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TURBULENT PIPE FLOW AS AN ACOUSTIC NOISE SOURCE

by

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Expressions are derived for the spectral density of acoustic power radiated from a pipe whose walls are vibrating as a result of excitation by an internal turbulent flow. The dependence of the power radiation on pipe geometry and material, internal and external fluids, and flow speed is discussed. The power radiated is found to be proportional to the fifth or higher power of flow velocity, pipe length, at most the fourth power of pipe radius, and inversely proportional to the square of the wall thickness. The results indicate that, for practical pipe-flow conditions, significant sound power levels can be generated by fully-developed turbulent flow, even in the absence of flow distrubances due to pipe fittings.

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1. Introduction

There are many cases in which acoustic radiation from an industrial piping installation is of sufficient intensity to cause considerable annoyance to nearby communities. The radiation results from vibration of the pipe walls which may be excited in a variety of ways - by fully developed turbulent flow in the pipe; by local flow disturbances at elbows, valves and other pipe fittings; by the internal acoustic field generated by the flow and disturbances to it; or by mechanical excitation by pipe fittings which have themselves been excited by the pressure field associated with the flow.

Here we shall be concerned only with the first of these mechanisms, excitation by undisturbed fully developed turbulent pipe flow, and shall attempt to make some assessment of the conditions for which this basic mechanism may be a significant noise source.

2. Theory for acoustic power radiation from a pipe excited by internal turbulent flow

The pipe flow model considered is a thin circular cylindrical shell with simply supported ends, of radius a, length ℓ, wall thickness h, and surface area $S(= 2\pi a\ell)$, with a fully-developed internal turbulent gas flow whose velocity on the pipe centre-line is U_o. The pipe is immersed in a fluid in which the speed of sound is c. The densities of the gas flowing in the pipe, the pipe material, and the ambient fluid are ρ_f, ρ_s, and ρ respectively.

The turbulent gas flow gives rise to a random fluctuating pressure field with power spectral density ϕ_p on the internal surface of the pipe. Vibration of the pipe wall excited by this pressure field results in radiation of acoustic power, of spectral density ϕ_{PR}, into the external ambient fluid.

For this model, Bull and Rennison[1] derived an expression for the power spectral density of the radial vibrational displacement of the pipe wall, averaged over the vibrating surface. In that derivation it was assumed that the pipe vibration is lightly damped; that the vibrational response is dominated by resonant modes; and that the resonant modes are sufficiently numerous to allow the joint acceptance of the pressure field and the pipe structure, which for the αth mode is defined as

$$j^2_{\alpha\alpha}(\omega) = \frac{1}{S^2} \int_S dS(\underset{\sim}{r}_1)\ \psi_\alpha(\underset{\sim}{r}_1) \int_S dS(\underset{\sim}{r}_2)\ \psi_\alpha(\underset{\sim}{r}_2) R_p(\underset{\sim}{\xi},o;\omega), \qquad (1)$$

(where $R_p(\xi,\tau;\omega)$ is the narrow-band correlation coefficient of the pressure field for points with a separation vector ξ and time delay τ, and $\psi_\alpha(\underset{\sim}{r})$ represents the shape of the αth mode) to be regarded as a continuous function, $j^2(\nu,\theta)$, of frequency and the angle θ which the structural wave vector makes with the coordinate axes.

With the further assumptions that significant acoustic radiation is produced only by those resonant modes whose phase velocities are supersonic with respect to the ambient fluid, and that these supersonic modes radiate with the same efficiency as an infinitely large piston, the following expression was obtained for the ratio of the non-dimensional spectral density of the acoustic power radiation Φ_{PR} $(= \phi_{PR}/\rho c^2 Sa)$ to the non-dimensional power spectral density of the pressure field of the internal flow $\Phi_p (= \phi_p U_o/q_o^2\ a)$*:

$$\frac{\Phi_{PR}}{(\Phi_p/u^3)} = \left(\frac{\pi\rho_{fs}^2}{12\beta\nu_{ac}^{\frac{1}{2}}}\right)\left(\frac{J_s\nu_{hc}^{3/2}}{\nu}\right)Q, \qquad (2)$$

where $\rho_{fs} = \rho_f/\rho_s$; $\beta = h/(2\sqrt{3}a)$; $\nu_{ac} = c^2/\beta c_{LP}^2$; $\nu_{hc} = U_c^2/\beta c_{LP}^2$; $q_o = \frac{1}{2}\rho_f U_o^2$; $U_c = uU_o$ is the convection speed of the internal pressure field; $c_{LP} = [E/\rho_s(1-\mu^2)]^{\frac{1}{2}}$; E and μ are respectively Young's modulus and Poisson's ratio for the pipe material; Q is the damping factor; $\nu = \omega/\omega_r$; ω is the radian frequency; and $\omega_r = c_{LP}/a$ is the 'ring' frequency. J_s is given by the following equation:

$$J_s(\nu) = \int_{\theta_1}^{\theta_2} j^2(\nu,\theta)n(\nu,\theta)d\theta, \qquad (3)$$

where $n(\nu,\theta)$ is a continuous modal density function given by

$$n(\nu,\theta) = \frac{\Lambda}{2\pi\beta}\left[1 - \frac{\sin^4\theta}{\bar{\nu}^2}\right]^{-\frac{1}{2}}, \qquad (4)$$

* The numerical constant in equation (2) is given incorrectly in reference 1.

$\bar{\nu} = \nu/\psi$, $\psi^2 = 1-\mu^2$, $\Lambda = \ell/a$, and $\theta_1 < \theta < \theta_2$ is the range of θ for which supersonic modes occur at a particular frequency.

An alternative form of equation (2) can be obtained by introducing a form of Mach number $M_o = U_o/c$, defined as the ratio of the flow velocity on the pipe centre-line to the speed of sound in the fluid outside the pipe. Then $\nu_{hc} = u^2 M_o^2 \nu_{ac}$, and we obtain

$$\frac{\Phi_{PR}}{\Phi_p} = \frac{\pi\rho_{fs}^2}{12\beta} \cdot \frac{\nu_{ac}}{\nu} \cdot M_o^3 J_s Q$$

$$= \frac{\pi}{12} \cdot \frac{\rho_{fs}^2 M_o^3 J_s Q}{\beta^2 \nu M_{LP}^2} , \qquad (5)$$

where $M_{LP} = c_{LP}/c$. Thus to obtain the complete functional form of the ratio Φ_{PR}/Φ_p it is necessary to examine the dependence of J_s (as defined by equation (3)) on the system parameters.

In accord with experiment, the narrow-band space-time correlation coefficient of the internal pressure field can be represented by

$$R_p(\xi,\eta,o;\omega) = \exp\left(- \frac{c_x\omega|\xi|}{U_c} - \frac{c_y\omega|\eta|}{U_c}\right) \cos\frac{\omega\xi}{U_c} , \qquad (6)$$

where ξ and η represent spatial separations on the pipe surface in the axial and circumferential directions respectively, and c_x and c_y are experimentally determined constants. The joint acceptance can then be expressed as (see Bull and Rennison [1])

$$j^2(\nu,\theta) = j^2_{mm}(\nu,\theta)\, j^2_{nn}(\nu,\theta), \qquad (7)$$

with *

$$j^2_{mm} = \{2[(K_m^2 - [1-c_x^2]K_c^2)^2 - 4c_x^2K_c^4][1-(-1)^m e^{-c_x\Lambda K_c} \cos \Lambda K_c]$$

$$+ 8c_xK_c^2(K_m^2 - [1-c_x^2]K_c^2)(-1)^m e^{-c_x\Lambda K_c} \sin \Lambda K_c$$

$$+ c_x\Lambda K_cK_m^2(K_m^2 + [1+c_x^2]K_c^2)\Delta\}/\Lambda^2K_m^6\Delta^2 , \qquad (8)$$

and

$$j_{nn}^2 = \frac{c_yK_c[1-(-1)^n e^{-c_y\pi K_c}]}{2\pi(c_y^2K_c^2 + K_n^2)} . \qquad (9)$$

Here

$$\Delta = [(K_m^2 + [1+c_x^2]K_c^2) - 4K_m^2K_c^2]/K_m^4 ; \qquad (10)$$

$K_c = k_c a$; $K_m = k_m a (= m\pi/\Lambda)$; $K_n = k_n a (=n)$; k_m and k_n are structural wave numbers in the axial and circumferential directions respectively, and m and n are the corresponding mode numbers; and $k_c = \omega/U_c$.

3. Hydrodynamic Coincidence

Hydrodynamic coincidence is the condition in which the axial wave number component K_m of a structural wave is equal to the "wave number" $k_c = \omega/U_c$ of the internal fluid flow ($K_c = K_m$) in which case the trace velocity of the structural wave in the flow direction is equal to the convection velocity U_c of the turbulent pressure field.

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* In ref rence 1, $\cos\Lambda K_c$ in the first term of j^2_{mm} is incorrectly given as $\cos K_{x1}$, and in the third term K_m^2 as K_m; the factor π is omitted from the denominator of j^2_{nn}.

According to equation (5) the spectral density of the acoustic power radiation is proportional to J_s, which in turn is strongly dependent on $j^2(\nu,\theta)$ as given by equations (7)-(10). It is therefore to be expected that large values of J_s, and consequently of Φ_{PR}, will correspond to situations for which the maximum possible value of j^2 occurs within the range of integration, $\theta_1<\theta<\theta_2$, of J_s at a given frequency. The maximum possible value of j^2 for any particular vibrational mode is determined essentially by that of j^2_{mm} which occurs for $K_c \simeq K_m$, that is very nearly at hydrodynamic coincidence. (j^2_{nn} also exhibits a maximum value, occurring at $c_y K_c \simeq K_n$, which is much broader than than of j^2_{mm}. In general this will tend to shift the peak value of j^2 away from $K_c/K_m = 1$, but the effect is not generally significant - typical examples of the variation of j^2_{mm}, j^2_{nn} and j^2 with K_c/K_m and K_c/K_n are shown in figure 1 of reference 1).

Hence large values of J_s and Φ_{PR} might be expected for flow conditions which give coincidence In the theory of section 2, the resonance frequencies of the pipe are assumed to be given by

$$\nu^2 = \beta^2 K^4 + \psi^2 \sin^4\theta, \qquad (11)$$

where $K^2 = K_m^2 + K_n^2$, in which case the required conditions for coincidence are

$$M_c \geqslant \sqrt{2} M_{LP} \sqrt{\beta\psi} \text{ for } \bar{\nu} \leqslant \sqrt{2}$$

and

$$M_c \geqslant \frac{\bar{\nu}}{(\bar{\nu}^2 - 1)^{\frac{1}{4}}} M_{LP} \sqrt{\beta\psi} \text{ for } \bar{\nu} \geqslant \sqrt{2} \qquad (12)$$

But, at coincidence the phase velocity of the structural wave is $U_c \sin\theta$; and since only supersonic modes contribute to J_s, it follows that the values of j^2 corresponding to coincidence will only contribute to J_s when the flow speed is supersonic and $M_c \sin\theta > 1$. Coincidence can occur for subsonic flow speeds but, according to the assumptions made in the present work, the structural wave excited would make no contribution to the acoustic radiation. Hence, in most practical situations, with subsonic flow speeds, coincidence as such will not play a part in determining the acoustic power radiated.

This raises the question of how close is the approach to coincidence in practical flow situations. Again using equation (11), it can be shown that the minimum value of K_c/K_m for supersonic modes (those for which $\nu^2 > \beta K^2 \nu_{ac}$) is given by $F_m M_{LP}\sqrt{\beta\psi}/M_c$, where

$$F_m = \begin{cases} \sqrt{2} & \text{for } \bar{\nu}_{ac} < 2,\ \bar{\nu} < \sqrt{2},\ \bar{\nu}/\bar{\nu}_{ac} > \sqrt{2}/2, \\ \dfrac{\bar{\nu}}{(\bar{\nu}^2 - 1)^{\frac{1}{4}}} & \text{for } \bar{\nu}_{ac} < 2,\ \bar{\nu} > \sqrt{2} \\ & \text{or } \bar{\nu}_{ac} > 2,\ \bar{\nu} > \bar{\nu}_A, \\ \left[\dfrac{\bar{\nu}^2}{\bar{\nu}^2_{ac}}\left(1 - \dfrac{\bar{\nu}^2}{\bar{\nu}^2_{ac}}\right)\right]^{-\frac{1}{4}} & \text{for } \bar{\nu}_{ac} < 2,\ \bar{\nu} < \sqrt{2},\ \bar{\nu}/\bar{\nu}_{ac} < \sqrt{2}/2, \\ & \text{or } \bar{\nu}_{ac} > 2,\ \bar{\nu} < \bar{\nu}_B,\ \bar{\nu}/\bar{\nu}_{ac} < \sqrt{2}/2, \end{cases} \qquad (13)$$

and

$$\bar{\nu}_{A,B} = \bar{\nu}_{ac}\{1 \pm [1 - 4/\bar{\nu}_{ac}^{\,2}]^{\frac{1}{2}}\}^{\frac{1}{2}}/\sqrt{2} \ . \qquad (14)$$

Hence, the minimum possible value of K_c/K_m for supersonic modes is $\sqrt{2}M_{LP}\sqrt{\beta/\psi}/M_c$. For a steel pipe in air (M_{LP}=15.4, μ=0.28, u=0.7), this has the value of 31.8 $\sqrt{\beta}/M_o$. For practical limiting values of, say, β = 0.005 and M_o = 0.5, the minimum value of K_c/K_m is 4.5. This would, for example, correspond to a j^2 value smaller than the maximum by a factor of over 10 for the mode numbers used in figure 1 of reference 1; and it is clear that, in general, the j^2 values contributing to J_s will be smaller than the maximum by very much larger factors.

K_c/K_m values much closer to unity are possible for pipe material quite dissimilar to steel; a P.V.C. pipe (M_{LP} = 5.8, μ = 0.4) would have a minimum K_c/K_m of 12.2 $\sqrt{\beta}/M_o$, which yields 1.7 for the same limiting conditions as for steel.

4. An approximation for acoustic power radiation

4.1 The joint acceptance and J_s

It can be concluded from the preceding discussion of hydrodynamic coincidence that for most cases of practical interest $K_c \gg K_m$.

If the minimum value, for supersonic modes , of the ratio of k_c to the other wave-number component/is expressed in a similar way to that of K_c/K_m, namely as $K_c/K_n = F_n\, M_{LP}\sqrt{\beta\psi}/M_c$ then

$$F_n = \begin{cases} \dfrac{\bar{\nu}_{ac}^{1/2}}{[1-\bar{\nu}(1-\bar{\nu}^2/\bar{\nu}_{ac}^2)^{1/2}]^{1/2}} & \text{for } \bar{\nu}/\bar{\nu}_{ac} \leqslant 1 \\ \bar{\nu}^{1/2} & \text{for } \bar{\nu}/\bar{\nu}_{ac} \geqslant 1 \, ; \end{cases} \tag{15}$$

and the minimum possible value of K_c/K_n is $\bar{\nu}_{ac}^{1/2} M_{LP}\sqrt{\beta\psi}/M_c = 1/M_c$. Thus, if the flow Mach number is restricted to M_o=0.5 (M_c = 0.35), as in section 3, K_c/K_n will always be greater than about 3, irrespective of the pipe material and the value of β. We can therefore expect the assumption $K_c >> K_n$ to be valid over a large part of the field of interest.

Assuming that these two inequalities are satisfied, that $c_x^2 << 1$ (experiment indicates c_x = 0.1), that c_y is of order unity (actually it is about 0.7), and that, in addition, $K_c >> 1/c_x\Lambda$ and $1/\pi c_y$, equations (8) and (9) can be replaced by the very much simpler approximate equations

$$j_{mm}^2 \simeq \frac{c_x}{\Lambda K_c} = \frac{u c_x M_o}{\Lambda \nu M_{LP}} \tag{16}$$

and

$$j_{nn}^2 \simeq \frac{1}{2\pi c_y K_c} = \frac{u M_o}{2\pi c_y \nu M_{LP}} \, . \tag{17}$$

Then

$$j^2(\nu,\theta) \simeq \frac{1}{2\pi}\frac{c_x}{c_y}\frac{u^2 M_o^2}{\Lambda \nu^2 M_{LP}^2} \, . \tag{18}$$

The inequality $c_x \Lambda K_c >> 1$ will be adequately satisfied except for short pipes at high flow speeds. The inequality $c_y \pi K_c >> 1$ is more difficult to satisfy; it is valid for large pipes at low flow speeds, but the approximation becomes rather poor for small-diameter pipes at high flow speeds.

The approximations, equations (16) and (17), were obtained previously by Clinch[2], even though his complete expression for the joint acceptance differs from ours (as a result of his analysis not taking account of symmetry conditions which must apply to the circumferential narrow-band correlation function of the pressure field - see reference 1).

When the joint acceptance given by equation (18), (which, it should be noted, is independent of θ) is substituted in equation (3), we obtain

$$J_s \simeq \frac{c_x}{c_y} \cdot \frac{u^2 M_o^2 N_s(\nu)}{2\pi\Lambda M_{LP}^2\, \nu^2} \, , \tag{19}$$

where

$$N_s(\nu) = \int_{\theta_1}^{\theta_2} n(\nu,\theta)\, d\theta . \tag{20}$$

4.2. The modal density of supersonic modes

We wish to obtain an approximate expression for the density of supersonic modes, and for analytical convenience we shall use an approximate resonance frequency equation similar to that proposed by Heckl[3], namely

$$\nu^2 = (\beta K^2 + \psi \sin^2\theta)^2 , \tag{21}$$

rather than equation (11). Following the same procedure used by Heckl for the total modal density we obtain

$$N_s(\bar{\nu}) = \begin{cases} \dfrac{\Lambda}{4\pi\beta}\left[\cos^{-1}(1-2\bar{\nu}) - \cos^{-1}\{[(1-\bar{\nu}^2) + 4\beta K_n^2/\psi]^{1/2} - \bar{\nu}\}\right] & \text{for } \bar{\nu}<1 \\ \dfrac{\Lambda}{4\pi\beta}\cos^{-1}\{\bar{\nu} - [(\bar{\nu}-1)^2 + 4\beta K_{n1}^2/\psi]^{1/2}\} & \text{for } \bar{\nu}>1 \, , \end{cases} \tag{22}$$

where K_{n1} is the maximum value of K_n at which supersonic modes occur at a given frequency.

For $\bar{\nu}<<1$, $\beta K_{n1}^2/\psi \simeq \bar{\nu}^2/\bar{\nu}_{ac}$, and we find

$$N_s(\nu) \simeq \frac{\Lambda\, \nu^{3/2}}{4\pi\beta\nu_{ac}\psi^{1/2}} = \frac{\Lambda\, \nu^{3/2} M_{LP}^2}{4\pi\psi^{1/2}} \, . \tag{23}$$

4.3 Acoustic power radiation

Using equations (6), (19) and (23) we can obtain the following low frequency approximations to J_s and the spectral density of the acoustic power radiated:

$$J_s \simeq \frac{1}{8\pi^2} \cdot \frac{c_x}{c_y} \cdot \frac{u^2 M_o^2}{\psi^{1/2} \nu^{1/2}} , \tag{24}$$

and

$$\frac{\Phi_{PR}}{\Phi_p} \simeq \frac{1}{96\pi} \cdot \frac{c_x}{c_y} \cdot \frac{\rho_{fs}^2 u^2 M_o^5 Q}{\psi^{1/2} \beta^2 \nu^{3/2} M_{LP}^2} . \tag{25}$$

5. Results and discussion

5.1. J_s

In this discussion, we shall refer to calculated results based on equations (3), (4), (5) and (7)-(10), without any further approximation, as exact values, and those obtained directly from equations (24) and (25) as approximate values.

As indicated in section 2, to define the functional form of the spectral density of the acoustic power radiated as given by equation (5), we have to consider the form of J_s. In general $J_s = J_s(\nu,\beta,\Lambda, M_{LP}, M_c)$, but the approximation, equation (24), is independent of β,Λ, and M_{LP} (indicating independence of pipe geometry and material).

Well away from hydrodynamic coincidence ($K_c/K_m >> 1$), $j_{mm}^2 \sim 1/\Lambda$, j_{nn}^2 is independent of Λ, and $j^2 \sim 1/\Lambda$; as coincidence is approached there is an increasing additional dependence of j^2 on $1/\Lambda^2$, but in general this remains weak. Hence, since $n(\nu,\theta) \sim \Lambda$ and the integration limits, θ_1 and θ_2, in equation (3) are independent of Λ, J_s can be expected to be essentially independent of Λ even when coincidence occurs. This has been confirmed by exact calculations of J_s for steel and P.V.C. pipes with flow speeds well in excess of practical values ($M_o = 2.75$) and sufficiently high to give coincidence for supersonic modes, for $10 \leqslant \Lambda \leqslant 100$ and for β values of 10^{-2} and 10^{-3}. It might be noted here that in the exact calculations the upper limit of integration, θ_2, is taken as the value of θ corresponding to the n = 1 mode at a given frequency for $\bar{\nu} < 1$ (since the n = 0 mode cannot occur for $\bar{\nu} < 1$), and as $\pi/2$ for $\bar{\nu} > 1$.

Consider next the dependence of J_s on β. j^2 remains almost independent of β except at K_c/K_m values quite close to unity, and $n(\nu,\theta) \sim 1/\beta$; so the integrand of J_s is $\sim 1/\beta$. For ν less than the acoustic coincidence frequency ν_{ac}, the range of integration, effectively determined by $\theta_1 = \sin^{-1}[\bar{\nu}^2(1 - \beta^2 M_{LP}^4 \bar{\nu}^2)]^{1/4}$, introduces a β-proportionality, resulting in J_s independent of β. For $\nu > \nu_{ac}$, the integration range is independent of β and $J_s \sim 1/\beta$. Thus, even if coincidence occurs, $J_s \sim \beta^o$ or β^{-1} according as ν is less or greater than ν_{ac}, and this is confirmed by exact calculations.

Having concluded that, at least for $\nu < \nu_{ac}$, J_s is essentially independent of both Λ and β, we can examine the remaining variables without having to consider wide ranges of β and Λ. The variation of exact values of J_s with ν for $\beta = 0.005$, $\Lambda = 20$, and $M_o = 0.2$ and 0.5 is shown in figure 1 for two values of M_{LP} (18 and 6) representative of steel and P.V.C. respectively. The results show that J_s is, at least, roughly independent of pipe material. Over the central part of the range $0 \leqslant \nu \leqslant 1$ the values of J_s do not differ by a factor of more than about 1.5 (equivalent to a 2 dB difference in power spectral levels) and the agreement extends to higher frequencies than those for which equation (24) might be expected to apply. At low frequencies the comparison is obscured by the different values of ν at which an n = 1 mode first becomes supersonic.

The dependence of J_s on M_o is determined by the variation of j^2 with M_o, since $n(\nu,\theta)$, θ_1, and θ_2 are each independent of M_o; and, except for cases very close to coincidence, $j^2 \sim M_o^2$. Exact values of J_s against ν for various Mach numbers are shown in figure 2 for a steel pipe with $\beta = 0.005$ and $\Lambda = 20$. The conditions represented do not very closely approach coincidence, and the curves show a very precise M_o^2-dependence of J_s.

Finally we compare the frequency dependence of J_s predicted by the approximate equation (24) with that obtained from exact calculation. Typical comparisons are shown in figure 1. Agreement is good at low frequencies, except where the effect of allowing no radiating modes for n<1 at $\bar{\nu} < 1$ in the exact calculations is evident. But, as might be expected since the approximation is basically a low-frequency one, the agreement deteriorates as ν increases. This results primarily from the fact that the modal density N_s varies as higher powers of ν as ν increases; and if account were taken of this, the approximation would be improved.

5.2. Spectral density of acoustic power radiation

Equation (25) for Φ_{PR}/Φ_p does not involve any additional approximation beyond that of equation (23), and the discussion in the preceding section applies equally well to it. Φ_{PR}/Φ_p can be obtained from J_s simply by applying the factor $(\pi\rho_{fs}^2 M_o^3 Q/12\beta^2 \nu M_{LP}^2)$.

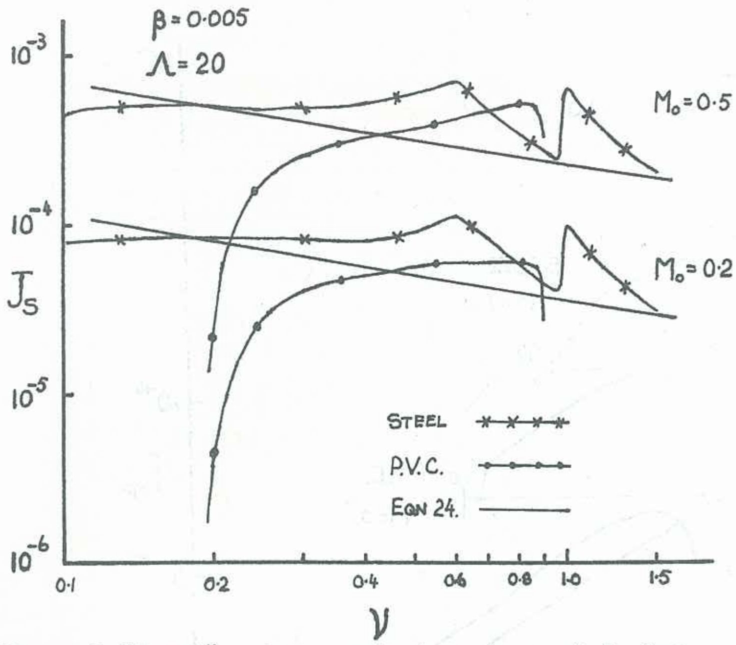

Figure 1 "Exact" and approximate values of J_s for different pipe materials

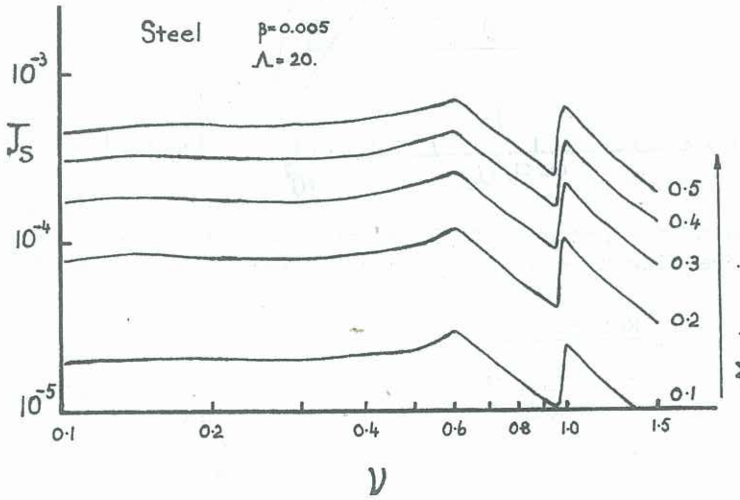

Figure 2 Effect of Mach number on frequency variation of J_s

The form of variation of the spectral density Φ_{PR} itself with ν will be additionally dependent on flow Mach number and pipe material (through M_{LP}), since the spectral density of the pressure field of the internal turbulent flow Φ_p is a function of $\omega a/U_o = \nu\, M_{LP}/M_o$ only. This is illustrated by figure 3, which shows spectra for steel and P.V.C. pipes at various values of M_o, based on the turbulent boundary layer data of Bull.(4) In general the effect of the shape of the pressure spectrum will be to increase the power of M_o on which the spectral density depends.

5.3. Total radiated power

The dependence of the non-dimensional power radiated on various parameters can be obtained directly from equation (25), apart from the modifying influence of Φ_p on the dependence on ν, M_{LP} and M_o.

In dimensional terms, the power will be proportional, in particular, to $U_o^5 a^4 \ell/h^2$, again with some modification of the variation with a and U_o depending on the form of Φ_p.

Overall sound power levels for steel and P.V.C. pipes with β = 0.005 and 0.01 as a function of the flow Mach number, are shown in figure 4. The values given are those from exact calculations; corresponding values based on the approximate spectral density relation are not given, since the relative values of the results from the two methods are very sensitive to the choice of low frequency limit in the case of the approximation.

Figure 4 indicates that quite high sound power levels can be generated by turbulent flow in thin-walled pipes for flow conditions which are not in any way extreme, even when the flow is not disturbed by pipe fittings.

6. Conclusion

From the analysis presented here it would appear that for practical piping systems, defined by say $\Lambda > 10$, $0.005 < \beta < 0.05$, and $0 < M_o < 0.5$, the spectral density of acoustic power radiation can be quite well represented by a simple approximate expression.

The significant results are that the radiated power is proportional to the fifth (or higher) power of flow speed, pipe length, the fourth (or perhaps lower) power of pipe radius, and inversely proportional to the square of the wall thickness.

The results obtained indicate that undisturbed fully-developed turbulent pipe flow in thin-walled pipes may well give rise to objectionably high radiated sound levels.

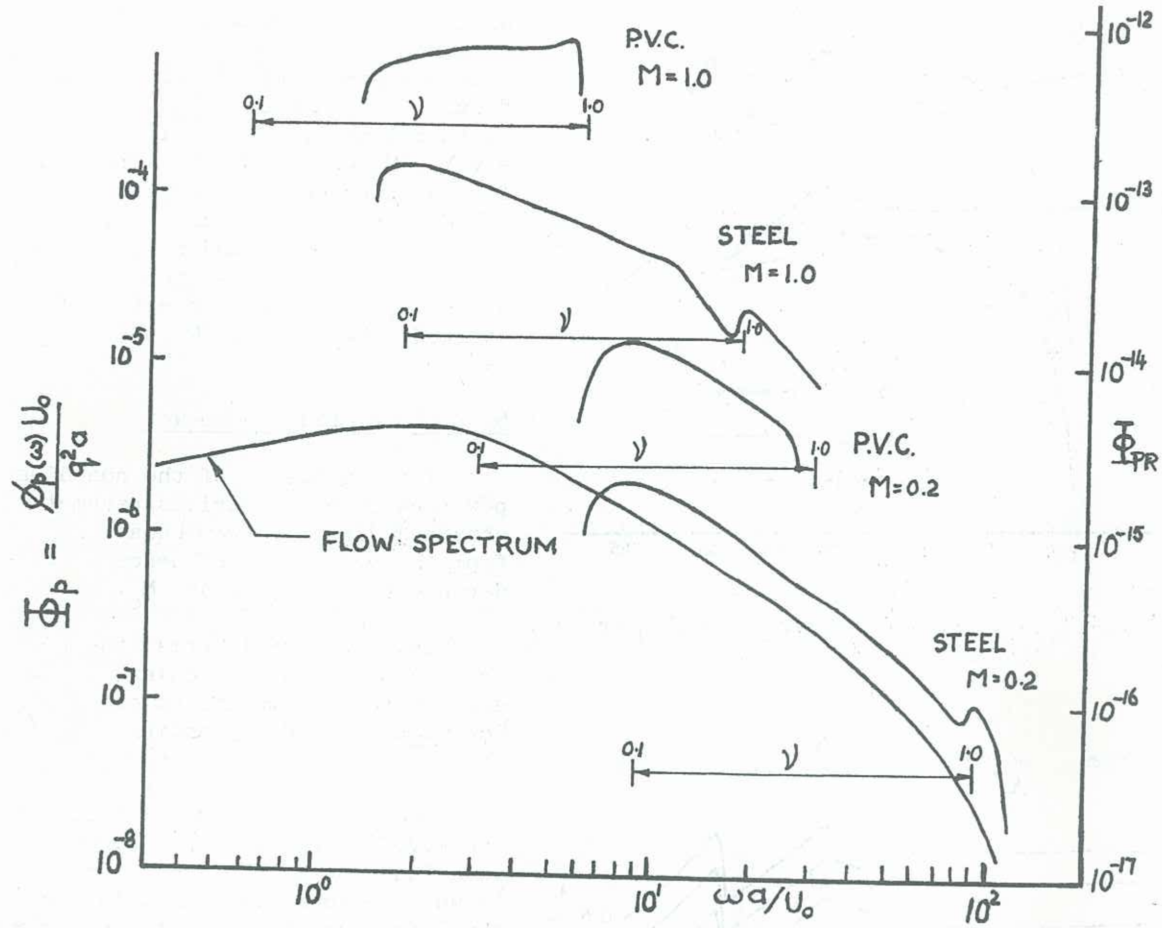

Figure 3 Comparison of non-dimensional spectra of the pressure field excitation and the radiated acoustic power, showing effect of increasing M_o and of different materials (β = 0.005 and Q = 500 for both materials).

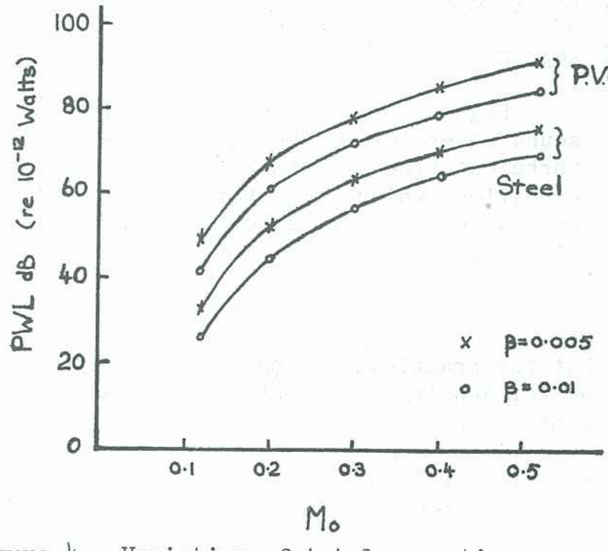

Figure 4 Variation of total acoustic power radiation with Mach number. Effect of different materials and wall thickness - to-diameter ratio.a = 50mm.

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