FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand

1974 December 9 to December 13

TRANSIENT FLOW INTO A WELL IN BOUNDED ARTESIAN MULTIPLE AQUIFERS

by

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SUMMARY

A general solution for flow into a well penetrating a system of two bounded confined aquifers is presented. Equations describing the transient and spatial drawdown in and around the well are derived. Equations are also obtained for the contributions of discharge from individual aquifers at any instant. The results are presented in the form of graphs which can be advantageously used for any field condition for the set of aquifer parameters. When the distance from the centre of the well to the recharge boundary is infinity the particular solution reduces to that of the case of two confined aquifer system given by Papadopulos. The analysis of the results reveals that a steady state is reached in all cases of finite recharge boundaries, the time to reach steady state depending on the boundary radius.

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INTRODUCTION

Wells are often drilled to penetrate more than one artesian aquifer. Such wells when screened in all the aquifers are herein referred to as multi-aquifer wells. Analysis of flow into such wells has appeared in groundwater literature recently. Sokol[5] gave a relation between the water level variation in a nonpumping multiaquifer well and the ratio of transmissibilities of the aquifers, under steady state conditions. Papadopulos[3] treated unsteady flow into a multiaquifer well and derived equation of draw-down in the case of a two unbounded confined aquifer system. Abdul Khader et al[1] proposed a mathematical model for flow into a two aquifer well with a confined aquifer overlain by an unconfined aquifer.

In all the unsteady flow solutions the aquifers are assumed to be of infinite areal extent. Unsteady flow into multiaquifer wells in bounded aquifers has not been analysed so far. In practice, the aquifers are mostly bounded in areal extent and at the recharge boundaries the head is not affected by pumping. In this paper a complete solution for unsteady flow into a well penetrating two bounded confined aquifers is presented. The aquifers are assumed to be homogeneous and isotropic, with circular recharge boundaries. Pumping at a constant rate is commenced immediately after the development of the well. Radius of the well is considered to be very small so that it is idealised to a line sink of constant strength, in the analysis. Well losses are neglected.

THEORETICAL DEVELOPMENT

The differential equations governing flow of groundwater towards a well in a bounded multiaquifer system (Fig.1), along with initial and boundary conditions are given as follows:

In aquifer 1 :

$$\frac{\partial^2 s_1}{\partial r^2} + \frac{1}{r}\frac{\partial s_1}{\partial r} = \frac{1}{\nu_1}\frac{\partial s_1}{\partial t} \quad \ldots \quad (1)$$

$$s_1(r, o) = o \quad \ldots \quad (2)$$

$$s_1(R_1, t) = o \quad \ldots \quad (3)$$

$$\text{and } 2\pi T_1 \, r \frac{\partial s_1}{\partial r} = -Q_1(t) \quad \ldots \quad (4)$$

$$\text{as } r \to o$$

in aquifer 2 :

$$\frac{\partial^2 s_2}{\partial r^2} + \frac{1}{r}\frac{\partial s_2}{\partial r} = \frac{1}{\nu_2}\frac{\partial s_2}{\partial t} \quad \ldots \quad (5)$$

$$s_2(r, o) = o \quad \ldots \quad (6)$$

$$s_2(R_2, t) = o \quad \ldots \quad (7)$$

$$\text{and } 2\pi T_2 r \frac{\partial s_2}{\partial r} = -Q_2(t) \quad \text{as } r \to o \qquad \dots \qquad (8)$$

and at the well,

$$H_1 - s_1(r_w,t) = H_2 - s_2(r_w,t) \quad .. \qquad (9)$$

$$\text{and } Q_1(t) + Q_2(t) = Q \qquad \dots \qquad (10)$$

in which

s_1, s_2 = drawdowns at any distance r from the centre of the well and time t, in aquifers 1 and 2 respectively

H_1, H_2 = initial piezometric heads in aquifers 1 and 2 respectively

ν_1, ν_2 = T_1/S_1, T_2/S_2 = diffussivities of aquifers 1 and 2 respectively

T_1, T_2 = transmissibilities of aquifers 1 and 2 respectively

S_1, S_2 = storage coefficients for aquifers 1 and 2 respectively

R_1, R_2 = radial distances from the centre of the well to the recharge boundaries of aquifers 1 and 2 respectively

r_w = radius of the well.

Applying Laplace transformation with respect to 't' to equations (1) to (10) the problem is reduced to the following:

$$\frac{d^2\bar{s}_1}{dr^2} + \frac{1}{r}\frac{d\bar{s}_1}{dr} - \frac{p}{\nu_1}\bar{s}_1 = o \qquad \dots \qquad (11)$$

$$\bar{s}_1(R_1, p) = o \qquad \dots \qquad (12)$$

$$2\pi T_1 r \frac{d\bar{s}_1}{dr} = -\bar{Q}_1(p) \quad \text{as } r \to o \qquad \dots \qquad (13)$$

$$\frac{d^2\bar{s}_2}{dr^2} + \frac{1}{r}\frac{d\bar{s}_2}{dr} - \frac{p}{\nu_2}\bar{s}_2 = o \qquad \dots \qquad (14)$$

$$\bar{s}_2(R_2, p) = o \qquad \dots \qquad (15)$$

$$2\pi T_2 r \frac{d\bar{s}_2}{dr} = -\bar{Q}_2(p) \quad \text{as } r \to o \qquad \dots \qquad (16)$$

$$\frac{H_1}{p} - \bar{s}_1 (r_w, p) = \frac{H_2}{p} - \bar{s}_2 (r_w, p) \quad \ldots \quad (17)$$

$$\bar{Q}_1(p) + \bar{Q}_2(p) = Q/p \quad \ldots \quad (18)$$

in which

$\bar{s}_1$, $\bar{s}_2$ = Laplace transform of s_1 and s_2 respectively

$\bar{Q}_1(p)$, $\bar{Q}_2(p)$ = Laplace transform of $Q_1(t)$ and $Q_2(t)$ respectively

and p = Laplace transform parameter.

Equations (11) and (14) can be identified as modified Bessel equations. At this stage it is useful to note the following properties of Bessel functions [2]:

$$\left.\begin{array}{ll} I_1(z) \approx .5z & \text{for } z < .1 \\ \text{and} \quad z\,K_1(z) \approx 1 & \text{for } z \rightarrow 0 \end{array}\right\} \quad \ldots \quad (19)$$

Making use of conditions (12), (13), (15) and (16) and the properties of Bessel functions in (19) the solutions of (11) and (14) are written as follows:

$$\bar{s}_1 = \frac{\bar{Q}_1(p)}{2\pi T_1} K_o(\lambda_1 r) - \frac{\bar{Q}_1(p)}{2\pi T_1} \frac{K_o(\lambda_1 R_1)}{I_o(\lambda_1 R_1)} K_o(\lambda_1 r) \quad \ldots \quad (20)$$

and

$$\bar{s}_2 = \frac{\bar{Q}_2(p)}{2\pi T_2} K_o(\lambda_2 r) - \frac{\bar{Q}_2(p)}{2\pi T_2} \frac{K_o(\lambda_2 R_2)}{I_o(\lambda_2 R_2)} K_o(\lambda_2 r) \quad \ldots \quad (21)$$

in which $\lambda_1 = (p/\nu_1)^{1/2}$ and $\lambda_2 = (p/\nu_2)^{1/2}$.

$\bar{Q}_1(p)$ and $\bar{Q}_2(p)$ in equations (20) and (21) are evaluated using the coupling conditions (17) and (18) at the well. Thus

$$\bar{Q}_1(p) = \frac{(H_1-H_2)\, 2\pi T_1 + T_1/T_2\, Q\left[K_o(\lambda_2 r_w) - \frac{K_o(\lambda_2 R_2)}{I_o(\lambda_2 R_2)} I_o(\lambda_2 r_w)\right]}{p.F} \quad \ldots (22)$$

and

$$Q_2(p) = \frac{-(H_1-H_2)\, 2\pi T_1 + Q\left[K_o(\lambda_1 r_w) - \frac{K_o(\lambda_1 R_1)}{I_o(\lambda_1 R_1)} I_o(\lambda_1 r_w)\right]}{p.F} \quad \ldots (23)$$

in which

$$F = K_o(\lambda_1 r_w) + T_1/T_2\, K_o(\lambda_2 r_w) - \frac{K_o(\lambda_1 R_1)}{I_o(\lambda_1 R_1)} I_o(\lambda_1 r_w) - \frac{T_1}{T_2} \frac{K_o(\lambda_2 R_2)}{I_o(\lambda_2 R_2)} I_o(\lambda_2 r_w)$$

Substituting eqs (22) and (23) in eqs(20) and (21) one gets

$$\bar{s}_1 = \frac{(H_1 - H_2)\left[K_o(\lambda_1 r) - \dfrac{K_o\,(\lambda_1 R_1)}{I_o\,(\lambda_1 R_1)}\; I_o(\lambda_1 r)\right]}{p.F}$$

$$+ \frac{Q}{2\pi T_2}\;\frac{\left[K_o(\lambda_2 r_w) - \dfrac{K_o(\lambda_2 R_2)}{I_o(\lambda_2 R_2)}\; I_o(\lambda_2 r_w)\right]\left[K_o(\lambda_1 r) - \dfrac{K_o(\lambda_1 R_1)}{I_o(\lambda_1 R_1)}\; I_o(\lambda_1 r)\right]}{p.F} \qquad \ldots (24)$$

$$\text{and } \bar{s}_2 = \frac{-\,T_1/T_2\;(H_1 - H_2)\left[K_o(\lambda_2 r) - \dfrac{K_o(\lambda_2 R_2)}{I_o(\lambda_2 R_2)}\; I_o(\lambda_2 r)\right]}{p.F}$$

$$+ \frac{Q}{2\pi T_2}\;\frac{\left[K_o(\lambda_1 r_w) - \dfrac{K_o(\lambda_1 R_1)}{I_o(\lambda_1 R_1)}\; I_o(\lambda_1 r_w)\right]\left[K_o(\lambda_2 r) - \dfrac{K_o(\lambda_2 R_2)}{I_o(\lambda_2 R_2)}\; I_o(\lambda_2 r)\right]}{p.F} \qquad \ldots (25)$$

Equations (24) and (25), on inversion for Laplace transform give the equations of drawdown in the two aquifers. The exact inversions for these equations are not readily available. A method of inversion for Laplace transforms developed by Schapery[4] is adopted for getting the final solution. According to this method,

$$f(t) \approx p\,\bar{f}(p)\Big|_{p=1/2t} \quad \text{if} \quad d\left[p\,\bar{f}(p)\right] / d\left[\log p\right]$$

is a slowly varying function of log p, $\bar{f}(p)$ being the Laplace transform of $f(t)$. Sternberg[6,7] adopted this method for a number of radial flow problems in groundwater and also compared the results with those obtained by exact inversion. The agreement is found to be good.

Making use of this method for inversion for Laplace transform, equations (24) and (25) give,

$$s_1(r,t) = \frac{(H_1 - H_2)\left[K_o(\omega_1 r) - \dfrac{K_o\,(\omega_1 R_1)}{I_o(\,\omega_1 R_1)}\; I_o(\omega_1 r)\right]}{F_1}$$

$$+ \frac{Q}{2\pi T_2}\left[K_o(\omega_2 r_w) - \frac{K_o(\omega_2 R_2)}{I_o(\omega_2 R_2)}\; I_o(\omega_2 r_w)\right] \cdot$$

$$\frac{\left[K_o(\;{}_1 r) - \dfrac{K_o(\;{}_1 R_1)}{I_o(\;{}_1 R_1)}\; I_o(\;{}_1 r)\right]}{F_1} \qquad \ldots (26)$$

$$\text{and } s_2(r,t) = \frac{-(H_1 - H_2)\, T_1/T_2 \left[K_o(\omega_2 r) - \frac{K_o\,(\omega_2 R_2)}{I_o\,(\omega_2 R_2)}\; I_o(\omega_2 r)\right]}{F_1}$$

$$+ \frac{Q}{2\pi T_2}\left[K_o(\omega_1 r_w) - \frac{K_o(\omega_1 R_1)}{I_o(\omega_1 R_1)}\, I_o(\omega_1 r_w)\right] \cdot$$

$$\frac{\left[K_o(\omega_2 r) - \frac{K_o(\omega_2 R_2)}{I_o(\omega_2 R_2)}\, I_o(\omega_2 r)\right]}{F_1} \qquad \ldots (27)$$

in which $\omega_1 = (\frac{1}{2\nu_1 t})^{½}$, $\omega_2 = (\frac{1}{2\nu_2 t})^{½}$

and $F_1 = K_o(\omega_1 r_w) + \frac{T_1}{T_2}\, K_o(\omega_2 r_w) - \frac{K_o(\omega_1 R_1)}{I_o(\omega_1 R_1)}\, I_o(\omega_1 r_w) - \frac{T_1}{T_2}\,\frac{K_o(\omega_2 R_2)}{I_o(\omega_2 R_2)}\, I_o(\omega_2 r_w)$

The equations (26) and (27) give the general solution for drawdown in a two aquifer system with finite recharge boundaries. It may be noted at this stage that if R_1 and R_2 go to infinity the solution reduces to that obtained by Papadopulos for the case of a well penetrating a two aquifer system in which the aquifers are assumed to be of infinite areal extent. The contributions of of discharge from the aquifers are obtained from equations (22) and (23) on inversion for Laplace transform.

Numerical Computations and discussions of Results:

A well of zero discharge with different initial heads and one with constant discharge with identical initial heads are separately considered for numerical computations. The results can be advantageously superposed to get solutions for any field condition. The sums of transmissibilities ($T_1 + T_2$) and storage coefficients ($S_1 + S_2$) are kept constant and the ratios T_1/T_2 and S_1/S_2 are varied to study the transient and spatial distribution of drawdown in the aquifers and the variation of discharge from each aquifer. Computations were carried out on IBM 370/155 digital computer at Indian Institute Technology, Madras.

The results are presented in the form of non-dimensional graphs. The distance to the radial boundary for both the aquifers is taken to be same ($R_1 = R_2$) in the calculations. Transient water level variations in the well in the two cases are shown in figures (2) and (3) respectively. Figure (4) indicates the circulation rate in the well when there is no pumping. Rate of contribution of the first aquifer with respect to time during pumping, is given in Fig.(5). Figure (6) is the drawdown variation in the well for various values of R/r_w.

CONCLUSIONS

The equations developed in this paper are of a generalised nature and are applicable to aquifers of any areal extent - infinite aquifers being a particular case. It is observed that a steady state is reached for all finite ratios of R/r_w, the time to reach steady state depending on the radial distance to the recharge boundary. The drawdown values around the well calculated for any value of time after attaining steady state are found to be the same as those obtained using Theim equilibrium equations[8] for different R/r_w ratios.

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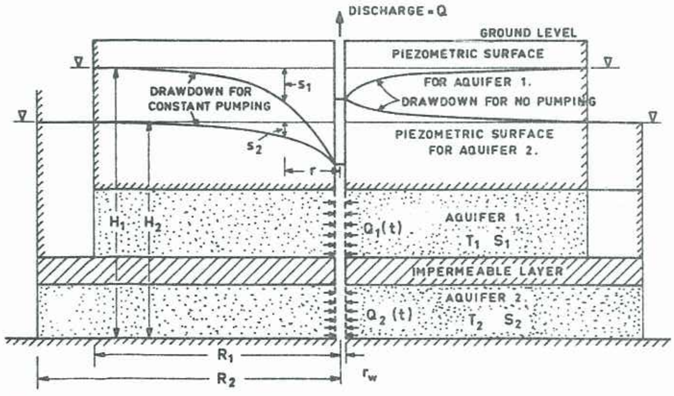

FIG. 1 WELL PENETRATING TWO BOUNDED CONFINED AQUIFER SYSTEM

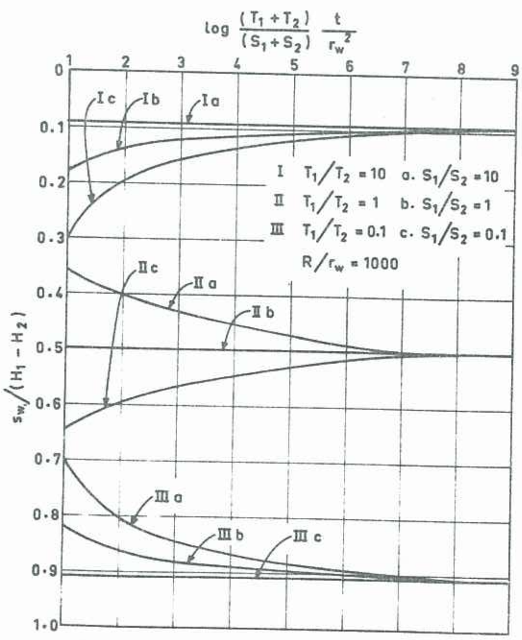

FIG. 2 VARIATION OF WATER LEVEL WITH TIME IN A WELL OF ZERO DISCHARGE PENETRATING TWO BOUNDED CONFINED AQUIFERS WITH DIFFERENT INITIAL HEADS.

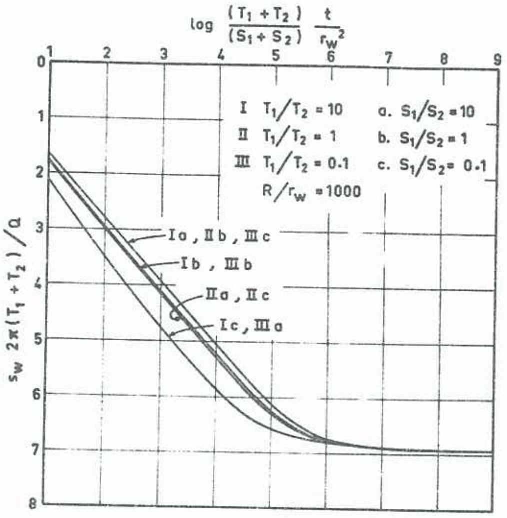

FIG. 3 VARIATION OF WATER LEVEL WITH TIME IN A WELL OF CONSTANT DISCHARGE PENETRATING TWO BOUNDED CONFINED AQUIFERS WITH IDENTICAL INITIAL HEADS.

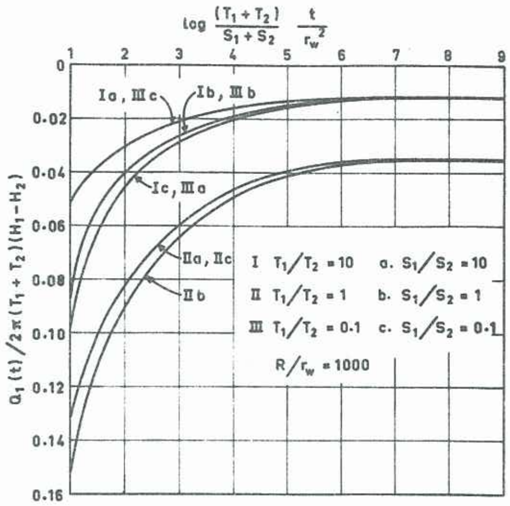

FIG. 4 VARIATION OF CIRCULATION RATE WITH TIME IN A WELL OF ZERO DISCHARGE PENETRATING TWO BOUNDED CONFINED AQUIFERS WITH DIFFERENT INITIAL HEADS.

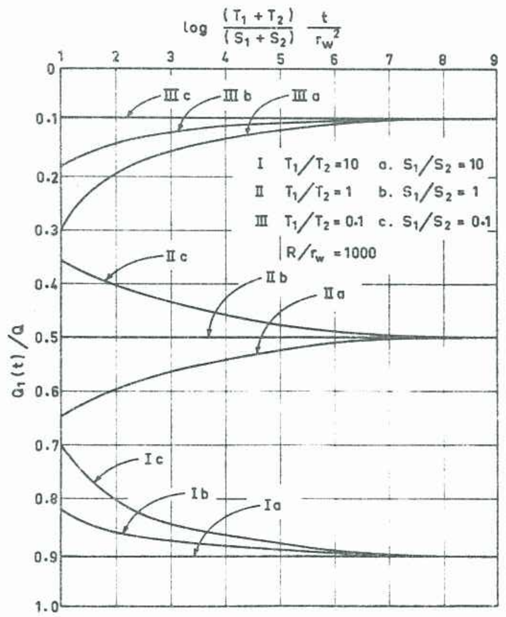

FIG. 5 VARIATION OF DISCHARGE WITH TIME FROM FIRST AQUIFER IN A WELL OF CONSTANT DISCHARGE PENETRATING TWO BOUNDED CONFINED AQUIFERS WITH IDENTICAL INITIAL HEADS.

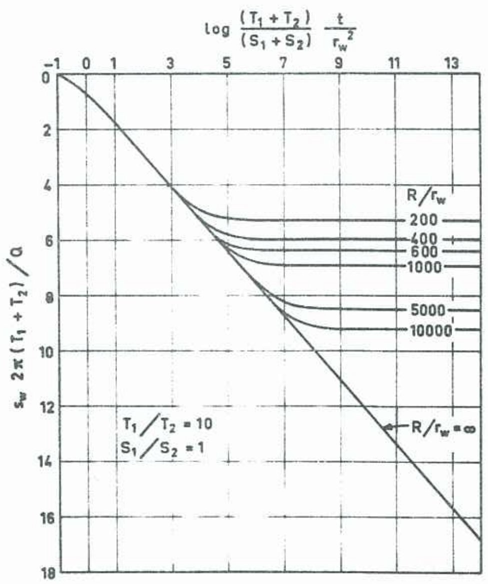

FIG. 6 VARIATION OF WATER LEVEL WITH TIME IN A WELL OF CONSTANT DISCHARGE PENETRATING TWO BOUNDED ARTESIAN AQUIFERS WITH IDENTICAL INITIAL HEADS.