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ACCELERATING AND DEACCELERATING FLOWS OF VISCOELASTIC FLUIDS

by

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S U M M A R Y

The results of a number of research projects dealing with accelerating and deaccelerating flow of inelastic and viscoelastic fluids through abrupt circular expansions and contractions are discussed. A numerical solution of the equations of motion for flow of inelastic power-law fluids through an abrupt circular expansion is experimentally verified. The influence of elasticity for viscoelastic fluids is investigated by comparing the observed flow field for the viscoelastic fluid in the expansion to the predicted flow field for the corresponding inelastic fluid. Results for the expansion are reported as the deviation from inelastic behaviour as a function of the shear wave-friction velocity ratio. These results are compared to similar data available for flow through an abrupt circular contraction. The conditions for deviation from inelastic behaviour and for the onset of flow instabilities are specified and coincide for the expanding and contracting flows.

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INTRODUCTION

Acceleration and deaccelerating flows of viscoelastic fluids are of considerable importance in the processing of polymer melts, where flow instabilities attributed to the elasticity can cause undesirable behaviour such as melt fracture in extrusion. This paper deals with the investigation of these instabilities and the prediction of the conditions at which they occur. It represents a current summary of a number of research projects both past (1,2,3) and present.

Weissenberg (4) first clearly noted that the behaviour of viscoelastic fluids in simple shear can be quite unlike that exhibited by inelastic fluids. For instance if a rod is rotated in a beaker containing a viscoelastic fluid, instead of a vortex being formed as is expected, the viscoelastic fluid pushes towards the centre of rotation of the rod and climbs up the rod. Centrifugal forces are overcome with a net centripetal force resulting. Weissenberg reasoned that this effect was due to the development of finite strains within the fluid which in turn result in the development of unequal normal stresses. These stresses are called deviatoric normal stresses because they represent a deviation from isotropy. The climbing rod effect is a result of the deviatoric normal stress in the direction of flow, P_{11}, being significantly greater than the other two, P_{22} and P_{33}. A tension in the direction of shear develops which increases towards the axis of rotation of the rod as the shear rate increases. The result of this tension is a hydrostatic pressure increase towards the axis of rotation of the rod. Therefore to completely specify or characterize an incompressible viscoelastic fluid it is necessary to measure the shear stress and two of the three deviatoric normal stresses. The results are usually reported as the shear stress, τ_{12}, the first normal stress difference, $P_{11} - P_{22}$, and the second normal stress difference, $P_{22}-P_{33}$, as a function of shear rate. Measurement of $P_{11}-P_{22}$ has become a standard practice but the techniques for measurement of $P_{22}-P_{33}$, which is considerably smaller than $P_{11}-P_{22}$, are not clearly defined.

In fully developed tube flow the presence of unequal normal stresses does not effect the energy requirements or the velocity profile (2,5). That is, the velocity profile can be predicted from the observed non-linear relationship between the shear stress and shear rate. In contrast, the large unequal normal stresses play a dominant role in accelerating and deaccelerating tube flows of viscoelastic fluids.

Developing and fully developed velocity profiles have been measured in an abrupt 2 to 1 circular contraction and in an abrupt 1 to 2 expansion. Streak photography was employed for determining point velocities and to record flow patterns. A detailed description of the apparatus employed is available elsewhere (6,7). The test fluids used were aqueous solutions of Methocel 90HG and Separan AP30. These polymer solutions were chosen because of the wide range of inelastic and viscoelastic properties that can be obtained with concentration variation, the absence of permanent shear and normal stress degradation upon testing in the flow system, and because of their optical clarity.

Samples of test fluid taken before and after a run on the flow system were rheologically characterized using an R16 Weissenberg Rheogoniometer. The shear stress and first normal stress difference were measured as a function of shear rate in the range of shear rates for which velocity profile measurements were made. Normal thrust measurements were used to determine the first normal stress difference. A fluid was termed viscoelastic at a particular shear rate when a measurable normal stress was observed with the Rheogoniometer. The observed shear stress-shear rate behaviour of all inelastic and viscoelastic fluids was fitted with an Oswald-de-Waele power-law model given by

$$\tau_{12} = K \dot{\gamma}^n \qquad (1)$$

where $\dot{\gamma}$ = shear rate
K = consistency factor
n = flow behaviour index

In order to investigate the influence of elasticity on the expansion and contraction flow fields, it is first necessary to understand the behaviour of the inelastic fluid. The literature abounds with theoretical attempts to predict the velocity field for flow through an abrupt contraction (8). Most of this work has been confined to Newtonian fluids in the downstream tube by the imposition of a flat entry velocity profile as a boundary condition. Two workers have considered the presence of the upstream tube (9,10). One realistic solution is available for inelastic power law fluids (11) at higher Reynolds numbers and there has been one attempt to consider flow of inelastic power law fluids through the entire contraction (9). No comprehensive study is available for inelastic power law fluids at low Reynolds numbers where the normal stress term in the equations of motion, $\frac{\partial \tau_{xx}}{\partial x}$, is significant even for inelastic fluids. This term, representing axial diffusion of momentum, becomes insignificant at higher Reynolds numbers (≈20) for inelastic fluids. For viscoelastic fluids axial diffusion of momentum is particularly significant at higher Reynolds numbers. The boundary layer solution available for inelastic power law fluids (11) has been experimentally verified for a 2 to 1 contraction for $0.585 \leq n \leq 1.00$ and for $20 \leq N_{Re'} \leq 1942$(2) where $N_{Re'}$ is the Reynolds number for a power-law fluid given by

$$Re' = \frac{D^n V^{2-n} \rho}{K\, 8^{n-1} (\frac{3n+1}{4n})^n} \qquad (2)$$

For flow through an abrupt expansion no theoretical solutions or experimental results are available for power-law fluids. Hung (13), investigating the laminar eddies in the corner of the expansion, solved the complete flow equations for a Newtonian fluid in a two dimensional planar expansion and in the axi-symmetric tubular expansion. Both geometries had a 1 to 2 expansion ratio and the Reynolds number ranged from 0 to 333 and 0 to 200 respectively. In this work a numerical solution of equations of motion was developed for inelastic power-law fluids for a circular 1 to 1 expansion. The numerical method, based on the Alternating Direction Implicit Method (ADI), was found to be fast, accurate and stable. The influence of axial diffusion of momentum was investigated by use of a stream tube-real tube model (ST-RT) and a real tube-real tube model (RT-RT). Only the results of this analysis will be quoted here. The interested reader is referred to Reference 7 and 12 for more detail about the mineral technique.

RESULTS AND DISCUSSION

Only the theoretical results for inelastic and experimental results for inelastic and viscoelastic fluids in the 1 to 2 abrupt expansion will be presented here as the experimental results for the 2 to 1 contraction are presented elsewhere (1,2). The two flow fields will however be compared when the viscoelastic fluid results are discussed.

INELASTIC FLUIDS

Comparison with Previous Work

The re-attachment length is the length of secondary cell formed in the corner of the larger tube of the sudden expansion. It is reported as L/D where L is the length of the secondary cell and D is the diameter of the smaller tube. Figure 1 shows the re-attachment length of the secondary cell predicted by the numerical scheme for a Newtonian fluid (n=1) in comparison to the results obtained by Hung (13). The excellent agreement shown in Figure 1 demonstrates in part the accuracy of the numerical scheme.

Experimental Re-attachment Lengths

Figure 2 shows the experimentally observed reattachment lengths in comparison to the computed lengths for $0.65 \leq n \leq 0.81$ and $6.7 \leq Re_{mod} \leq 158.8$. Note that Re_{mod}, defined by,

$$Re_{mod} = \frac{\rho D^n V^{2-n}}{K} \qquad (3)$$

differs from Equation 2 which has been used to define the Reynolds number in the tube entry flow work. Excellent agreement is seen between the experimental data and the numerical predictions. The re-attachment length is found to increase with Re_{mod} and decrease with increasing n. Exact relationships are available (7).

Figure 3 shows the experimentally observed flow patterns for an inelastic fluid characterized by n = 0.76 and at $Re_{mod} = 11.2$. The numerical solution for the stream function using the RT-RT model is superimposed on the upper half of the photograph. The agreement between the observed and predicted streamline is excellent.

Figure 4 shows a similar comparison for an inelastic fluid characterized by n = 0.73 and at a higher Re_{mod} ($Re_{mod} = 21.6$). At this Re_{mod} the ST-RT model was found to be adequate. Axial diffusion of momentum was found to be significant for $Re_{mod} < 20$. Since the RT-RT model takes axial diffusion into account it was used for $Re_{mod} < 20$ while the ST-RT model was used for $20 \leq Re_{mod} \leq 200$.

Experimental Velocity Field Measurements

To judge the accuracy of the experimental techniques employed for point velocity measurement fully developed axial velocity profiles were measured both upstream and downstream of the plane of the expansion for all fluids tested. Typical profiles are shown in Figure 5 for n = 0.73 and

$21.6 \leq Re_{mod} \leq 121.0$. V is the average velocity in the upstream tube of radius R. The open and closed points represent the local velocity on either side of the centerline of the tube, thus showing the symmetry of the profile. The experimental profile is compared to the fully developed profile predicted for a power-law fluid of n = 0.73. The small error (≃ 3%) near the centerline is due to the polymer solution tending to Newtonian behaviour at the centerline where the shear rate approaches zero.

The development of the centerline velocity from fully developed flow in the small tube to fully developed flow in the large tube was also measured. Figure 6 shows the excellent agreement that was obtained between the numerical prediction and the experimental data; in this case for n = 0.73 and $21.6 \leq Re_{mod} \leq 121$. Similar agreement between the numerical prediction and the experimental data were obtained for developing centerline velocities and for entire developing velocity profiles for $0.65 \leq n \leq 0.81$ and for $6.7 \leq Re_{mod} \leq 158.8$. This data in addition to complete fluid characterizations are available (7).

Being now able to predict the behaviour of inelastic power-law fluids in an abrupt expansion, one can justifiably observe the behaviour of viscoelastic fluids of similar flow behaviour index, n, and in the same range of Re_{mod} for which the behaviour of inelastic fluids is understood. A comparison of the numerical predictions for the inelastic fluid to the experimentally observed behaviour of the viscoelastic fluid of the same n and at the same Re_{mod} allows the influence of elasticity to be determined.

VISCOELASTIC FLUIDS

A number of parameters can be used to quantitatively evaluate the degree of elasticity of a viscoelastic fluid. Two of these are the Weissenberg number and the ratio of the friction velocity to the shear wave velocity. The Weissenberg number for tube flow is given by

$$W_e = \frac{V\theta}{D} \tag{4}$$

where θ is the relaxation time of the fluid which can be defined by (14)

$$\theta = \frac{P_{11}-P_{22}}{2\tau_{12}\dot{\gamma}} \tag{5}$$

The ratio of the friction velocity to the shear wave velocity is given by (15)

$$\eta = \frac{\left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}}}{\left(\frac{\mu_{app}}{\rho\theta}\right)^{\frac{1}{2}}} \tag{6}$$

where for tube flow the apparent viscosity is

$$\mu_{app} = \frac{\tau_w}{\frac{8V}{D}} \tag{7}$$

and τ_w is the shear stress at the tube wall. Then using Equations 6, 7 and 4 yields

$$\eta = \left(\frac{8V\theta}{D}\right)^{\frac{1}{2}} = (8W_e)^{\frac{1}{2}} \tag{8}$$

Equation 8 indicates that the Weissenberg number and the friction velocity - shear wave velocity are equivalent as correlating parameters. The friction velocity-shear wave velocity ratio is used in this work.

In order to evaluate η a shear rate in the flow field must be chosen. The usual choice is the wall shear rate and since the most likely point to generate any unusual behaviour is at the corner of the expansion just at the inlet, the shear rate at this point should be used. The value of the shear rate at this point may be approximated by the fully developed value of the shear rate in the small tube which is given for a power-law fluid by

$$\dot{\gamma}_w = \frac{3n+1}{4n}\left(\frac{8V}{D}\right) \tag{9}$$

η was then evaluated by using the average velocity, V, in the upstream tube of diameter D and by determining $P_{11}-P_{22}$ and τ_{12} from the fundamental flow property measurements at $\dot{\gamma}_w$.

In the flow field the re-attachment length was found to be the most sensitive parameter to the elasticity of the fluid. The discussion is therefore confined to this parameter. Details of the velocity field for the viscoelastic fluid are available elsewhere (7). The observed re-attachment lengths for viscoelastic fluids are presented in Figure 7 as a deviation from inelastic behaviour, δ, where

$$\delta = \frac{\left(\frac{L}{D}\right)_{EST} - \left(\frac{L}{D}\right)_{MEAS}}{\left(\frac{L}{D}\right)_{EST}} \times 100\% \qquad (10)$$

$\left(\frac{L}{D}\right)_{EST}$ is the re-attachment length predicted from the inelastic numerical solution for a fluid of the same n and at the same Re_{mod} as that for which $\left(\frac{L}{D}\right)_{MEAS}$ was observed for the viscoelastic fluid. For $n < 0.58$ it was not possible to obtain a numerical solution for the inelastic fluid due to stability problems arising at the smallest computing mesh size which could be used with the available computer core. $\left(\frac{L}{D}\right)_{EST}$ for these cases was estimated by linearly extrapolating to the relevant power-law index from the available numerical results. This is a conservative estimate as the true computed value of $\frac{L}{D}$ was a slightly increasing non-linear function of n (7).

Also shown in Figure 7 are the results of Boger and Rame Murthy (2) for the reversed flow through an abrupt 2 to 1 contraction. δ' represents the deviation of the observed entry length for the viscoelastic fluid from that predicted for the inelastic fluid by the analysis of Collins and Schowalter (11). η was again evaluated at the conditions in the smaller tube.

The entry lengths and the re-attachment lengths for all viscoelastic fluids examined were always less than the inelastic predictions and in both cases the deviation from inelastic behaviour was independent of the Reynolds number and flow behaviour index. For $\eta < 0.6 (W_e = 0.045)$ no significant deviation from inelastic behaviour was observed in either the expansion or contraction flow. Further increase in η resulted in a rapid and continuous increase in the deviation from inelastic behaviour. Associated with this rapid increase is a predevelopment of the velocity field, i.e. a rapid decrease in the size of the secondary cell for the expansion and a rapid decrease in the entry length for the contraction until at $\eta = 1$ ($W_e = 0.125$) for the contraction, the entry length was reduced to zero, the entry velocity profile was fully developed, and the viscoelastic fluid was able to completely anticipate the contraction. Unfortunately there is not sufficient data for the expansion to determine if the secondary cell is reduced to nonexistence as η goes to 1 although a maximum reduction of 66% from inelastic behaviour was observed at $\eta = 0.96$.

The results in Figure 7 show that for a Weissenberg number is less than 0.045 the flow field for the viscoelastic fluid in the expansion or contraction does not deviate from inelastic behaviour. For Weissenberg numbers greater than 0.045 the viscoelastic fluid anticipates the contraction or expansion with the upper limit of anticipation (predevelopment)occurring when the shear wave and friction velocity are of the same order. Since the shear wave velocity is a measure of the velocity of propagation of information in a viscoelastic fluid, it may not be surprising that predevelopment in both flow fields occurs when the shear wave velocity is greater than a characteristic system velocity. η might then be regarded as analogolous to a Mach number in compressible flow.

A reversible and perhaps a discontunity in the deviation from inelastic behaviour occurs for both the expansion and contraction flows for $\eta > 1$. Also beyond this point, the functional relationships between the deviation from inelastic behaviour and the friction velocity-shear wave velocity ratio depart from each other. However for $\eta > 1$, the two flows have one additional common characteristic; each flow field gradually becomes unstable. The precise conditions for the instabilities have not been determined and are the subject of continuing work. Since it is not the purpose of this paper to discuss the nature of these instabilities, the interested reader is referred to the photographs and descriptions available (2,7).

Denn and Porteous (15) on theoretical grounds first suggested that the onset of a striking number of viscoelastic phenomena may be correlated by the equality of the shear wave velocity and a characteristic kinematic velocity. The phenomena included : drag reduction in dilute and concentrated polymer solutions, melt fracture in polymers and anonalous effects in éxternal flows. This condition has also been used to predict the critical wall shear stress for melt fracture (3). Jones and Day (16) also observed an abrupt decrease in the discharge coefficient for flow of dilute polymer solutions through an orifice. Their results, consistent with those of Giles (17), supported the arguments of Denn and Porteous.

CONCLUSION

The data presented here in comparison with the experimental and theoretical results available elsewhere suggest that flow anonalies in viscoelastic fluids may be expected when the shear wave velocity and a characteristic kinematic velocity of the system are of the same order. In particular for accelerating flows in an abrupt contraction and deaccelerating flows in an abrupt expansion, unstable flow can be expected when the shear wave velocity exceeds the friction velocity. In addition for $\eta < 0.6$ no deviation from inelastic behaviour for the viscoelastic fluid is expected and for $0.6 < \eta \leqslant 1.0$ anticipation by the fluid of the contraction or expansion can be expected. Further work is required to precisely define the nature of the instabilities and to investigate the influence of contraction and expansion ratios.

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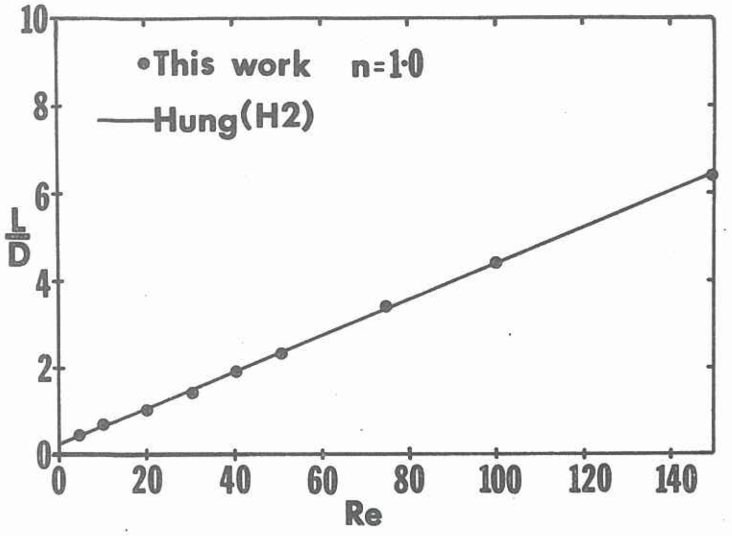

FIGURE 1 : Comparison of re-attachment length with previous numerical work.

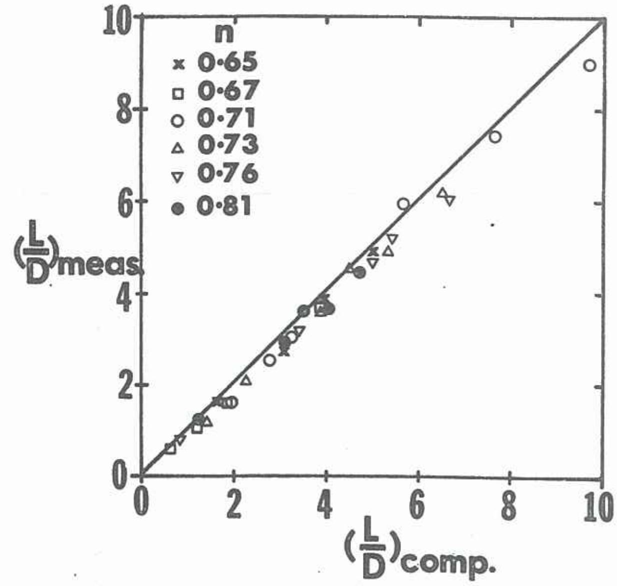

FIGURE 2 : Comparison of experimentally measured and numerically predicted re-attachment lengths.

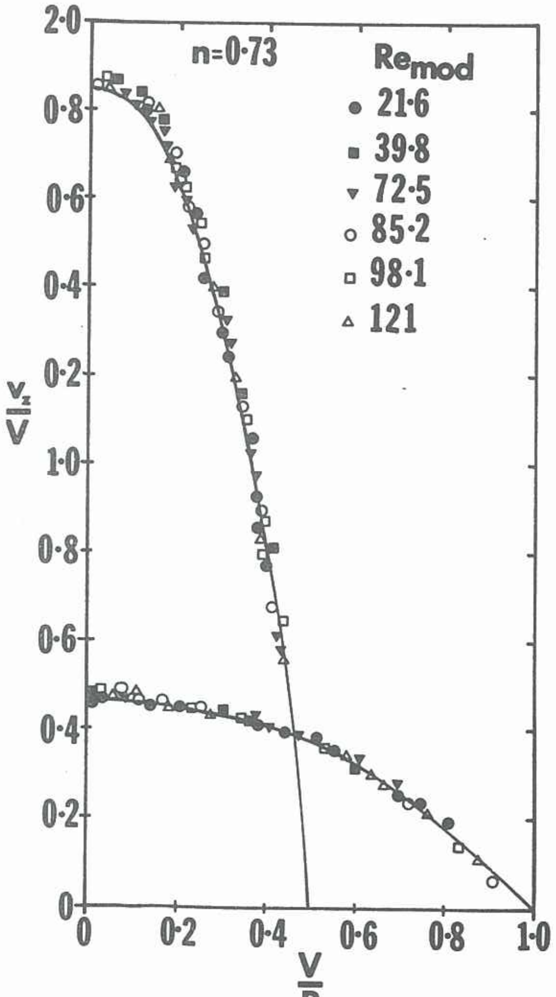

FIGURE 5 : Fully developed velocity profiles upstream and downstream of the expansion

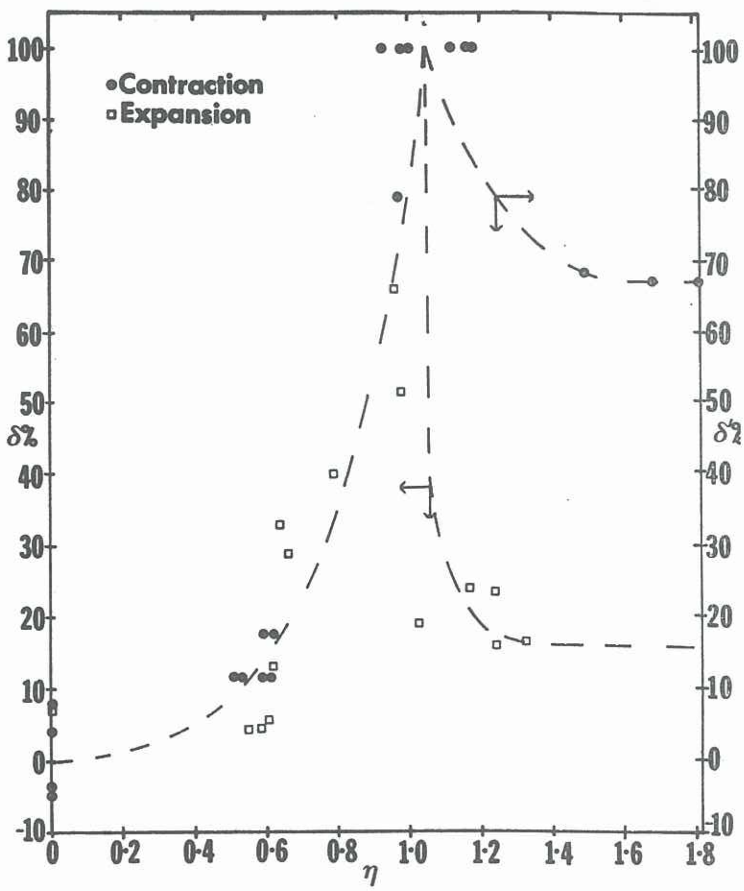

FIGURE 7 : Deviation from inelastic behaviour versus friction velocity-shear wave velocity ratio for the expansion and contraction.

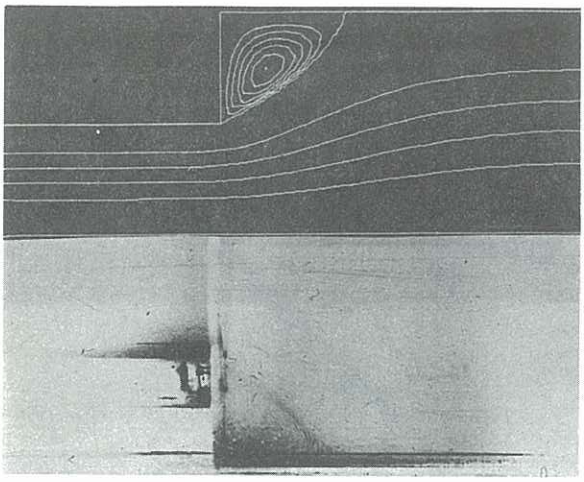

FIGURE 3 : Stream lines for n = 0.76, Re_{mod} = 11.2 with numerical comparison.

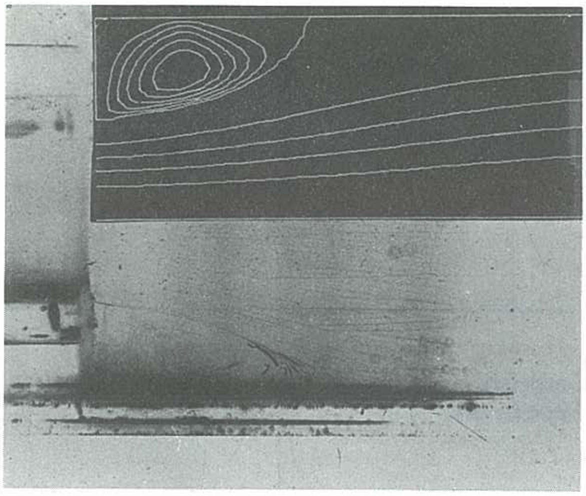

FIGURE 4 : Stream lines for n = 0.73, Re_{mod} = 21.6 with numerical comparison.

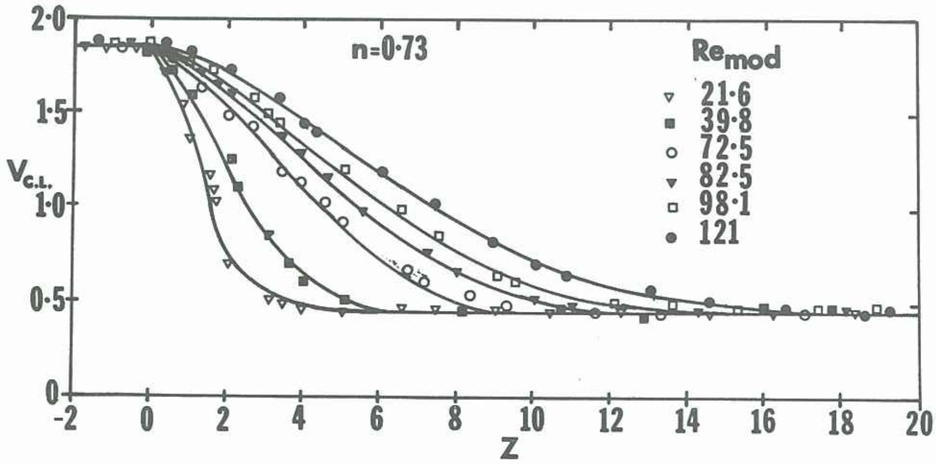

FIGURE 6 : Centre line velocity development.