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MOISTURE MOVEMENT IN PINE SAPWOOD UNDER KILN-SEASONING CONDITIONS

by

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SUMMARY

Lumped-parameter models, which can successfully describe the process conditions during kiln-seasoning, do not provide information about moisture-content profiles normal to the exposed surface of the boards. Such profiles may be generated by solving the diffusion equation with the appropriate concentration-dependent "diffusivity" for the movement of moisture through the timber. For movement below the fibre-saturation point, the authors use an improved version of Stamm's model of multiple diffusional paths through resin-free softwood material which permits a more detailed description of the flow through the pores. For movement above the fibre-saturation point, the authors estimate a capillary-flow coefficient predicted from pore-size distributions and other microscopic data on the structure of pinewood.

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INTRODUCTION

Previous work in this department (1) has shown how a lumped-parameter model, based on the concept of the characteristic drying curve (2), can be used to predict the variations of process conditions in a timber kiln. The calculation provides information about board-to-board changes in moisture content, but cannot determine the moisture-content profiles normal to the exposed surface of the board. Such profiles are needed in the development of drying schedules that represent an economic minimization of degrade, kiln-time, and energy requirements.

This paper develops equations for moisture movement within softwoods and predicts an effective concentration-dependent diffusivity over the complete range of moisture content from the green condition to the dried state. The moisture-gradient patterns are obtained by solving the one-dimensional equation of diffusion for a generalised convective boundary condition at the exposed surface.

We may distinguish between two regions of flow:

1. Capillary-flow regime, $X > X_{hygr}$;
2. Hygroscopic-moisture flow regime, $X < X_{hygr}$.

In general, the moisture is only weakly attached to its host material in the capillary-flow regime, and the energy of bonding is assumed to be negligible. The flow is caused by capillary-tension forces acting on liquid threads in the macrocapillaries of the material. The end of this regime is defined as the moisture content corresponding to 1% depression of the vapour pressure, as given by the moisture isotherm. The corresponding moisture content (X_{hygr}) is known as the fibre-saturation point. Below the fibre-saturation point, moisture is energetically bound to its host material, and the moisture flow is caused by diffusive-capillary flow through the micropores in the cell-walls and a vapour flow through the macroporous structure.

CAPILLARY-FLOW REGIME

The flow of liquids through wood at steady state has been shown to obey Darcy's law (3,4,5). From the equation of continuity and Darcy's law, Klute (6) has derived the equation of moisture flow through a porous, unsaturated slab:

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial z} \left(\frac{\rho_w K}{\rho_m \mu_w} \left(\frac{dp_z}{dx} \cdot \frac{\partial X}{\partial z} - \rho_w g \sin \theta \right) \right) \quad \dots (1)$$

where the symbols have the meaning indicated at the back of the paper.

Several models have been provided to describe porous media. One early model was that of a bundle of straight capillary tubes (7,8). Fatt (9) has constructed network models which include non-uniformities of the void space to predict permeability and capillary pressure, while Haring and Greenkorn (10) postulated a model which includes non-uniformities by allowing the pore radii and pore lengths to be statistically distributed. However, these distributions are assumed to be independent, which is not the case for wood where each class of pore radii has a distinctive distribution of length. The interconnecting vessels in softwood may be regarded as a bundle of parallel capillaries, but the capillaries are of varying diameters and lengths, so one must take into account the statistical distribution of effective pore radii.

1. Permeability Debye and Cleland (11) have modified the Hagen-Poiseuille's law for flow of liquid through microporous adsorptive media by postulating a slip flow along the capillary surface of the adsorbed layers. This effect should be present also in the larger capillaries. Although this slip-flow may contribute little to the total flow, it does overcome the boundary-condition anomaly noted by Ludviksson and Lightfoot (12) for deforming menisci within capillaries. These considerations lead to the expression:

$$Q = uA = \frac{\pi r^4}{8\mu_w} \left\{ (1 - \delta/r)^4 + 4\mu_w/fr \right\} \frac{dp}{dz} \quad \dots (2)$$

in which δ is the thickness of the slip layer and f is the friction factor for the slip-flow.

A force balance on the flowing liquid thread gives:

$$\frac{dp_z}{dz} \cdot Adz = \rho_w \sin \theta Adz + \frac{dp}{dz} \cdot Adz \quad \dots (3)$$

The flux is obtained from equations (2) and (3) and on noting that $\delta \ll r$ for this regime:

$$W_A = \frac{-\rho_w r^2 \epsilon}{8\mu_w} \{1 + 4\mu_w/fr\} \{-\rho_w g \sin \theta + \frac{dp_z}{dz}\} \quad \dots(4)$$

where ϵ is the void ratio available for the slip-flow.

On comparing equations (1) and (4), we obtain the permeability

$$K = \frac{r^2 \epsilon}{8} \{1 + 4\mu_w/fr\} \quad \dots(5)$$

for a circular capillary of radius r . In general, there will be a spectrum of capillary radii. The largest capillaries will be emptied first, and the effective area for flow diminishes as moisture is removed from the material. If the cumulative radius-distribution function g is defined as ratio of the number of capillaries with a radius less than or equal to r to the total number of capillaries, then

$$K = \int_{r_{\min}}^{r'} \frac{r^2}{8} \{1 + 4\mu_w/fr\} \frac{dg}{dr} \cdot dr \quad \dots(6)$$

Although pore-size distributions for natural materials can be approximated by normal distribution functions (13), beta functions were chosen in this case on the grounds of flexibility and finite range:

$$\text{i.e. } g(r^*) = \frac{(p+q-1)!}{(p-1)!(q-1)!} \int_0^{r^*} r^{p-1} (1-r)^{q-1} dr \quad \dots(7)$$

where $r^* = (r-r_{\min})/(r-r_{\max})$ and $0 < g \leq 1$.

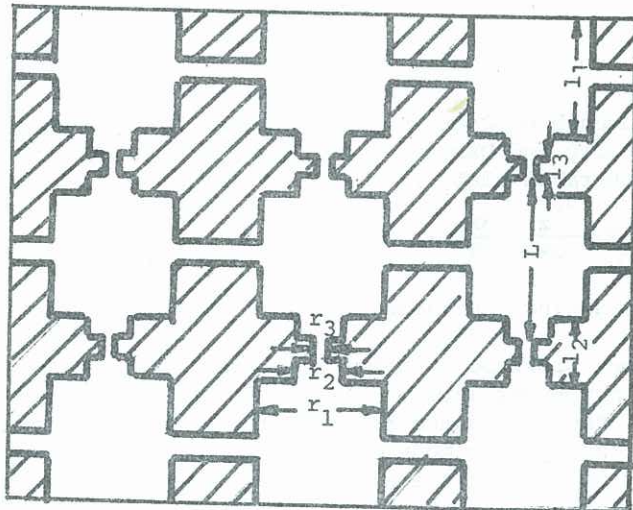


Fig. 1 Capillary model for softwood timber

Equation 6 describes the permeability of a material if it be considered as a bundle of parallel capillaries with radii independent of length. However, the flowpath for liquid in wood is through a set of unit cells, each consisting of large capillaries interconnected by two small pores of differing diameters and lengths. A corrected radius is now calculated for each capillary. If each unit cell is regarded as three simple flow resistances in series, and expansion and contraction losses may be ignored, then the volumetric flow becomes

$$Q = \frac{\pi r_e^4}{8\mu_w} \cdot \frac{\Delta p}{L} \quad \dots(8)$$

in which $r_e^4 = L / \{ l_1/r_1^4 + l_2/r_2^4 + l_3/r_3^4 \}$

In general, the path is tortuous because of non-alignment of connecting pores, blocked pathways due to extraneous materials, and the effect of dead-end pores. The final expression for the permeability thus includes a tortuosity factor α :

$$K = \int_{r_{\min}}^{r'} \frac{r_e^2 \epsilon \alpha}{8} \{1 + 4\mu_w/fr_e\} \frac{dg}{dr} \cdot dr \quad \dots(9)$$

Kininmonth (14) has obtained experimental values for the permeability of green Pinus radiata, and the tortuosity factor α can be estimated from these data.

2. Suction-potential curve For a capillary of radius r , the suction potential p is given by

$$p_z = 2\sigma/r \quad \dots(10)$$

so for a bundle of capillaries with a distribution of radii $g(r^*)$ the function $p_2(r)$ is simply obtained. These radii must be corrected for the non-uniformity of the cells. However, for a given unit cell, it may be assumed that the probability of the meniscus being at any position is uniform. This assumption leads to the definitions:

$$p_z = 2\sigma/r_e \quad \dots(11)$$

where $r_e = L/[l_1/r_1 + l_2/r_2 + l_3/r_3]$. Typical dimensions of a unit cell for Pinus radiata are: $l_1 = 30 \mu\text{m}$, $r_1 = 15 \mu\text{m}$, $l_2 = 6 \mu\text{m}$, $r_2 = 5 \mu\text{m}$, $l_3 = 1 \mu\text{m}$, $r_3 = 0.25 \mu\text{m}$ and $L = 37 \mu\text{m}$, so that $r_e = 5.1 \mu\text{m}$.

Pearse, Oliver and Newitt (15) found from experiments on drying glass beads that gravitational forces could be ignored when the beads were less than $10 \mu\text{m}$ radius, which implies a pore radius of $4 \mu\text{m}$. The above calculation shows that the effective suction-radius r_e is of a similar magnitude. Hence, equation (1) can be simplified to give

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial z} (D \frac{\partial X}{\partial z}) \quad \dots(12)$$

where the diffusivity is given by

$$D = \frac{\rho_w K}{\rho_m \mu_w} \cdot \frac{dp_z}{dX} \quad \dots(13)$$

Now, X , K , p_z are known as functions of r , so the functional dependence of K and p_z on moisture content X can be found.

HYGROSCOPIC-MOISTURE FLOW REGIME

1. Liquid movement The flow of liquid under these conditions is not well understood. Neither Hagen-Poiseuille's law nor the Kelvin's equation are valid for flow through very small pores, since both rest on the assumption that the liquid molecules are surrounded only by similar molecules. In very narrow capillaries, however, a considerable fraction of the liquid flow will be influenced by interaction with the walls. Nevertheless, Foster's investigations (16) on the adsorption of liquids on to ferric oxide show that Kelvin's equation,

$$\ln \frac{p_A}{p_S} = \frac{-2\sigma}{r} \cdot \frac{M}{\rho_w RT} \quad \dots(14)$$

may hold for small diameters provided an apparent capillary radius, corrected for the thickness of the adsorbed layer, is used. If this layer were two molecules thick, then Forster found that equation (14) was valid to a pore radius of $0.13 \mu\text{m}$.

The corrected Kelvin equation can be used to obtain a relationship between moisture content and suction potential, as the vapour-pressure depression is known as a function of moisture content from the desorption isotherm. We thus find

$$p_z = \frac{2\sigma}{r} = \rho_w RT/M = (1 - r_o/r) \ln p_S/p_A \quad \dots(15)$$

$$\text{and } \frac{dp_z}{dX} = \frac{-\rho_w RT}{M p_A} \left\{ 1 - \frac{r}{a} \right\} \frac{dp_A}{dX} \quad \dots(16)$$

in which r_a is the radial thickness of the adsorbed layer.

The Hagen-Poiseuille equation has been modified for this flow regime by Debye and Cleland, as shown in equation (2). However, they assumed that the slip layer δ was only monomolecular, whereas in general a multimolecular layer is likely (17,18). The concept of "surface diffusion" provides another way of describing this flow. Stamm (19) argues that the diffusion coefficient of this bound water may be approximated by the self-diffusion coefficient of the moisture. At low relative humidities (up to 0.2), moisture is held mainly as a monomolecular layer on the accessible surface, and at relative humidities between 0.2 and

0.6 it is supposed that successive molecular layers accumulate on this layer. At higher relative humidities the moisture is disposed as a more mobile layer that differs in character from the underlying immobilized layers (20). Hence, the movement may be regarded as the diffusion of the multimolecular layers of water through the monomolecularly adsorbed layer. This picture ignores the influence of adsorptive hindering of the motion. The energy of adsorption, however, does not become significant compared with the activation energy of diffusion until a moisture content of 2% is reached, well below final moisture content attained in kiln drying (ca. 15%).

From Debye and Cleland's definition of the slip factor f , it can be shown that

$$f = \frac{r \rho_w \epsilon^2}{2 \rho_w \rho_m} \frac{dp}{dx} \quad \dots (17)$$

where \mathcal{D}_w is the diffusion coefficient for the adsorbed layer. Hence the flux, as described by equation (2), is

$$W_A = \frac{-r^2 \rho_w}{8 \mu_w} \left\{ (1 - \delta/r)^4 + \frac{8 \mu_w \mathcal{D}_w \rho_m}{\rho_w \epsilon^2 r^2} \cdot \frac{dx}{dp} \right\} \cdot \frac{dp}{dz} \quad \dots (18a)$$

for $r \geq \delta$, and when $r < \delta$, the flux simplifies to

$$W_A = \frac{-\mathcal{D}_w \rho_m}{\epsilon} \frac{dx}{dp} \cdot \frac{dp}{dz} \quad \dots (18b)$$

From a force balance on the capillary thread

$$\frac{dp_z}{dz} = \frac{dp}{dz} = \frac{\rho_w RT}{M} \{1 - r_{a/r}\} \frac{dp_A}{dx} \cdot \frac{dx}{dz} \quad \dots (19)$$

since gravitational forces can be neglected. The effective diffusion coefficient is defined by

$$\mathcal{D}_e = W_A / \rho_m \frac{dx}{dz} \quad \dots (20)$$

so we find for \mathcal{D}_e , when $r \geq \delta$

$$\mathcal{D}_e = \frac{\rho_w r^2 \epsilon}{8 \rho_m \mu_w} \left\{ (1 - \delta/r)^4 + \frac{8 \mu_w \mathcal{D}_w \rho_m}{\rho_w \epsilon^2 r^2} \cdot \frac{dx}{dp} \right\} \frac{\rho_w RT}{p_A M} (r - r_{a/r}) \frac{dp_A}{dx} \quad \dots (21a)$$

and when $r < \delta$

$$\mathcal{D}_e = \mathcal{D}_w / \epsilon \quad \dots (21b)$$

In general, there is a range of capillary radii r and, as the pores empty, the void space shrinks, reducing the area for flow. Thus

$$\mathcal{D}_{e_{tot}} = \alpha \int_{r_{min}}^{r'} \mathcal{D}_e(r) dh = \alpha \int_{\psi_{min}}^{\psi'} \mathcal{D}_e(\psi) \frac{dh}{d} \cdot d\psi \quad \dots (22)$$

where $h = h(r)$ is the distribution of cell-wall capillary radii, which can be obtained from the moisture-desorption isotherm. The tortuosity α can be evaluated from measurements of cell-wall permeability.

2. Vapour movement The movement of vapour from a pore can be regarded as diffusion of a single component, and the corresponding flux from Fick's first law of diffusion becomes

$$W_A = \frac{-\epsilon \rho_A \mathcal{D}_{AB}}{(1 - y_A)} \frac{dy_A}{dz} \quad \dots (23)$$

However, the outflow of vapour from a pore encounters a resistance and the total pressure diminishes along the flow path. Fryer (21) corrects the Poiseuille equation for flow through microcapillaries to permit the transmission to the tube wall of the momentum generated by the pressure gradient. This modified equation predicts a diffusional flux of

$$W_A = -\rho_A r \left(\frac{2P}{\pi \rho_v} \right)^{1/2} \left[\frac{\pi}{32} \cdot \frac{P}{P_1} \{1 - \exp(-\sinh^{-1} \frac{P}{P_1})\} + \frac{\pi}{3} \frac{P}{P+P_1} + \frac{4}{3} \frac{P_1}{P+P_1} \right] \frac{dP}{dz} \quad \dots (24)$$

in which $P_1 = \lambda P / 2r$, the pressure at which the mean free path of the molecules equals the

capillary diameter. The first term of equation (24) describes the flow due to Poiseuille motion, the second term for slip-flow and the final term includes Knudsen flow.

The pressure gradient dP/dz follows from equation (23) and (24):

$$\frac{dP}{dz} = \left[\frac{\mathcal{D}_{AB}^P}{P-p_A} \cdot \frac{dp_A}{dz} \right] / \left[\frac{\mathcal{D}_{AB}^P}{P-p_A} + \mathcal{D}_f \right] \quad \dots (25)$$

in which

$$\mathcal{D}_f = r \left[\frac{2P}{\pi \rho_v} \right]^{1/2} \frac{\pi}{32} \frac{P}{P_1} \{1 - \exp(-\sinh^{-1} \frac{P}{P_1})\} + \frac{\pi}{3} \frac{P}{(P+P_1)} + \frac{4}{3} \frac{P_1}{(P+P_1)} \quad \dots (26)$$

Comparison with equation (20) yields the effective diffusion coefficient:

$$\mathcal{D}_e = \frac{\rho_v \epsilon \mathcal{D}_f \mathcal{D}_{AB} / (1-y_A)}{\rho_m \mathcal{D}_{AB} y_A / (1-y_A) + \mathcal{D}_f} \cdot \frac{dy_A}{dx} \quad \dots (27)$$

The overall vapour-phase diffusivity is obtained by integrating \mathcal{D}_e over the range of the capillary-size distribution as given by equation (22).

3. Network for diffusion

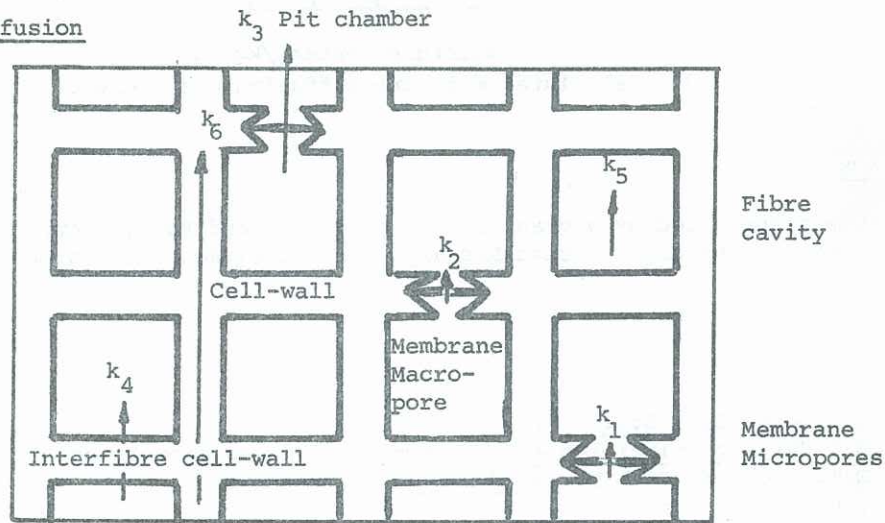


Fig. 2 Multiple flow paths for moisture movement, after Stamm (22)

Wood may be considered as a network of parallel and series paths along which moisture flows. The equivalent conductance of the network, illustrated in Fig. 2, is

$$\mathcal{D}_e = \frac{k_3(k_6+k_5)(k_1+k_2) + (k_1+k_2+k_3)(k_4k_5 + k_5k_6 + k_4k_6)}{k_3(k_1+k_2) + k_4+k_5(k_1+k_2+k_3)} \quad \dots (28)$$

The conductances for each path are obtained by factoring the effective diffusion coefficient for the particular path by the fractional cross-sectional area and path length for flow. As the moisture is removed from the cell-walls, the wood shrinks. Hence the conductances are complex functions of moisture content. Furthermore, the shrinkage is weakly dependent upon the rate of moisture loss, but this effect has been ignored. The dependence of the foregoing parameters on moisture content has been evaluated for softwoods by Stamm (19).

EFFECTIVE DIFFUSION COEFFICIENT

From the equations developed above, it is possible to calculate the effective diffusion coefficient as a function of temperature and moisture content, and the predicted variations are shown in graphical form in Fig. 3. As may be expected, the coefficient increases with temperature and the diffusion coefficient varies approximately a hundredfold over the range of moisture contents that pertain in a kiln. While generally moisture is less mobile as the material dried out, the curves do exhibit a maxima and minima in the hygroscopic-moisture range as pointed out by Evans and Keey (23). The use of these coefficients in predicting moisture-content profiles in the boards under kiln-drying conditions is detailed by one of us (JCA) elsewhere (24).

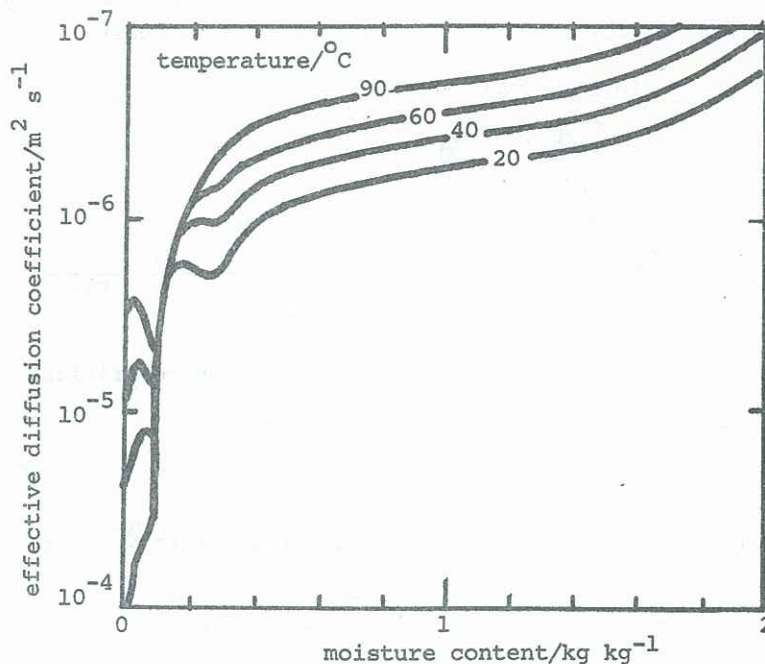


Fig. 3 Calculated moisture-diffusion coefficient

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NOMENCLATURE

a	area	m^2
D	diffusion coefficient,	$m^2 s^{-1}$
D_{AB}	diffusion coef. of vapour A	$m^2 s^{-1}$
D_e	diffusion coef., effective	$m^2 s^{-1}$
D_f	diffusion coef., flow	$m^2 s^{-1}$

f	slip-flow friction factor	1
g,h	radius-distribution functions	l_2
k_i	conductance	$m^2 s^{-1}$
K	permeability	m
l_i	capillary lengths	m
L	capillary-cell length	m
M	molar mass	$kg mol^{-1}$
P_i	partial pressure	Pa
P_s	vapour pressure	Pa
P_z	suction potential	Pa
P	total pressure	Pa
P_l	$\lambda P/2r$	Pa^3
Q	volumetric flow rate	$m^3 s^{-1}$
r,r'	radius	m
r_a	radial thickness of adsorbate	m
r_e	capillary radius, effective	m
r_{min}	capillary radius, minimum	m
R	gas constant	$J mol^{-1} K^{-1}$
t	time	s
T	temperature	K
v	fluid velocity	$m s^{-1}$
W_A	moisture flux	$kg m^{-2} s^{-1}$
X	moisture content	$kg kg^{-1}$
X_{hygr}	fibre saturation point	$kg kg^{-1}$
Y_i	mole fraction	1
α	tortuosity	1
δ	slip-layer thickness	m
ϵ	void-space ratio	1
ψ	relative humidity	1
μ_w	moisture viscosity	$N s m^{-2}$
ρ_m	material density	$kg m^{-3}$
ρ_v	vapour density	$kg m^{-3}$
ρ_w	moisture density	$kg m^{-3}$
σ	surface tension	$N m^{-1}$
θ	angle of inclination	1