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NUMERICAL MODELLING OF TIDAL PHENOMENA IN BAYS AND ESTUARIES WITH INTER-TIDAL FLATS

by

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S U M M A R Y

A numerical model for simulation of tidal dynamics in bays and estuaries containing large areas of shallows which dry at lower stages of the tide is described. The model incorporates two spatial dimensions in the horizontal plane and the effects of wind shear, of frictional resistance and of Coriolis force are included. The processes of drying and wetting of the inter-tidal flats are simulated automatically by dynamic alteration of the boundary conditions. Some results from a number of case studies carried out with the model are presented and it is shown that the presence of extensive areas of inter-tidal flats within a basin gives rise to significant and complex effects on the tidal dynamics.

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LIST OF SYMBOLS

x,y spatial coordinates in the horizontal plane.
z the vertical coordinate.
t time.
$\Delta x, \Delta y$ mesh lengths in the horizontal plane.
Δt time step in numerical integration.
z_o elevation of the sea bottom above datum plane.
a_o depth to the sea bottom measured below mean water level.
h height of water surface above mean water level.
$\hat{h} = h + a_o$ (see Figure 1).
$h^* = h + a_o + z_o$ (see Figure 1).
$u = \frac{1}{\hat{h}} \int_{z_o}^{h^*} u(z)dz$ and $v = \frac{1}{\hat{h}} \int_{z_o}^{h^*} v(z)dz$ are the vertically averaged velocity components in the x and y directions, respectively.
$V = (u^2 + v^2)^{\frac{1}{2}}$, is the magnitude of the local velocity vector.
g the acceleration due to gravity.
Ω the Coriolis parameter.
W_x, W_y the components of wind shear stress on the water surface.
C Chezy coefficient
r linear friction coefficient.
c pollutant concentration.
k_1, k_2 pollutant dispersion coefficients.
S generalised term to represent sources and/or sinks in pollutant equation.

INTRODUCTION

A programme of research into methods for computer simulation of the processes of dispersion and flushing of pollutants in estuaries and bays being carried out in the Department of Civil Engineering of the University of Queensland has led to the development of a numerical model which simulates unsteady phenomena having spatial variations in two dimensions in the horizontal plane. Variations in the vertical direction are accounted for by integration over the depth of flow. The effects of wind shear, of frictional resistance and of Coriolis force on the tidal dynamics are included and the transport and dispersion of pollutants are simulated. Certain aspects of the numerical model have been described by Apelt, Gout and Szewczyk (1). The model is applicable to situations in which there is no significant horizontal stratification and it can be used to represent any tidal region whether it be an estuary, bay or a large sea. The main thrust of the research programme, however, has concentrated on simulation of phenomena in estuaries and bays. In many estuaries and bays there are extensive areas of inter-tidal flats which are dry at low stages of the tide and there is some evidence which indicates that their presence has significant effects on the tidal dynamics and, therefore, on the dispersion and transport of pollutants. The details of tide levels and currents over these flats are also important when wider ecological considerations must be taken into account. It became apparent that the model must be able to simulate the wetting and drying of inter-tidal flats during the tidal cycle if its applicability was not to be severely restricted and, consequently, a method has been developed which achieves this simulation automatically.

In the following sections the way in which the wetting and drying of inter-tidal flats is modelled is described and some results from a number of case studies are presented. The effects of inter-tidal flats have been incorporated into the modelling of the tidal dynamics and of the pollutant behaviour. However, in what follows, attention will be restricted to the tidal dynamics.

THE MATHEMATICAL MODEL

The equations of motion are derived from the Navier-Stokes equation by integration over the depth of flow and they take the form,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial h^*}{\partial x} - g\frac{Vu}{C^2\hat{h}} + \frac{W_x}{\hat{h}} + \Omega v \quad (1),$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial h^*}{\partial y} - g\frac{Vv}{C^2\hat{h}} + \frac{W_y}{\hat{h}} - \Omega u \quad (2).$$

The continuity equation integrated over the depth of flow is,

$$\frac{\partial h}{\partial t} + \frac{\partial(\hat{h}u)}{\partial x} + \frac{\partial(\hat{h}v)}{\partial y} = 0 \quad (3),$$

and the pollutant equation is,

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = \frac{1}{\hat{h}}\frac{\partial}{\partial x}(k_1\,\hat{h}\,\frac{\partial c}{\partial x}) + \frac{1}{\hat{h}}\frac{\partial}{\partial y}(k_2\,\hat{h}\,\frac{\partial c}{\partial y}) + S \qquad (4).$$

The symbols used are defined in the List of Symbols.

For computer solution, the differential equations are converted into finite difference approximations and the resulting difference equations are integrated numerically. Time derivatives are approximated with forward difference expressions and spatial derivatives with central difference expressions. The discretisation of the spatial domain creates two sets of grid points which are displaced relative to each other as illustrated in Figure 2. Velocity components are computed at stream points for times, $t_o + 2n\,\Delta t$, and water surface elevations are computed at elevation points for times, $t_o + (2n + 1)\Delta t$, where n takes on values 0, 1, 2... etc. The numerical integration procedure used is of explicit, "leap-frog" type. The detailed finite difference equations are given in Apelt, Gout and Szewczyk (1), and are not repeated here. The pollutant concentration is computed at elevation points, either simultaneously with the computation of the dynamical variations or separately by using stored results from the tidal simulation as input parameters.

To ensure stability in the numerical integration of the hydrodynamic equations it is necessary to restrict the magnitude of the time step, Δt, and the stability criteria which have been found to be satisfactory are $\Delta t < \min(\Delta x, y)/2(g\,\hat{h}_{max})^{\frac{1}{2}}$, $\Delta t < r/\Omega^2$, where r is a linear friction coefficient. The second of these criteria is relevant only when Coriolis effects are included and it is the more severe restriction when Δx and Δy are very large, as happens when the region being modelled is very large in dimension. The first criterion must always be satisfied and it is the determining one in cases with medium to fine scale spatial resolution. As a further protection against amplification of truncation errors smoothing, in the sense of Harris and Jelesnianski (2), is used to prevent a spurious increase in the energy of the system, which would otherwise arise from the effects of the discretisation. It is interesting to note that, as a consequence of the stability restrictions, the more extensive the area being modelled the less expensive is the cost per tide cycle of computer simulationfor the same number of grid points, because the larger values of Δx, Δy associated with representation of extensive areas permit correspondingly larger values of Δt to be used.

BOUNDARY CONDITIONS

The three types of boundary conditions which can occur are the specification of either the velocity or the water surface elevation or a relationship between them for some boundary curve in the horizontal plane. The two specific cases most commonly encountered are the requirement of zero velocity component normal to a coast line or "land boundary" and the specification of water surface elevation as a function of time. Whether the latter involves the ocean tide at deep sea points or a hydrograph input at the landward end of an estuary, it is described here as an "open sea boundary". These boundary conditions can be satisfied with relative ease if the boundary is arranged so that land boundaries are defined by stream points and open sea boundaries are defined by elevation points. A general boundary curve is approximated by straight line segments drawn between grid points in directions parallel to the grid axes, as illustrated in Figure 2. A consequence of the versatility so achieved in representing a general boundary shape is that there are seventeen different variants of the finite difference formulation of the boundary conditions at points on land boundaries. The formulation of these different cases follows closely the approach of Heaps (3) and some details are provided by Apelt, Gout and Szewczyk (1). The details are not repeated here but the number of different variants is significant because it contributes to the magnitude of the task involved in modelling of inter-tidal flats.

MODELLING OF INTER-TIDAL FLATS

As the areas of inter-tidal flats dry or are covered during the different stages of the tidal cycle the geometry of the boundary defining the tidal basin changes. Representation of the effects of inter-tidal flats has been achieved in the numerical model by providing for dynamic alteration of boundary conditions, this being triggered automatically during the computations in response to the local water surface levels. Alteration of boundary conditions will involve changes in the location of the boundary, changes in the type of boundary point at a specific location or a combination of both. All of these possibilities have been provided for by defining the types of grid points which may exist within a tidal basin, on its boundaries or external to it. The total number of such types is twenty-two, including the seventeen variants of land boundary points referred to above. All possible changes of boundary conditions associated with wetting and drying of inter-tidal flats are accomplished by changes in the type numbers of grid points which are affected. However, if the attempt is made to provide automatically for wetting and drying in any arbitrary sequence, the number of options which must be examined at each time step in the computation becomes so large as to make the task not feasible. In order to avoid this difficulty it is necessary to define in advance the sequence of drying and wetting. In the simulation of historical tidal cycles in existing basins this sequence is readily determined from visual observation. However, if the simulation involves prediction of future events the user of the program is required to judge in advance, approximately, those areas in the tidal basin which will be subjected to wetting and drying and, if the wetting and drying of an inter-tidal area is to be simulated as a multistage process, the order in which these stages will occur must be specified in advance.

A simple example of a basin with inter-tidal flats is shown in Figure 3, which also illustrates the sequence of drying and wetting of the flats. The inter-tidal flats in a basin may be sub-divided into independent regions and each region itself may be sub-divided into different "levels" corresponding to the different stages in a multi-stage drying and wetting process. The different "levels" correspond to different stages in the drying and wetting sequences; the physical elevations of different levels within a region may be the same or they may differ as required by the modelling. In the example of Figure 3 there are two distinct inter-tidal regions and in each region there are four levels. In the drying-wetting sequences illustrated in Figure 3, the shapes which the boundary of the tidal basin takes on during the stages of drying are different from those shapes taken on during the stages of wetting. This is an example of a "first-wet, first-dry" sequence which might occur, for example, in a long estuary with inter-tidal flats along each bank. In such an estuary, those flats nearer the seaward end of the estuary will be dry earlier during the ebb tide and, also, they will be wet earlier during the flood tide than flats which are further up the estuary. In other circumstances, inter-tidal flats may be subjected to a "first-wet, last-dry" sequence. This sequence will occur, for example, on a sloping bank where there is no significant shift in the phase of the tide at different locations on the bank with the same bottom elevation. In a "first-wet, last-dry" sequence, the shapes which the boundary of the tidal basin takes on during successive stages of drying are the same as those which occur during wetting but they occur in reverse order. The computer program deals with each type of wetting-drying sequence automatically. Of course, the user must be aware of the type of sequence which will apply to a specific inter-tidal flat because the sequence of boundary shapes must be specified as part of the program input data.

The process of drying of a level within an inter-tidal region is triggered automatically when the water surface elevation at any one of a number of specified elevation points within the level becomes lower than a specified value. In Figure 3 the points which trigger drying from level 1 to level 2 are identified. The process of wetting a level is triggered automatically when the water surface elevation at every one of a number of specified elevation points in the vicinity of the level becomes higher than a specified value. The points which trigger wetting from level 2 to level 1 in the basin of Figure 3 are indicated. For accuracy of modelling it is necessary to dry a level when the receding water surface is only marginally higher than the bottom elevation of the level and likewise it is necessary to cause a level to become wet as soon as the water surface at points adjacent to the level is only marginally higher than the bottom elevation of the level. On the other hand, it is also necessary to ensure that the program does not improperly permit negative values of water depth over the inter-tidal flat. There is nothing in the modelling of the inter-tidal flats which requires that the bottom elevations of points within a specific level should have the same value. What is necessary, however, is that drying must be triggered before the water surface elevation falls below bottom elevation at any point within the level and that wetting must not be triggered before the water surface elevation at the edge of the level about to become wet is above the bottom elevation of all points within the level. Experience has indicated that satisfactory results are obtained with margins of 0.03 m for drying and 0.05 m for wetting. These figures may not be optimal but it has been found that reductions in the margins by an order of magnitude have led to substantial effects from enhanced rounding errors.

EFFECTS OF INTER-TIDAL FLATS ON TIDAL DYNAMICS

The program has been used to simulate tidal phenomena in a number of basins, some simple in plan form and some complex. A difficulty encountered here, as in many other aspects of simulation of natural systems, is the lack of any accurate data against which the model can be tested. Partly for this reason it was decided to explore fairly generally the ways in which the the tidal dynamics in a basin of simple plan form with inter-tidal flats are affected by variations in the geometric proportions and physical scale. The basins which have been studied in most detail are of the type illustrated in Figure 3, i.e. they are of rectangular plan-form and they have a central deeper channel with inter-tidal flats along each side. Some of the effects of inter-tidal flats on the tidal dynamics in rectangular basins are shown in Figures 4 to 7. The following details apply to all cases treated in Figures 4 to 7:-

Depth to bottom of central channel measured from mean sea level, 1.52 m; width of central channel, 2.44 km; half-amplitude of sinusoidal tide applied at ocean end, 0.915 m; Chezy coefficient, 71 $m^{1/2}$ s^{-1}; $\Delta x, \Delta y$, 610 m; W_x, W_y and Ω each zero, i.e., effects of wind stresses and of Coriolis force have been neglected. The x axis has been set parallel to the longitudinal axis of the basin and the positive direction is from the ocean towards the landward end. The details of a basin which distinguish it from other basins are specified by a code involving four separate numbers, in the form, X/Y/A/L. The meaning and values of the elements are as follows:-

X - the length of the basin - takes values 5 and 10,corresponding to lengths of 5.49 and 10.35 km, respectively.

Y - the width of inter-tidal flats along the sides of the basin - takes values 1 and 2 corresponding to widths of 1.22 and 2.44 km, respectively.

A - the depth to the bottom of the flats below M.S.L. - takes values .3 and .6 corresponding to depths of 0.305 and 0.61 m, respectively.

L - the number of levels in the drying sequence and in the wetting sequence.

If only the first number is present in the code, the basin has no inter-tidal flats.

In Figure 4, time histories of the water surface elevation and of the velocity component, u, are shown for three basins in order to illustrate some of the differences between the tidal dynamics in a basin with inter-tidal flats and those in a basin without flats and also to illustrate some of the differences between results for single stage and multi-stage drying and wetting on extensive regions of inter-tidal flats. The basins modelled are types 5/1/.6/1, 5/1/.6/4 and 5, and the time histories are plotted for the mid-length of the basin in each case. The results for the two cases with inter-tidal flats demonstrate the importance of using a number of levels for modelling extensive areas of flats; the two cases differ from each other by amounts comparable to the differences between either of them and the case without any flats. As might be expected, the significant differences between results for multi-stage and single stage modelling of the flats occur only during that part of the tidal cycle for which some or all of the flats are dry. Some of the changes in the tidal dynamics associated with the existence of the flats can be seen from comparison of results for basins 5 and 5/1/.6/4. The presence of the flats causes a lengthening of the duration of the ebb tide but a raising of low water, an increase in the rate of rise of the flood tide and increases in the magnitude of peak velocities for both ebb and flood tides.

Some results for two basins which are identical, except that the depth to the flats below M.S.L. differs, are shown in Figure 5. The basins are types 5/1/.6/4 and 5/1/.3/4. The time histories of water surface elevation in Figure 5 are for a location 610 m from the landward end of the basin. The corresponding plot for the simple basin, type 5, is also included. It can be seen from comparison between the water surface elevation histories in Figure 5 that the depth to the flats is a significant parameter. The results for basin type 5/1/.3/4 differ from those for basin type 5/1/.6/4 more than the latter differ from the results for the simple basin, type 5. Comparison between Figures 4 and 5 shows the variation between water surface elevation histories at mid-length and at the landward end of the basin for basin type 5 and basin type 5/1/.6/4. The tide curves at the two locations in the simple basin show little difference, apart from a phase shift, whereas there are significant differences in the shape of the low tide segment between the two locations in the basin with inter-tidal flats.

The influence of the length of the tidal basin is illustrated in Figure 6. Time histories of water surface elevation at a location 610 m from the landward end of the basin are shown for types 5/1/.3/4 and 10/1/.3/8 and also for the corresponding simple basins, types 5 and 10. The differences between the tide curves are most pronounced for the low tide segment but the high tide segments also display significant differences. The sudden dip in the water surface elevation history for basin 10/1/.3/8 at low tide results from the combination of effects from the inter-tidal flats and from the reflections off the barrier at the landward end of the basin. It has been demonstrated that the dip is not present in the case of a basin which has inter-tidal flats at the landward end as well as along the sides, but the details are not included here.

The time histories in Figure 7 illustrate the large influence of the width of the inter-tidal flats on the tidal dynamics. The basins represented are types 10/1/.3/8 and 10/2/.3/8. The water surface elevation is plotted for a location 610 m from the landward end of the basin and the u-component of velocity is plotted for a location 1220 m from the landward end. A large proportion of the effects of inter-tidal flats in basins of this type can be attributed to the mass transfer which takes place between the deep channel and the shallow flats, especially about the times of drying and wetting. For example, just before drying is about to occur water surface gradients in the y-direction (across the basin) are significant and, in some places, they are of the same order as the gradients in the x-direction (along the basin).

CONCLUSION

The case studies which have been carried out have shown that the presence of extensive areas of inter-tidal flats within a basin gives rise to significant and complex effects on the tidal dynamics. The effects are more pronounced during the lower half of the tidal cycle but are not confined to it. Some of the more obvious effects include changes in the height of low water, changes in the maximum tidal currents and changes in the shapes of the time histories at a specific location for both velocity and water surface elevation. It has been shown that the length of the basin, the width of the inter-tidal flats and the bottom elevation of the flats are all significant parameters. It would appear that, in the case of a basin with extensive areas of inter-tidal flats, any attempt to simulate the tidal dynamics and associated phenomena, such as pollutant behaviour, which does not directly model the processes of wetting and drying of the flats will omit significant elements of the system characteristics.

ACKNOWLEDGEMENT

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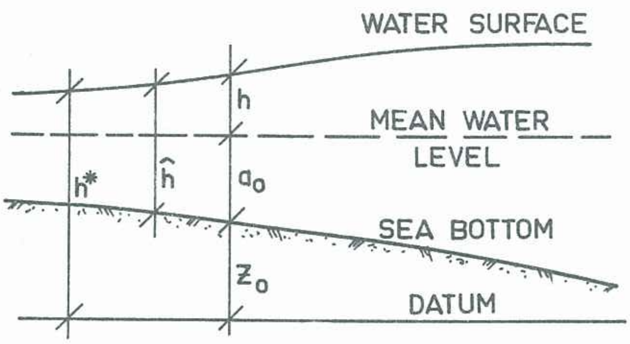

FIGURE 1

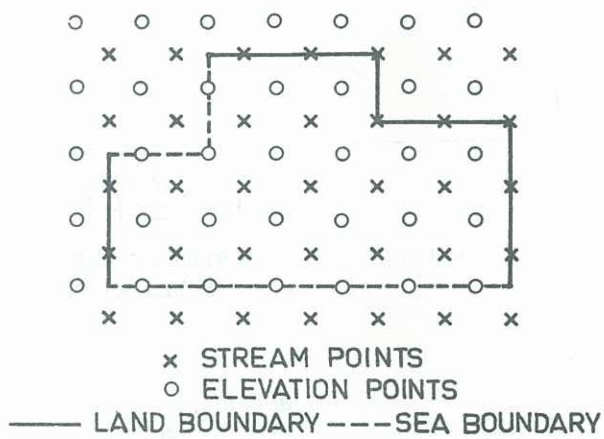

FIGURE 2

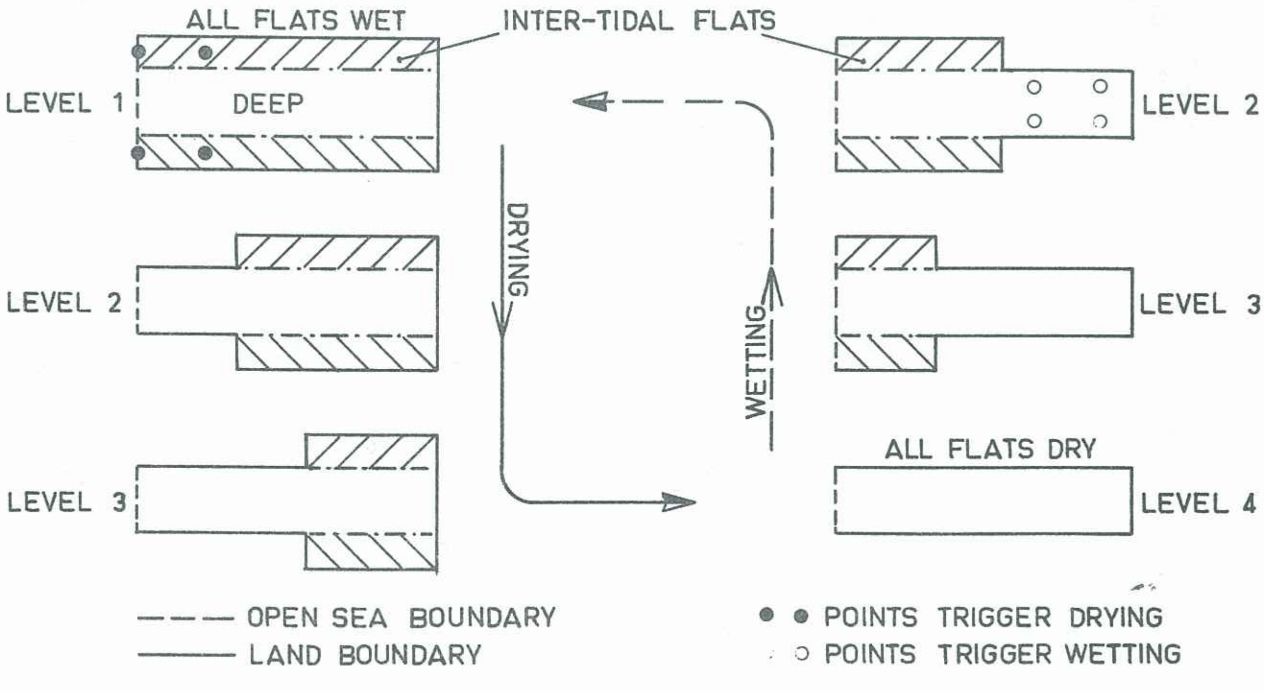

FIGURE 3. BASIN WITH INTER-TIDAL FLATS.

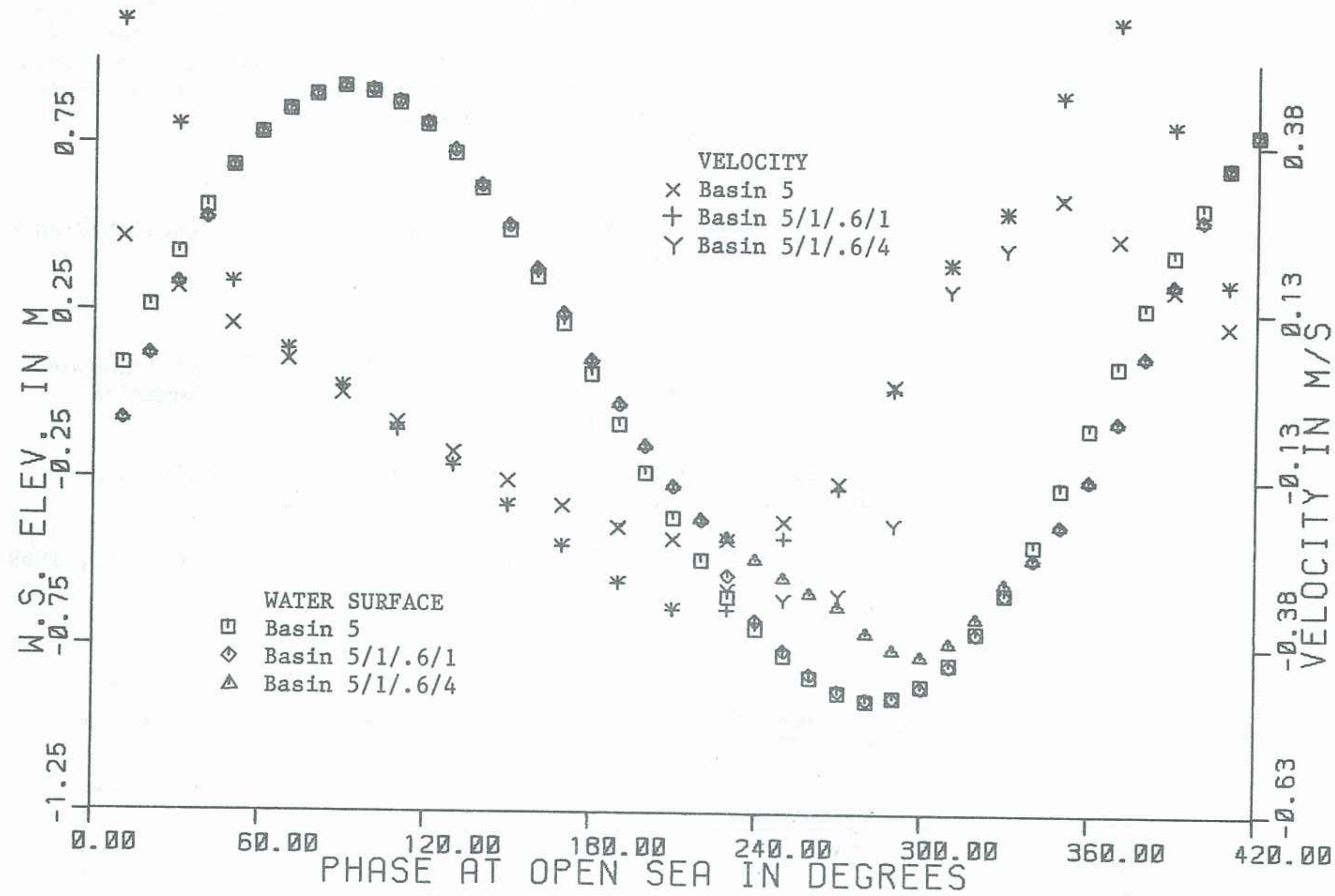

FIGURE 4. TIME HISTORIES OF WATER SURFACE ELEVATION AND OF VELOCITY AT MID-LENGTH OF BASIN; NUMBER OF LEVELS AS PARAMETER.

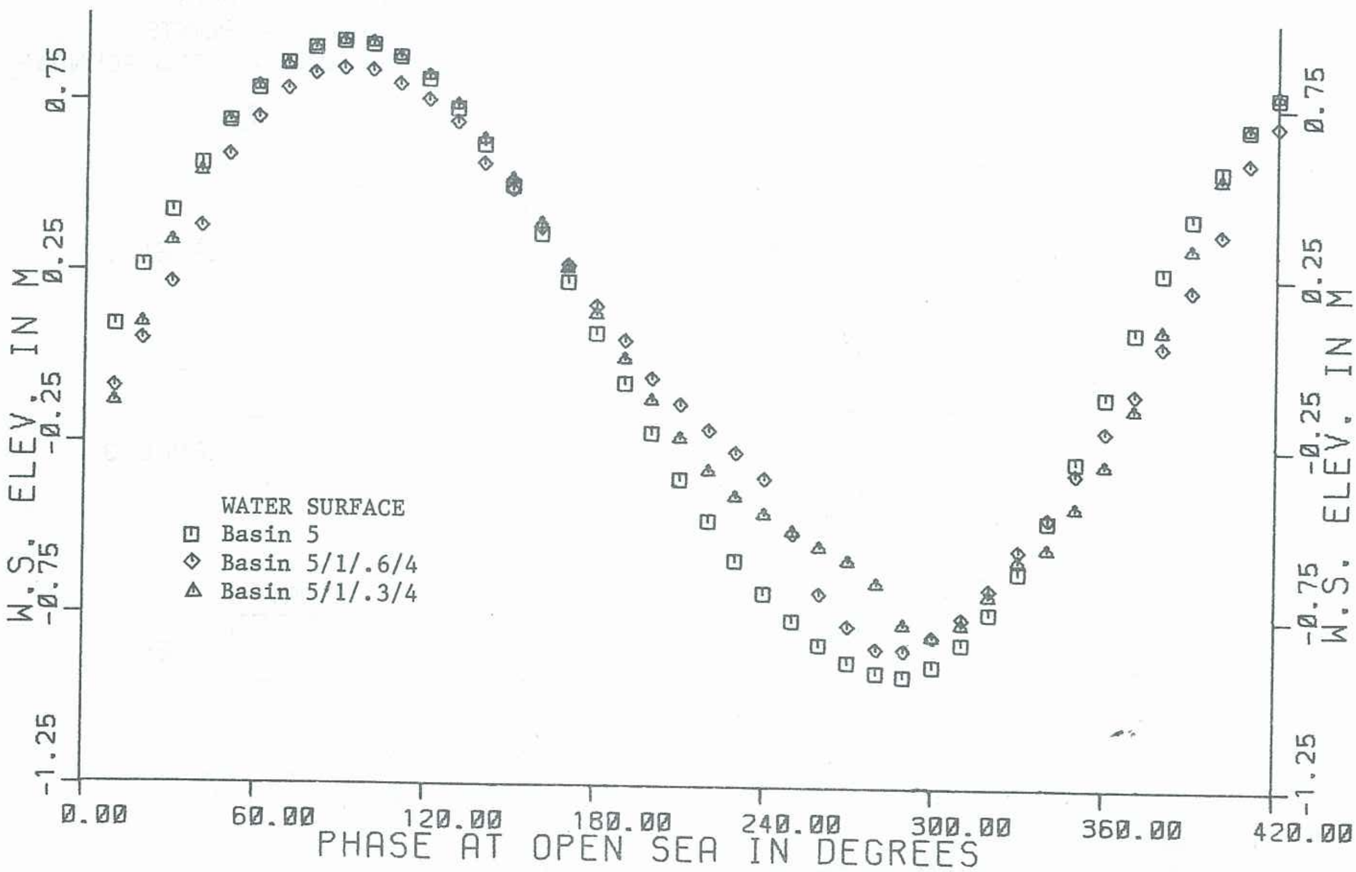

FIGURE 5. TIME HISTORIES OF WATER SURFACE ELEVATION NEAR LANDWARD END OF BASIN: DEPTH TO FLATS AS PARAMETER.

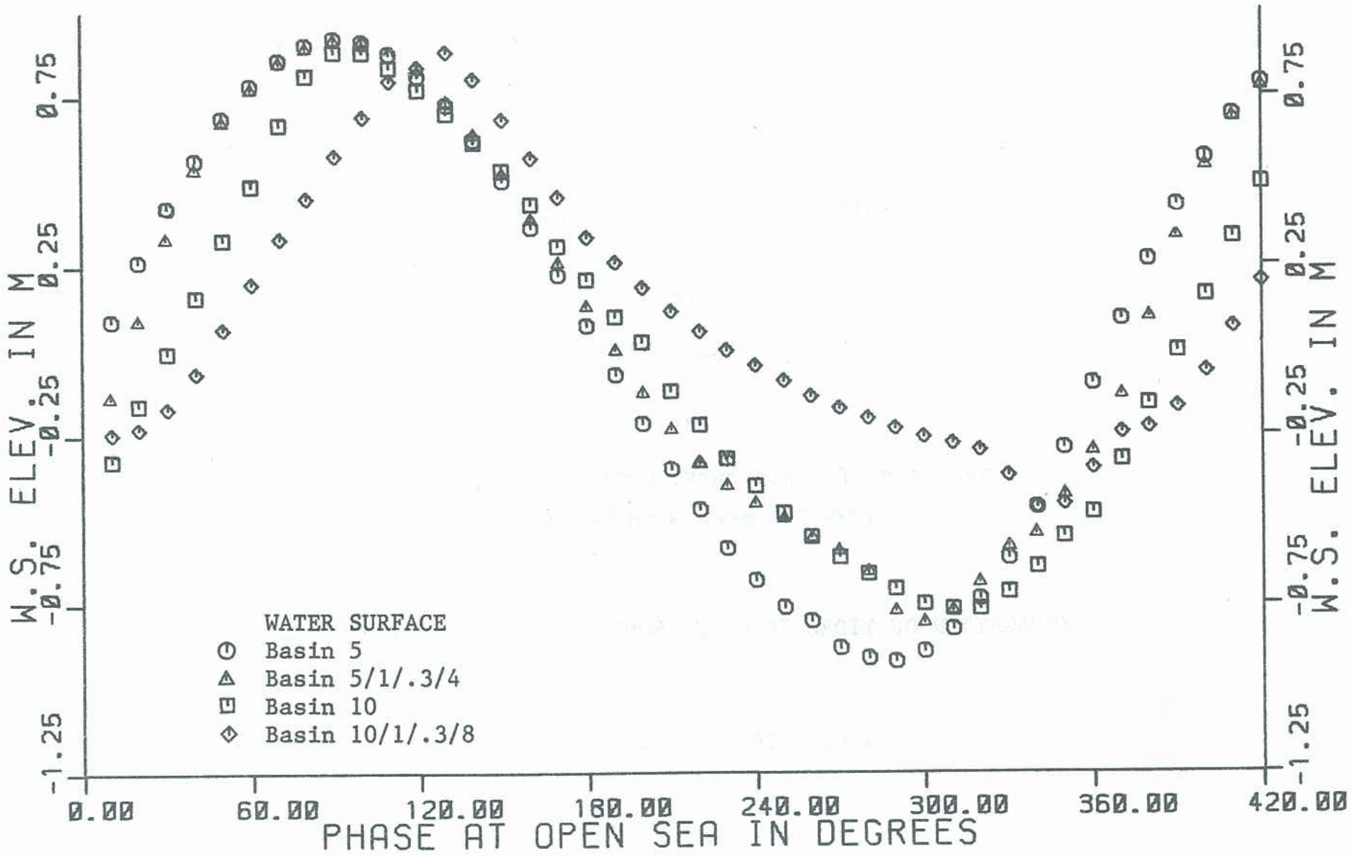

FIGURE 6. TIME HISTORIES OF WATER SURFACE ELEVATION NEAR LANDWARD END OF BASIN: LENGTH OF BASIN AS PARAMETER.

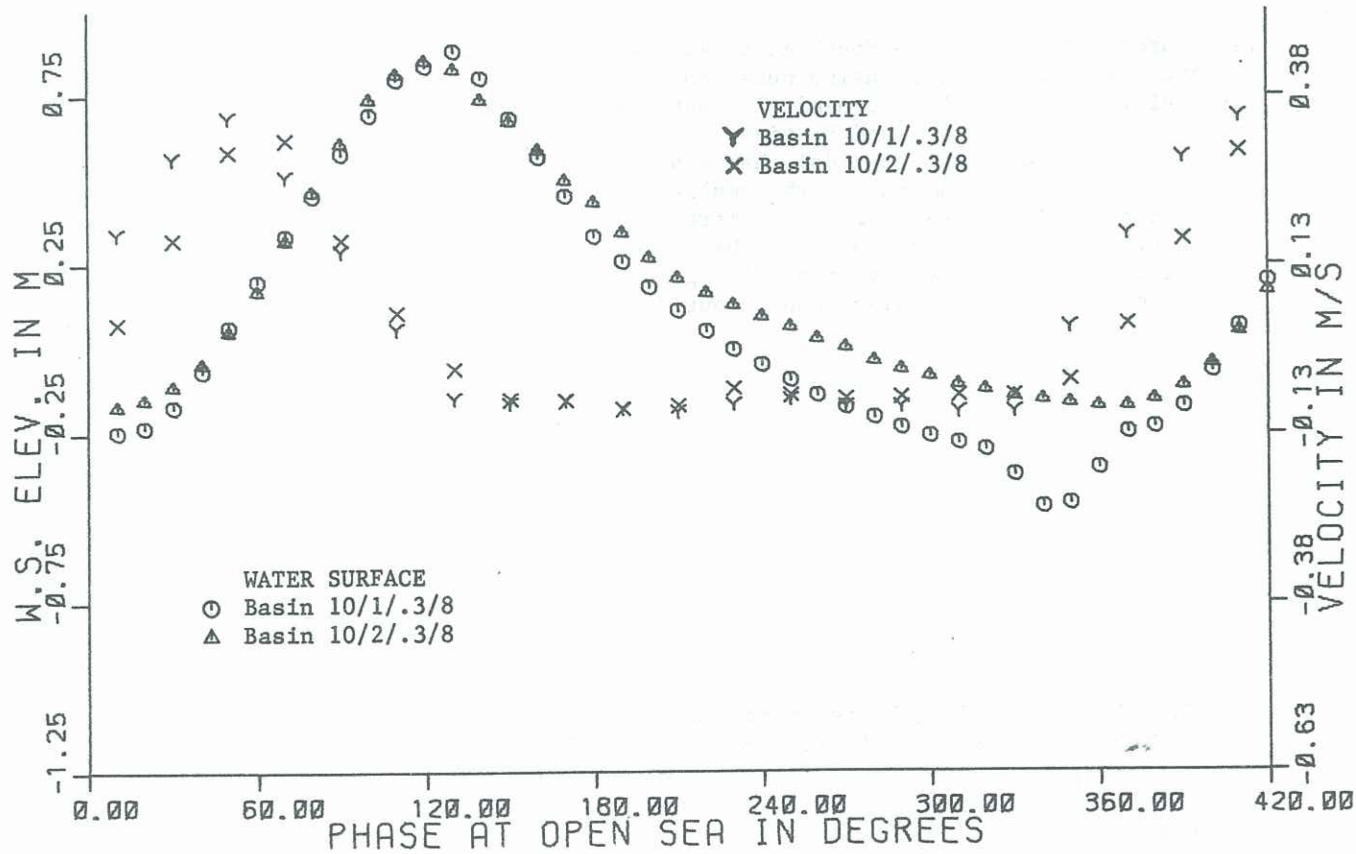

FIGURE 7. TIME HISTORIES OF WATER SURFACE ELEVATION AND OF VELOCITY NEAR LANDWARD END OF BASIN: WIDTH OF FLATS AS PARAMETER.