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EXTREME CASES OF COLLAPSE AND REBOUND OF A CAVITATION BUBBLE

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SUMMARY

A study of collapse and rebound of a cavitation bubble for three extreme cases on the density and pressure of the fluid inside the bubble is presented. The case, when the density of the fluid in the bubble reaches the limiting density as in the shock front inside the bubble is analyzed in detail and discussed in comparison with the other two cases. For this case, the study indicates that during rebound the effectiveness of pressures at $r/r_m = 5.0$ is only 0.8 per cent of that at $r/r_m = 1.0$.

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GLOSSARY OF TERMS

a = equilibrium radius of cavitation bubble
B = constant
c = acoustic velocity
C_p = specific heat of test liquid at constant pressure
D = shock front velocity
E = total energy
h = enthalpy
k_1, k_2, k_3 = variables used in Eqs. (31)-(33)
n = constant
p = pressure in the shock front
p_b = pressure of the fluid in bubble at any instant
r = radius of bubble or spherical shock at any instant
t = time
T = temperature in the shock front
u = flow velocity behind the shock
γ = ratio of specific heats of the fluid in the cavitation bubble
δ = shock front thickness
ϵ = internal energy
ξ = nondimensional parameter
ρ = density of the test liquid
ρ_b = density of the fluid in cavitation bubble
σ = Riemann function

subscripts

c = during collapse
o = initial condition or outside the shock front
m = at minimum radius or limiting case
r = during rebound

INTRODUCTION

The shock or pressure pulse mechanism is generally believed to be one of the sources of cavitation damage (1). The shock radiates outwards once the implosion of the cavitation bubble is arrested. In the case of collapse also, there is every possibility of shock forming in the fluid of the cavitation bubble due to tremendous radial velocities. Depending upon the thermodynamic and physical parameters existing, the diameter of the bubble reaches a particular minimum value. This limiting diameter is governed by the ratio of specific heats of the fluid in the cavitation bubble which also influences the limiting density ratio in the shock front. The pressure of the shock thus formed varies as $1/r^3$ (2) considering self-similar flow either for explosion or implosion.

The investigations presented in this paper pertain to a study of the shock resulting from rebound. The variation of shock pressure with radius is investigated for one limiting case for the fluid inside the cavitation bubble and is compared with those of two other possible limiting cases of collapse.

The strength of shock during rebound depends upon the extent of collapse motion of the bubble. In the present study the following three extreme cases of collapse motion which arise under different conditions of pressure and density of the fluid inside the cavitation bubble are analysed. These are: (a) the density of the fluid inside the cavitation bubble reaches the limiting density behind the shock front inside the cavitation bubble, (b) the density of fluid inside the bubble reaches the density of the surrounding liquid and (c) the pressures both inside and outside the bubble equalize. The cases (a) and (b) are associated with high pressures since reaching the limiting density and condensing the liquid vapour to liquid are possible only at very high pressures. In these cases, rebound is inevitable due to these pressures in the fluid of the bubble. But in case (c), because of the pressure equalization, there is no scope of rebound even though the phases are different. So the three cases mentioned above ultimately truncate to two prevailing models only. The present study is confined to the case (a) where the fluid in the bubble reaches a limiting density as the limiting density behind the shock front.

FORMULATION OF THE PROBLEM : a. Motion During Collapse

1. **Minimum Radius:** The pressure of the fluid in the cavitation bubble at any instant can be written as (3)

$$p_b = p_{b_o} (a/r)^{3\gamma} \quad (1)$$

where p_b is the pressure of the fluid in the cavitation bubble, p_{b_o} is the initial pressure in the bubble just before collapse, a is the equilibrium radius of the bubble, r is the radius at any instant and is the ratio of the specific heats of the fluid in the bubble. Once the bubble starts collapsing, the radius r decreases rapidly up to a certain limiting value depending upon the value of γ. Because of collapsing motion, the possibility of formation of a spherically symmetric shock inside the bubble exists. The maximum density that can be attained behind this shock front is $\rho_{b_m} = \rho_{b_o}(\gamma+1)/(\gamma-1)$, where ρ_{b_o} and ρ_{b_m} are the initial and limiting densities of the fluid in the cavitation bubble. Hence, using Eq.(1), the the minimum radius r_m that would occur for a particular bubble during the collapse motion can be derived by assuming adiabatic compression.

For the minimum radius, r_m, Eq.(1) can be re-written as

$$p_{b_m}/p_{b_o} = (a/r)^{3\gamma} \quad (2)$$

since,

$$p_b \propto (\rho_b)^\gamma \quad (3)$$

We obtain from the limiting density condition,

$$p_{b_m} \propto [(\gamma+1)\rho_{b_o}/(\gamma-1)]^\gamma \quad (4)$$

Replacing ρ_{b_o} in Eq. (4) with p_{b_o} using Eq. (3),we obtain

$$p_{b_m}/p_{b_o} = [(\gamma+1)/(\gamma-1)]^\gamma \quad (5)$$

Equating Eqs(2) and (5), we get

$$(a/r_m) = [(\gamma+1)/(\gamma-1)]^{1/3} \quad (6)$$

2. Energy Considerations: For calculating the total energy generated by the shock, during implosion, we shall introduce a nondimensional parameter ξ, which connects the total energy E, produced by the spherical shock, the time t, radius r and density ρ_{b_o} as (2)

$$\xi = r(\rho_{b_o}/Et^2)^{1/5} \quad (7)$$

For $r = r_m$, Eq. (7) gives

$$\xi_m = r_m(\rho_{b_o}/E)^{1/5}\, t^{-2/5} = \xi\, r_m/r \quad (8)$$

Substituting for r_m from Eq.(6) in Eq.(8) and simplifying, we obtain

$$\xi_m = a\,[(\gamma+1)/(\gamma-1)]^{-1/3}/(E/\rho_{b_o})^{1/5}\, t^{2/5} \quad (9)$$

After the shock formation, the pressure p_c, density ρ_{b_c} and the flow velocity u_c behind the shock are given by (2)

$$p_c/p_{b_o} = [(\gamma+1)(\rho_{b_c}/\rho_{b_o}) - (\gamma-1)]/[(\gamma+1)(\rho_{b_o}/\rho_{b_c}) - (\gamma-1)] \quad (10)$$

$$\rho_{b_c}/\rho_{b_o} = [(\gamma+1) + (\gamma-1)p_{b_o}/p_c]/[(\gamma-1) + (\gamma+1)(p_{b_o}/p_c)] \quad (11)$$

and

$$u_c^2 = [(\gamma+1)p_{b_o}/p_c + (\gamma-1)]^2\, p_c/2[(\gamma-1)(p_{b_o}/p_c) + (\gamma+1)]\rho_{b_c} \quad (12)$$

The value of E produced by the collapse motion can be calculated by the strength of shock using equation

$$E = 4\pi r_m^2 \delta_e (h_c + u_c^2/2)\, \rho_{b_e} \qquad (13)$$

where δ_c is the shock front thickness and enthalpy, $h_c = \int dp_c/\rho_{b_e}$, can be calculated using Eqs.(10) and (11). In this case, since the collapse takes place instantaneously with very high radial velocities, the value of ρ_{b_c} can be taken as limiting density for Eq.(13), and other parameters can be evaluated accordingly. For very strong shocks, during collapse motion, the density reaches a limiting value $\rho_{b_c} = (\gamma+1)\rho_{b_o}/(\gamma-1)$ and the pressure, p_c and flow velocity, u_c behind the shock front are given in (2) in terms of the shock front velocity, D_c, as

$$p_c = 2\rho_{b_o} D_c^2/(\gamma+1) \qquad (14)$$

and,

$$u_c = 2D_c/(\gamma+1) \qquad (15)$$

b. Motion During Rebound

Once the bubble reaches the minimum radius r_m, the rebound starts and a spherically symmetric shock forms in the liquid outside the bubble. The rebound is approximated to a point detonation occurring a short distance from the boundary. The shock travels with a velocity D_r given by

$$D_r = 2\xi (E/\rho_o)^{1/5} t^{-3/5}/5 \qquad (16)$$

$$= 2\xi^{5/2}(E/\rho_o)^{1/2} r^{-3/2}/5 \qquad (17)$$

where ξ, E and ρ_o represent quantities for liquid corresponding to the quantities already mentioned for the fluid in the bubble, and t is time measured from the commencement of rebound.

The flow velocity u, density ρ, pressure p and temperature T behind the shock are given (4) by

$$u = (1 - \rho_o/\rho) D_r \qquad (18)$$

$$(\rho/\rho_o)^n = (p + B)/(p_o + B) \qquad (19)$$

or

$$p/p_o = \{B\,[(\rho/\rho_o)^n - 1]/p_o\} + (\rho/\rho_o)^n \qquad (20)$$

and

$$T/T_o = \{D_r^2[1 - (\rho_o/\rho)^2]/2\,C_p T_o\} + 1 \qquad (21)$$

where the subscript 'o' indicates the quantities outside the shock, and n and B are constants.

But from Rankine-Hugoniot relationship for shock in water

$$p/p_o = [\rho_o(1 - \rho_o/\rho) D_r^2/p_o] + 1 \qquad (22)$$

Equating Eqs. (20) and (22), we obtain

$$B\,[(\rho/\rho_o)^n - 1] + p_o(\rho/\rho_o)^n = \rho_o(1 - \rho_o/\rho) D_r^2 + p_o \qquad (23)$$

The energy, E, during rebound is given by

$$E = 4\pi r_m^2 \delta_r (h + u^2/2)\rho = 4\pi r_m^2 \delta_r (h + \sigma^2/2)\rho \qquad (24)$$

where δ_r is the shock front thickness, h is enthalpy, and σ is Riemann function to approximate the flow velocity, u. Because of this approximation, the errors inherent in the propagation theory of Kirkwood and Bethe(5) are actually reduced. The quantities h and are defined by:

$$h = \int dp/\rho$$

$$= n(p_o + B)\,[(\rho/\rho_o)^{n-1} - 1]/\rho_o(n-1) \qquad (25)$$

and,

$$u = \int c\, d\rho/\rho$$

$$= 2[n(p_o + B)/\rho_o]\,[(\rho/\rho_o)^{(n-1)/2} - 1]/(n-1) \qquad (26)$$

where, the acoustic velocity c is given by

$$c = [n(p_0 + B)/ \rho_0]^{1/2} (\rho/ \rho_0)^{(n-1)/2} \quad (27)$$

Using the equation of state $\rho^n \propto (p + B)$.
Substituting Eq. (17) in Eq. (22) and rewriting, we obtain

$$[p/p_0 - 1] = 4(1 - \rho_0/\rho) \xi^5 E/25 r^3 p_0 \quad (28)$$

Similarly substituting Eq. (17) in Eqs.(18) and (21), we obtain

$$u = 2(1 - \rho_0/\rho) \xi^{5/2}(E/ \rho_0)^{1/2}/5 r^{3/2} \quad (29)$$

and, $(T/T_0 - 1) = 2 \xi^5 E [1 - (\rho_0/\rho)^2] /25 r^3 \rho_0 C_p T_0$ (30)

Eqs. (28) to (30) can be rewritten in non-dimensional form as,

$$[(p/p_0) - 1] /k_1 E = 1/(r/r_m)^3 \quad (31)$$

$$[(T/T_0) - 1] /k_2 E = 1/(r/r_m)^3 \quad (32)$$

$$(u/c_0)^2/k_3^2 E = 1/(r/r_m)^3 \quad (33)$$

where

$$k_1 = 4(1 - \rho_0/\rho) \xi^5/25 p_0 r_m^3 \quad (34)$$

$$k_2 = 2[1 - (\rho_0/\rho^2] \xi^5/25 T_0 \rho_0 C_p r_m^3 \quad (35)$$

$$k_3 = 2(1 - \rho_0/\rho) \xi^{5/2}/5 r_m^{3/2} \sqrt{n(B + p_0)} \quad (36)$$

and

$$c_0 = \sqrt{n(B + p_0)/ \rho_0} \quad (37)$$

The right hand side of Eqs. (31) to (33) is the same so that the variation of the non-dimensional quantities on the left hand side with the ratio r/r_m should also be the same.

The minimum value for pressure(p_0) behind the shock front which occurs after a certain time interval, t, can be obtained by differentiating p with respect to time and equating it to zero. This gives an idea of shock vanishing time and stress relaxation period.

$$dp/dt = (\partial p/ \partial \rho)(\partial \rho / \partial t) + (\partial p/ \partial t) \quad (38)$$

DISCUSSION OF RESULTS

Fig.1 shows the variation of the ratio (a/r_m) with γ according to Eq. (6) for the cavitation bubble during collapse motion. As the value of γ increases the ratio (a/r_m) decreases, i.e., for adiabatic compression of the fluid in the cavitation bubble, the ratio (a/r_m) tends towards lower values. If the compression is isothermal(γ =1), the ratio (a/r_m) tends to infinity, i.e., bubble collapses to almost zero radius. Hence, during isothermal compression the pressures produced are very high compared to those produced during adiabatic conditions, and the shock produced during rebound is very strong. This is in agreement with Plesset(1). Hence, for isothermal compression, the assumption of point detonation is justified.

Fig.2 presents the non-dimensional variation of pressure, temperature and flow velocity of the shock during rebound as a function of $\log(r/r_m)$ using Eqs. (31) to (33). Theoretically, the density, pressure and temperature in the shock reach the values in the surrounding test liquid after traversing an infinite distance, i.e., as $(r/r_m) \rightarrow \infty$. But figure indicates that for $(r/r_m) \geq 5$, the values of pressure and temperature reach almost the surrounding values, i.e., $p \rightarrow p_0$ and $T \rightarrow T_0$. From this we may conclude that, during the rebound, the shock pressure, density and temperature reduce considerably in magnitude within a short range of (r/r_m), so that, even at small distances from the bubble the shocks may not be effective in causing damage to the adjoining material. The same figure shows that the effective pressure at a ratio of (r/r_m) = 5.00 is only 0.8 per cent of the effective pressure at (r/r_m) = 1.00. Because of this fact, the protective methods such as air injection and cathodic protection are very effective in reducing cavitation damage due to the separation provided by them in between metal surface and the cavitation bubbles. These methods provide cushioning in one phase, and separation and repulsion on the other so that the effectiveness of the shock is very much reduced.

The two additional extreme cases of collapse mentioned in the introduction are briefly discussed here. Case (b): the density of the fluid in the bubble reaching the density of the surrounding liquid; and case(c): equilization of the pressures both inside and outside the bubble.

For the case(b), $\rho_{b_m} \longrightarrow \rho_o$, so that Eq.(2) can be rewritten as

$$(a/r_m) = (p_{b_m}/p_b)^{1/3\gamma} = (\rho_o/\rho_{b_o})^{1/3} \qquad (39)$$

for adiabatic compression.

Similarly, for case(c), $p_{b_m} \longrightarrow p_o$, and Eq.(2) reduces to

$$(a/r_m) = (p_o/p_{b_o})^{1/3\gamma} = (\rho_{b_m}/\rho_{b_o})^{1/3} \qquad (40)$$

Fig.3 shows the variation of (a/r_m) as a function of the density ratio for case(b) from Eq.(39), which indicates that as the ratio (ρ_o/ρ_{b_o}) increases, (a/r_m) also increases; or, for a given value of 'a' the value of r_m decreases. This indicates that the strength of the shock produced during rebound will increase with the density ratio.

Fig.4 presents the variation of (a/r_m) as a function of pressure ratio for case(c) from Eq.(40). For this case, there is no scope for rebound and shock formation even though the phases are different. The ratio (a/r_m) increases with decreasing γ for a particular (p_o/p_b) indicating that the bubble collapses to smaller size in isothermal compression than in adiabatic compression.

The following two particular conditions can arise in case (a): (i) when the minimum radius, r_m, equals to the shock front thickness, and (ii) where the limiting density ρ_{b_m} approaches the surrounding liquid density, ρ_o, which is case(b) mentioned earlier.

At high pressures ionization of the test liquid can also occur, e.g., water undergoes a smooth conversion from a largely unionized state to an almost fully ionized state between 150 to 200 kbars (5). For the extreme cases considered in the present study high pressures are likely to be produced in case(b) than in case(a), if the limiting density in case(a) would not reach the density of the surrounding liquid.

The shocks formed during the rebound compress some of the grains of the material and thus develop an emf and Peltier heat. The high pressures associated with shock wave may induce several other phenomena, such as, thermal and electrolytic action, mechanical damage, electric and electronic effects, ionization, radiation, stress corrosion, etc., which are responsible for the total damage and pitting of the materials.

In conclusion it may be pointed out that the assumed point detonation of self-similar flow may fail due to good order of the size of rebounding bubble compared to r_m and other real liquid properties. The equations which are taken for spherical shock hold good.

CONCLUSIONS

A study of collapse and rebound of a cavitation bubble for the following three extreme cases is presented. Case(a): when the density of the fluid inside the bubble reaches the limiting density as in the shock front inside the bubble. Case(b): when the density of the fluid in the bubble reaches the density of the surrounding liquid, and case(c) when the pressures both inside and outside the bubble equalize.

1. The study of case(a) indicates: (i) that the pressures produced during collapse motion are infinitely large for isothermal compression when compared with those for adiabatic compression, and (ii) that the pressure of

the shock during rebound decreases rapidly with r/r_m. At $r/r_m = 5.0$, the effectiveness is only 0.8 per cent of the pressure at $r/r_m = 1.0$.

2. For cases(b), the bubble collapses to smaller and smaller sizes as density ratio, (ρ_o / ρ_{b_o}) increases.

3. For case(c), the bubble collapses to smaller size in isothermal compression than in adiabatic compression.

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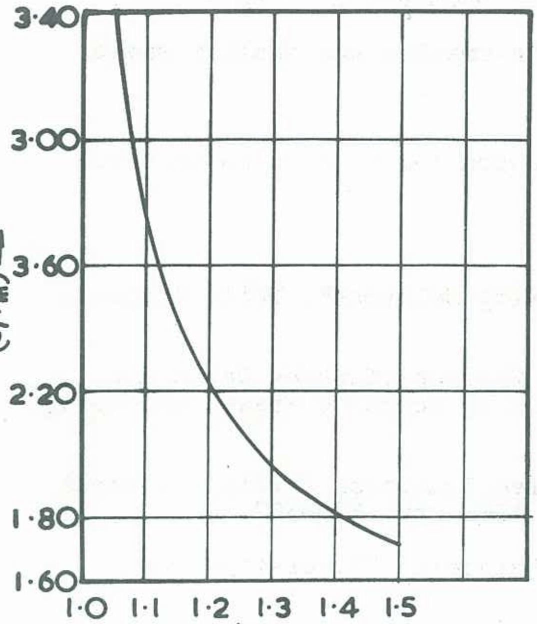
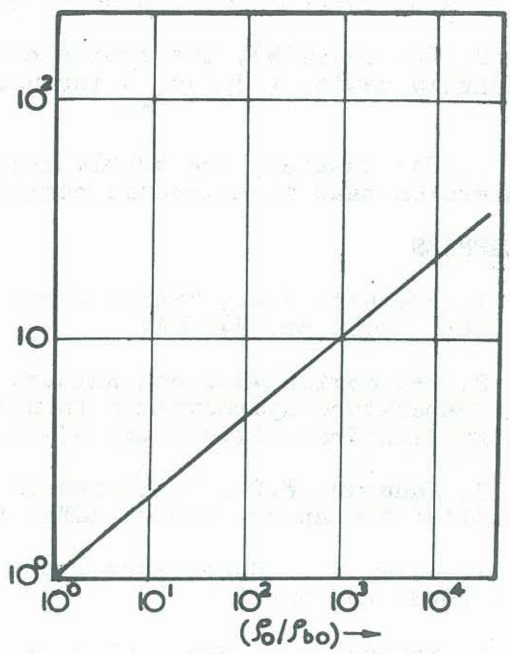


FIG.1–VARIATION OF ATTAINABLE MINIMUM RADIUS OF BUBBLE DURING COLLAPSE MOTION



FIG·3–VARIATION OF MINIMUM RADIUS OF BUBBLE FOR CASE (b)

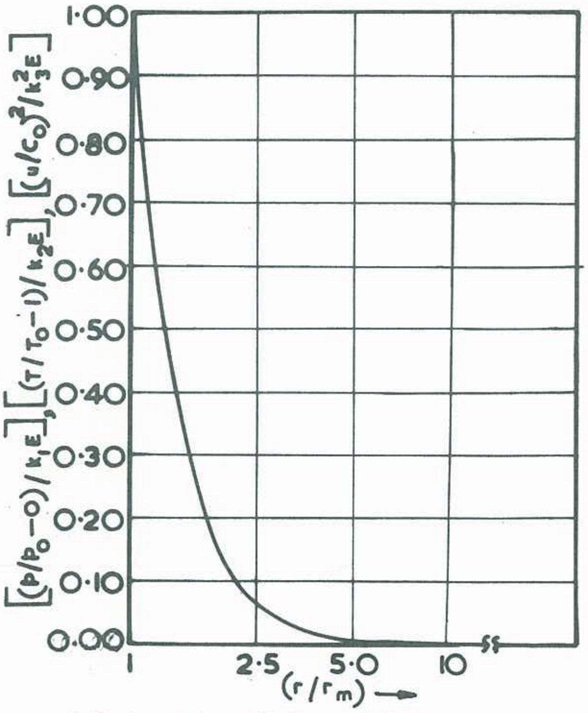
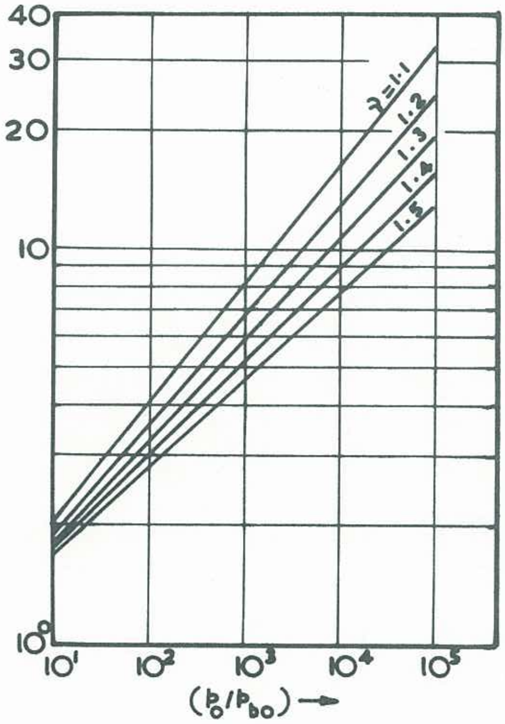


FIG.2–VARIATION OF NON-DIMENSIONAL PRESSURE TEMPERATURE AND FLOW VELOCITY WITH TIME DURING REBOUND



FIG·4–VARIATION OF MINIMUM RADIUS OF THE BUBBLE WITH PRESSURE RATIO FOR CASE (C)