes. Not only is there buoyancy due to the temperature excess of the plume over the ambient air, but there is the release of latent heat with the condensation of water vapour to form cloud drops. This latent heat adds to the buoyant energy of the plume.

2. MODELLING ATMOSPHERIC CONVECTION

With the availability of high speed computors with large memories it has been possible to develop sophisticated numerical models of convection. For example Steiner (1973) developed a three dimensional model in which he could watch the growth of a cumulus cloud in the presence of wind sheer and veer. The model allowed for condensation but did not allow the drops to precipitate out or to freeze. Even using the large McGill University computor it was an expensive proposition as the model ran at up to three times slower than real time. This meant it cost about $1000 for a model run which gave the cloud 20 mins to develop. In addition the storage restricted the model size to a 3 km cube with 200 m grid spacing. Although this model gives valuable insights into the effects of variations of many of the parameters it is impractical for use in day to day forecasting. As far back as the nineteen twenties Shaw (1925) demonstrated the value of the thermodynamic chart the tephigram, in detecting instability in the air and calculating the potential energy of an adiabatic parcel model. This type of model can easily be programmed for use on a computor and can be a valuable aid to forecasting. It is quick, requires little storage and is easy to handle and to interpret.

3. A MODIFIED ADIABATIC MODEL OF BUOYANT CONVECTION

We consider a parcel of air. The buoyancy force on it per unit mass is given by

$$F_B = g\left[\frac{T}{T'} - (1 + m_1)\right] \qquad (1)$$

The water loading term m is commonly assumed to be the adiabatic liquid water content, that is it is assumed that all the condensed water reamins in the parcel or the loss due to fall out is balanced by the gain due to drops falling into the parcel from above. For dry convection m is zero. This force is equated to the parcel's vertical acceleration, which is then integrated over height to give the vertical velocity profile,

$$w^2(z) = w_o^2 + 2\int_{z_o}^{z} F_B(z)\, dz \qquad (2)$$

F_B is obtained as a function of height from the ambient temperature profile and by assuming that the parcel ascent adiabatically. That is, the parcel ascends dry adiabatically up to the lifting condensation level (LCL) and ascends pseudoadiabatically (allowing for the release of latent heat) thereafter.

The effect of entraining air can be included by computing modified parcel temperature and liquid water profiles. The temperature profile is computed at height intervals of dz so that

$$T(z) = \frac{T(z-dz) - dz\Gamma + \bar{T}'(z)}{1 + \alpha} \qquad (3)$$

The adiabatic liquid water content must be modified because some of the water that has condensed, because the parcel cools as it expands with the ascent, is evaporated to maintain the entrained ambient air at saturation. Also the mass of the parcel is increasing so the liquid water per unit mass must be recalculated.

4. DRY BUOYANT PLUMES

It is often required to compute the height to which a smoke plume from a chimney will rise and what its vertical velocity profile will be. A particular case of interest was the proposed thermal power station at Evans Bay, Wellington. This was to be just to one side of the approach flight path to Rongatai airport. The N.Z. Meteorological Service was asked to investigate the possible effects of plumes at the heights that aircraft would fly across the power station. The project was halted, perhaps because Civil Aviation thought that a plume several tens of metres wide with core vertical velocities in excess of 20 m sec^{-1} would be hazardous for aircraft on their final approach to land.

Analytic models have been used, for example by Priestley and Ball (1955) (P&B) and Morton, Taylor and Turner (1956) (MT&T). They used the equations of continuity, conservation of vertical momentum and heat conservation (P&B) or density deficiency (MT&T). They assumed that the plume expands with the radius in constant proportion to the height above the virtual source. It was shown by Spurr (1959) that their expressions for the ceiling height of the plume were essentially the same. This may be written as

$$z_c = 1.15\ c^{-\frac{1}{2}} \left(\frac{g\,Q}{T_o \rho\, c_p}\right)^{\frac{1}{4}} \left(\frac{g\,\Gamma}{T_o}(1 + n)\right)^{-\frac{3}{8}} \qquad (4)$$

This may be simplified, as shown by MT&T, assuming that the source level temperature is 288 °K,

$$z_c = 31\ (1 + n)^{-\frac{3}{8}}\ Q^{\frac{1}{4}} \qquad (5)$$

Here z_c is in metres and Q is in kilowatts.

Standard atmospheres often take 6.5 °K km^{-1} as the tropospheric lapse rate. Using this gives the following typical plume ceiling heights:

(a)	A household chimney, 8 kW,	80 m (40m)
(b)	A bonfire, 450 kW,	214 m (110m)
(c)	A power station chimney, 500MW,	1240 m (636m)

The figures in brackets are for an ambient lapse rate of - $10^{o}K\ km^{-1}$. Generally the temperature gradient cannot be approximated well by a constant over this range of heights. This points out a major limitation of these analytic models.

The numerical model outlined in section 3 overcomes this difficulty as computations are done in discrete steps. At each level the ambient temperature and the rate of mixing can be specified. It is commonly observed that for low emission velocities a smoke plume expands at a constant rate with dR = c dz, where c is about 0.1. When accelerations take place, such as just above a bonfire, convergence is observed and when the plume is slowed down, say by a stable layer, it diverges. Initially this discussion will neglect these effects. The entrainment rate is then given by

$$\alpha = \frac{1}{m}\frac{dm}{dz} = \frac{2c}{R} = \frac{2}{z} \qquad (6)$$

Thus the greatest dilution occurs in the first few metres above the source. It becomes less than 10 % m^{-1} at heights greater than 20 m above the source and less than 2 % m^{-1} for heights greater than 100 m above the virtual source.

The parcel temperature and vertical velocity profiles for a source with a strength of about 11.5 kW is shown in Fig.1. The parcel is thermally buoyant as it leaves the source. It mixes rapidly with the environment, being neutrally buoyant at a height of about 33 m where the vertical velocity reaches its maximum value of 1.52 m sec^{-1}. Above this height the parcel is negatively buoyant so that it slows down and its vertical velocity returns to zero at a height of 112 m. The ascent takes about 110 secs. The parcel then undergoes damped oscillations and settles down around 76 m.

5. PENETRATION OF A STABLE LAYER ALOFT

Commonly the boundary layer is mildly stable (say $\frac{dT}{dz} = -5\ ^{o}K\ km^{-1}$) and is capped by a stable layer (say $\frac{dT}{dz} = 50\ ^{o}K\ km^{-1}$). Above this the temperature decreases with height. This situation may occur because of low level subsidence but is usually a result of surface heating during the morning after there has been a radiation inversion overnight. Such a situation is illustrated in Fig. 2. The effect of the depth of the inversion layer is shown by considering a parcel that mixes with the ambient air at a constant rate of 10 % m^{-1}. The temperature profiles are shown in Fig. 2a and the velocity profiles in Fig. 2b. When the stable layer is 10 m deep the plume penetrates it with a significant amount of vertical momentum and ascends to about 140 m. When the layer is 15 or 20 m deep the plume is slowed right down by the layer and settles down into the layer. The conical plume mixes very quickly near the source, becomes neutrally buoyant at about 18 m and rises to just within the stable layer. The vertical velocity profiles reflect this behaviour. The conical plume reaches a very much lower vertical velocity then the plume with 10 % mixing per unit height.

6. MOIST CONVECTION, CONVECTIVE CLOUDS

To model moist convection for forecasting severe local storms the parcel is assumed to rise dry adiabatically until condensation takes place at the LCL. Above this it rise pseudoadiabatically. Stommel (1947) suggested that observations made by aircraft of the temperatures within cumulus clouds that were less than the adiabatic value could be explained by the mixing in of cooler, drier ambient air. The effect of mixing at a constant rate is shown in Fig. 3 using a typical radiosonde sounding from Alberta for a mid-summers day on which hailstorms were observed. While the parcel is positively buoyant the temperature, modified by mixing, is less than the unmixed parcel. Thus the total buoyant energy gained is less. Above the level of neutral buoyancy (LNB) the air being entrained is warmer than the parcel air so that the modified lapse rate is less than pseudoadiabatic. This reduced the magnitude of the negative buoyancy and helps to offset the reduction in positive buoyant energy, known as the positive area. The area on a tephigram is proporional to energy so the area between the ambient temperature profile and the parcel temperature profile between the LCL and the LNB is proportional to the buoyant potential energy.

The corresponding vertical velocity profiles are shown in Fig. 4. In Fig. 4b water loading equal to the adiabatic liquid water content has been included. We can see that the effect of both mixing and water loading is to decrease the total energy, decreasing the maximum vertical velocity and lowering the storm top. On this day storms were observed to have maximum tops between 11.5 and 12.5 km. This would result from mixing in about 7 to 20 % km^{-1} or, if loading was included, up to about 12 % km^{-1}. It should be noted that mixing and loading are not independent (Fig. 5). The greater the proportion of ambient air entrained into the parcel the more liquid water is required to be evaporated to saturate the ambient air. Stommel found that for single cell cumulus clouds mixing in about 10 % km^{-1} accounted for the observed temperature reduction from the adiabatic value. This is consistent with our experience in Alberta which is typified by this sounding for a day on which the majority of the storms observed were multicellular. The loaded model without mixing corresponds to the maximum observed storm top. This would probably be from a well developed storm in which there is a strong, relatively steady updraft. The core of the updraft would experience little entrainment of ambient air. On the otherhand the majority of storms would ascent to 1 to 1.5 km below this. The smaller the cell, the greater would be the effect of mixing on the core of the updraft. Fig. 4b shows that this height reduction in the model storm top would occur with mixing in 10 to 15 % km^{-1} on the average, and would result in the maximum vertical velocity decreasing from about 40 m sec^{-1} to between 20 and 25 m sec^{-1}.

A storm cell is generally broader at the base and the core narrows as it ascends. In a multicell storm several cells in various stages of development exist at any one time. Each successive cell moving into air that has been modified by the previous cell mixes in little ambient air until it ascends out of the modified air and mixes strongly with the ambient air. To approximate this sort of effect some arbitrary mixing profiles, shown in Fig. 6, were chosen. The resulting loaded vertical velocity profiles are shown in Fig. 7. Although there is a considerable drop in the maximum vertical velocity there is very little effect on the storm top. This suggests that the loaded, unmixed model can be used to forecast the maximum storm tops for multicell and steady state storms even though the actual temperatures and vertical velocities inside the cloud may be less than the adiabatic values.

Forecasting can go further and predict the type of storm activity, its severity and the maximum size of hail. It is reasonable to expect that there should be a correlation between the amount of buoyant energy available for convection and the resulting storm activity. Chisholm (1970), through a study of many storms, empirically divided soundings according to their loaded positive area into low, medium and high energy storms. Some of their characteristics are given below.

(a) Low energy storms ($\leq$ 200 J kg^{-1}): Maximum storm top between 6 and 7 km (AGL), maximum vertical velocity between 15 and 18 m sec^{-1} and producing grape-walnut sized hail.

(b) Medium energy storms (200 to 450 J kg^{-1}): Maximum storm top will be within 750m above the tropopause, with vertical velocities up to about 25 m sec^{-1} and producing walnut - golfball sized hail.

(c) High energy storms (> 450 J kg^{-1}): Storm tops penetrate into the stratosphere by between 500m and 1.5 km, have vertical velocities in excess of 35 m sec^{-1} and produce golfball and larger sized hail.

7. CONCLUSIONS

This simple adiabatic parcel model was found to be very useful in forecasting severe local storm activity and severity. It enables a study to be made of the effects of water loading, mixing and the ambient temperature profile. The sensitivity of the storm activity to the forecast maximum surface temperature could also be determined. It can be used to study the gross characteristics of dry convection and has been applied to the penetration of a stable layer of a plume. There appears to be no reason why, given the characteristics of a plume in terms of its mixing profile, the model should not be used in pollution studies or smoke stack investigations using real temperature profile data.

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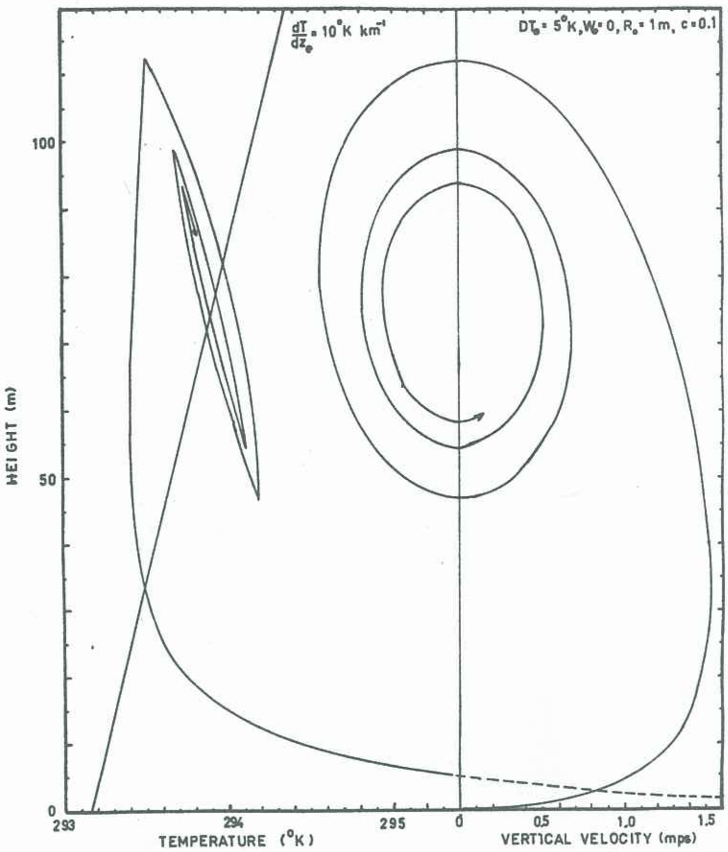


Fig. 1: The parcel temperature and vertical velocity profiles for a conical plume with an approximate source strength of 11.5 kW. The ambient temperature gradient is 10 ^{o}K km^{-1} which is mildly stable.

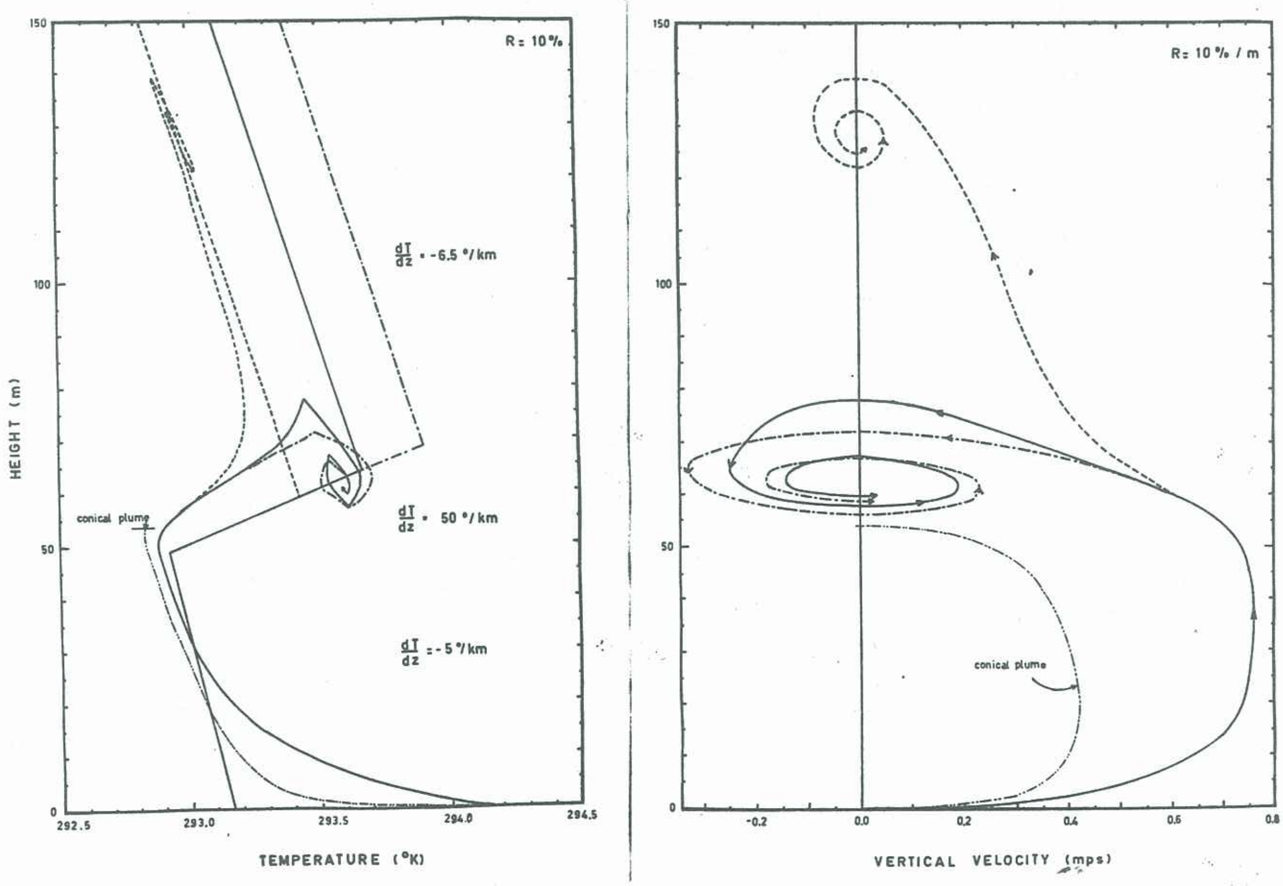
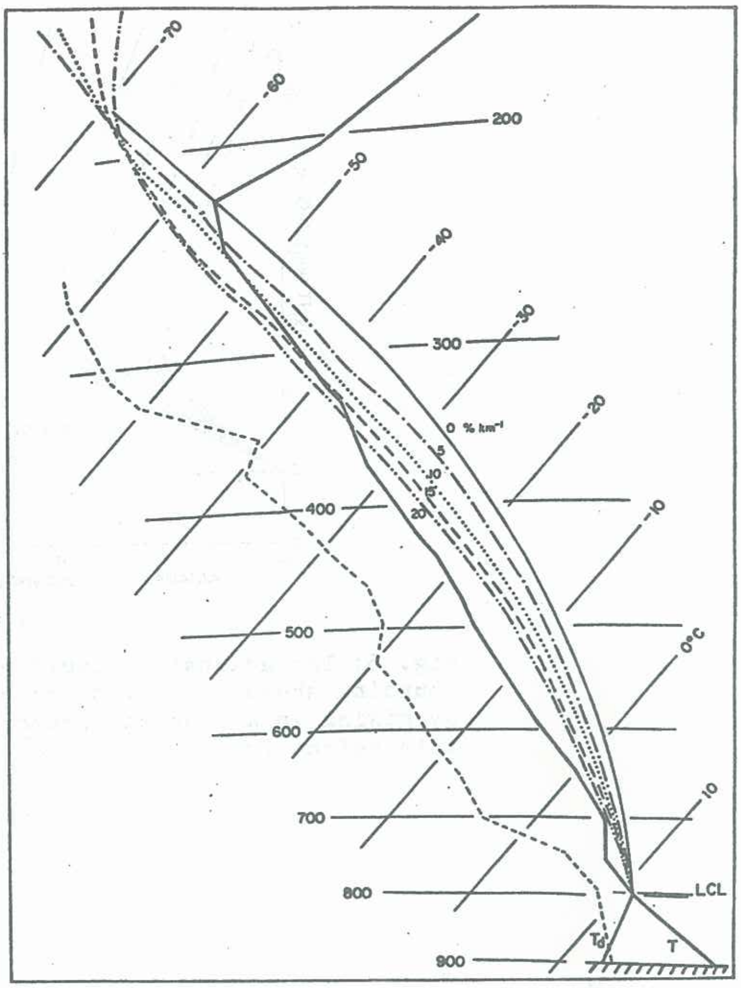


Fig. 2: The effect of a stable layer aloft (temperature inversion) illustrated by the model with constant mixing with height (10% m^{-1}) with three thicknesses of layer, broken line - 10m, solid line - 5m and dot/dash line - 15m. The conical plume profiles have been added.

Fig. 3: A typical radiosonde sounding from Alberta in mid-summer, showing the ambient temperature (T) and dew-point (T_d) profiles with the model temperature profiles with constant mixing with height from 0 - 20 % km^{-1}.



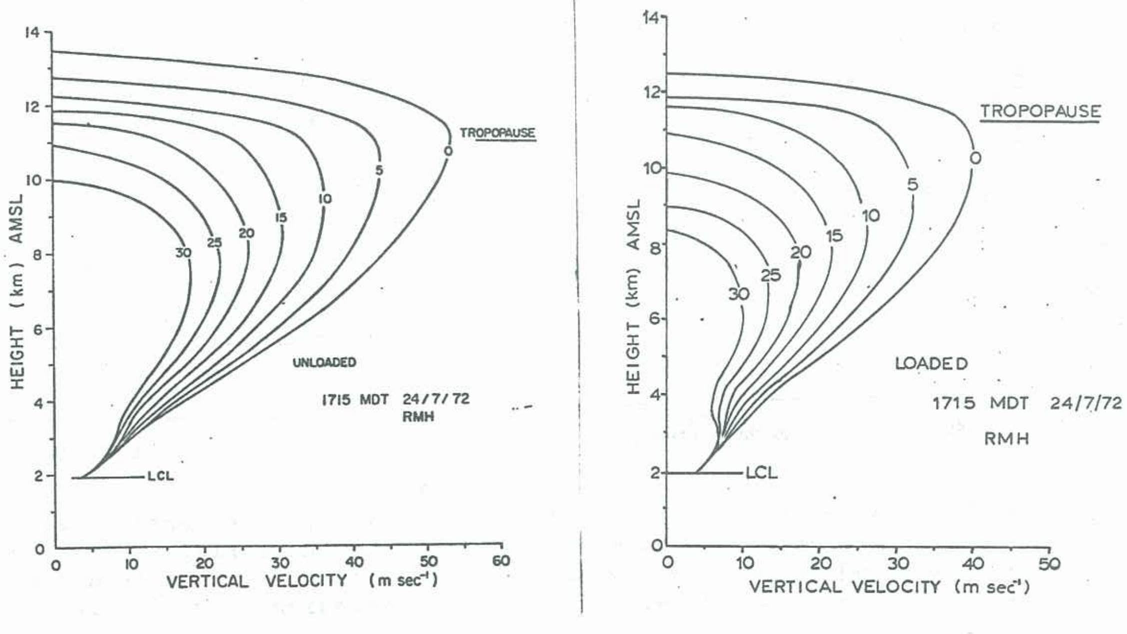


Fig. 4: The vertical velocity profiles for the sounding shown above (a) without water loading and (b) with water loading equal to the adiabatic liquid water content, for constant mixing with height from 0 - 30 % km^{-1}. LCL refers to the lifting condensation level.

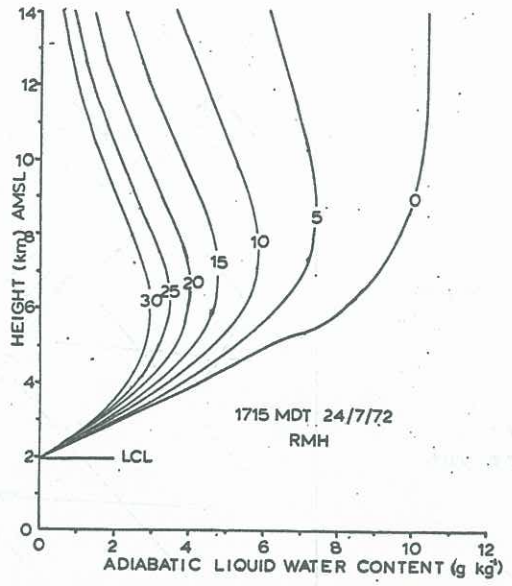


Fig. 5: The adiabatic liquid water content for the sounding shown in Fig. 3, showing how it is modified by mixing in a constant proportion of ambient air with height from 0 - 30 % km^{-1}.

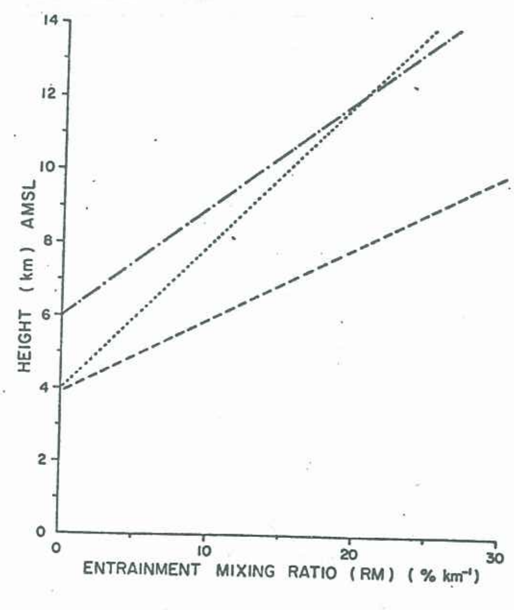


Fig. 6: Some arbitary mixing profiles to look at the effect of no mixing at low levels and increasing mixing near the top of the troposphere.

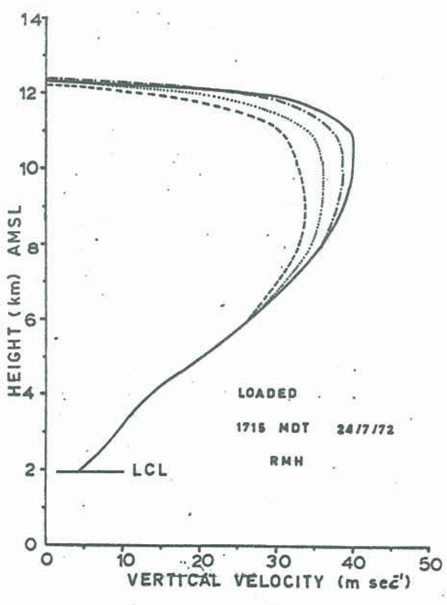


Fig. 7: The vertical velocity profiles for the mixing profiles given in Fig. 6, for the loaded model of the sounding shown in Fig. 3.