FOURTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

Monash University, Melbourne, Australia
1971 November 29 to December 3

FLUTTER INSTABILITY OF RECTANGULAR BUILDINGS

by

J.W. Saunders*

Quasi-steady analysis is applied to oscillation in the transverse (cross wind) mode and the torsional mode (about the vertical axis) of a segment of a tall rectangular building subjected to wind loading for the wind direction near normal to one face. Previous theory on the one degree of freedom galloping instability of square prisms is extended to include the torsional degree of freedom. This dynamically coupled system is then considered for stability, and the conditions that define the regions of stable and unstable "static" divergence and divergent oscillation are determined. If wind tunnel tests of a model of a tall, slender rectangular building are being considered, the necessity to model the torsional rotation appears to depend primarily on the magnitude and sign of the rate of change of aerodynamic moment with angle about the vertical axis.

* J.W. Saunders, Graduate Scholar, Department of Mechanical Engineering, Monash University.

GLOSSARY OF TERMS

The following symbols are used in this paper:-

A_1, A_2 - amplitudes associated with the complex root λ;
a_i - ith coefficient of the characteristic equation;
b - cross-wind building breadth;
C_D - drag coefficient of building segment;
C_L - lift coefficient of building segment;
C_M - moment coefficient of building segment;
C_y - cross-wind aerodynamic force coefficient;
c_2, c_3 - equivalent viscous damping coefficients in degree of freedom 1 and 2;
D - aerodynamic drag on the building segment;
d - streamwise building depth;
e - base of natural logarithms;
F - resultant aerodynamic force on segment;
F_y - aerodynamic force component is the y direction;
F_3, F_4 - test functions;
I - rotational moment of inertia of the building segment about its vertical centre of gravity axis;
i - unit imaginary number;
K_2, K_3 - building stiffness for the segment in degree of freedom 2 and 3;
L - aerodynamic lift on the building segment;
ℓ - a characteristic length;
M - mass of the building segment;
M_g - aerodynamic moment about the vertical centre of gravity axis of building segment;
q_i - generalised co-ordinate of the building segment;
r - real part of complex root λ;
S - cross-wind reference area of building segment;
t - time;
U - wind velocity vector;
U_{rel} - wind velocity vector relative to the building segment;
X - reference earth axis from the base of the vertical centre gravity axis of the building towards the wind vector U;
X' - body axis rotated Θ from X;
Y - reference earth axis perpendicular to X;
Y' - body axis perpendicular to X';
Y_1 - maximum vibrational amplitude of y at time t = 0;
y - cross-wind displacement relative to earth;
$\dot{y}$, $\ddot{y}$ - cross-wind velocity and acceleration relative to earth;
α - angle of attack of the body axis X' relative to U_{rel};
ζ_2, ζ_3 - fraction of critical damping in degree of freedom 2 and 3;
Θ - rotation of the building referenced to the X axis;
Θ_1 - maximum vibrational amplitude of Θ at time t = 0;
$\dot{\Theta}$, $\ddot{\Theta}$ - rotational velocity and acceleration relative to earth;
λ - exponent, root;
ρ - air density;
ω_2, ω_3 - undamped natural frequency of building segment in degree of freedom 2 and 3 (rad/sec)
∂ - partial derivative; and
2, 3 - lateral and rotational degree of freedom subscripts in the y and Θ modes of vibration, respectively.

1. INTRODUCTION

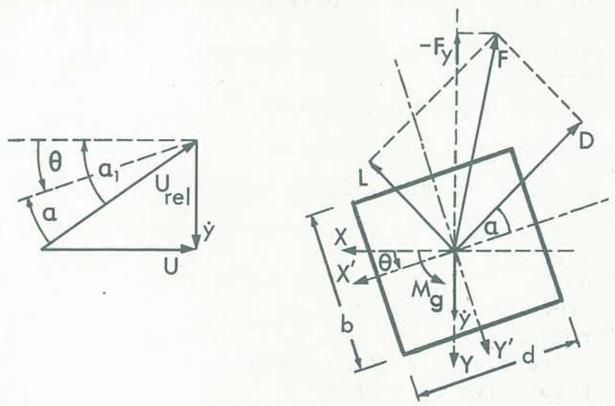
Of the several forms of aeroelastic behaviour of bluff bodies that have been identified, the best known perhaps, is the vortex-excited oscillation, where the fundamental frequency of the Karman vortex street coincides with the natural frequency of the transverse mode. Galloping transverse vibrations represent another form of self-excited phenomena which will occur in a single degree of freedom body. Galloping was first described by Den Hartog (1,2). Since then it has been extensively studied, and summarised by Scruton (3), Richardson, et al. (4), and others.

For square prisms, Scruton (3) determined a procedure for evaluating the steady amplitudes due to galloping, based on a quasi-steady approach. Parkinson, et al, (5) who solved the problem using nonlinear analysis, considered the instability using phase-plane analysis and validated the use of the quasi-steady approach in the wind tunnel for sharp edged, symmetric cross sections (5,6). Davenport (7) suggested that galloping instability may become a problem in some types of light tall buildings.

This paper forms part of a continuing study of wind effects on tall buildings and extends the previous work on galloping to flutter by incorporating the torsional degree of freedom about the vertical axis. The stability of the building under wind action is then considered.

2. TRANSVERSE AERODYNAMIC FORCE

It has been shown (3,5,6) that a rectangular building is most sensitive to cross-wind oscillations, generated by galloping excitation, when the wind direction is near normal to one face.



X and Y are earth fixed axes with X directed towards the freestream wind direction.

FIG. 1. - BUILDING CROSS SECTIONAL SEGMENT IN AIRSTREAM

If a horizontal segment of a building with one face normal to the mean wind direction U, is considered in a uniform, two dimensional wind flow with an instantaneous cross-wind velocity of $\dot{y}$ and a small rotation Θ, the resulting angle of attack to the relative wind direction is

$$\alpha = \frac{\dot{y}}{U} - \Theta \qquad (1)$$

as shown in Figure 1.

In the quasi-steady approach, it is assumed that for every instant during the oscillation, the aerodynamic force on the body is the same as for a static test on a rigid body at the same angle of attack. This allows the aerodynamic force on the segment to be determined from the sectional drag and lift characteristics for a given angle of attack, α. Then the drag and lift force components are,

$$D = C_D \tfrac{1}{2}\rho U_{rel}^2 S \qquad (2a)$$

$$L = C_L \tfrac{1}{2}\rho U_{rel}^2 S \qquad (2b)$$

where
D is the drag force,
L is the lift force,
C_D is the sectional drag coefficient,
C_L is the sectional lift coefficient,
ρ is the air density,
U_{rel} is wind velocity relative to the building, and
S is the cross-wind reference area which may be taken as the across wind building breadth b by the sectional height.

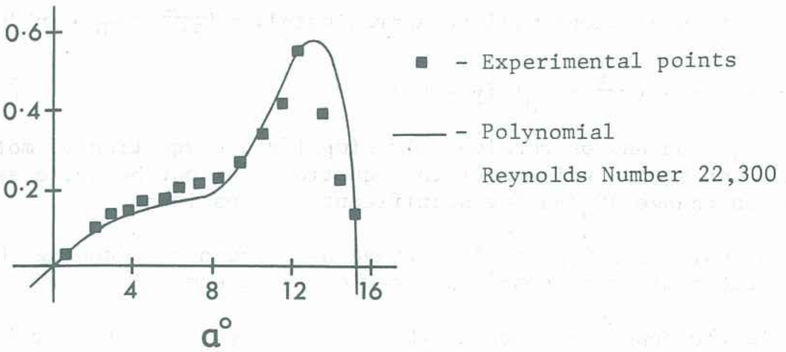


FIG. 2. - LATERAL FORCE COEFFICIENT IN THE Y' BODY AXIS DIRECTION PLOTTED AGAINST ANGLE OF ATTACK FOR A SQUARE PRISM IN UNIFORM TWO-DIMENSIONAL TURBULENT FLOW

These wind axis forces may be resolved in the Y axis force of the earth axes as

$$F_y = -(L \cos \alpha + D \sin \alpha) \cos \Theta - (D \cos \alpha - L \sin \alpha) \sin \Theta \qquad (3)$$

This equation may be non-dimensionalised by using

$$C_y = \frac{F_y}{\frac{1}{2}\rho U^2 S}, \quad C_L = \frac{L}{\frac{1}{2}\rho U_{rel}^2 S}, \quad C_D = \frac{D}{\frac{1}{2}\rho U_{rel}^2 S}$$

Noting that $U = U_{rel} \cos (\alpha + \Theta)$, then C_y, the lateral force coefficient is

$$C_y = -(C_L \cos \alpha + C_D \sin \alpha) \sec^2 (\alpha + \Theta) \cos \Theta$$
$$- (C_D \cos \alpha - C_L \sin \alpha) \sec^2 (\alpha + \Theta) \sin \Theta \qquad (4)$$

To obtain an estimate of the order of the angles involved, only the cross-wind motion was considered of a building of height 500ft (150m) with a typical period of 5 sec., where the sway at the top is arbitarily taken as 0.25ft (0.075m), due to a wind profile that has a velocity of 130ft/s (39m/s) at the building top. Tall slender buildings deflect in a fundamental mode shape that can be reasonably approximated to a straight line pivoting about the base, which is due to the combination of cantilever and sway deformations within the building. So the sway deflection divided by the building height is 1 in 2000 which is well within stress limits for a typical building. If the motion is considered to be S.H.M., the maximum acceleration at the top floor will be $0.4 ft/s^2$ ($0.12 m/s^2$) which is above the acceleration perception threshold level for 90% of the population reported by Robertson and Chen (8). So, if the torsional rotation is neglected, this motion will generate a maximum velocity of 0.32ft/s (0.096 m/s) and a maximum angle according to eq. 1 of 0.15 deg. This means $\dot{y}/U$ can be taken as small and infers that Θ in a building would have an allowable maximum of a similar value.

Therefore, if $\cos \alpha \simeq 1$, $\cos^2 (\alpha + \Theta) \simeq 1$ and Θ is small, then eq. 4 becomes,

$$C_{y'} = -(C_L + C_D \sin \alpha) \qquad (5)$$

The lateral force coefficient in the Y' body axis direction has been experimentally determined by Parkinson and Smith (2) for a square prism in uniform flow and is shown in Fig. 2. To within 0.1%, the $C_{y'}$ curve will be the same as C_y curve, if the maximum allowable Θ is less than 2 deg.

Parkinson and Smith (2) approximated these experimental results by a seventh order, odd polynomial (the solid line in Fig. 2), which when substituted in the equation of motion for the transverse oscillation will create a non-linear differential equation.

As the change in angle of attack is small compared with the fairly smooth curve of Fig. 2, particularly in the negative region from 0 to 13 deg., it appears reasonable for Engineering purposes, to consider a linear approximation the gradient in the range of oscillation, except perhaps for the metastable region around 13 to 14 deg. This gives, using eq. 1,

$$F_y = \frac{1}{2}\rho U S \frac{\partial C_y}{\partial \alpha} (\dot{y} - U \Theta) \qquad (6a)$$

where $\partial C_y/\partial \alpha$ is the gradient associated with the angle of attack about which the oscillation is occurring.

For small α, this gradient will be approximately $-\left(\frac{\partial C_L}{\partial \alpha} + C_D\right)$, so we have

$$F_y = -\tfrac{1}{2}\rho US\left(\frac{\partial C_L}{\partial \alpha} + C_D\right)(\dot{y} - U\Theta) \qquad (6b)$$

The use of eqs. 6a and 6b considerably simplify the equation of motion, with probably little loss in accuracy for square prisms, but the equation will not be quite as flexible as previously due to the need to change $\partial C_y/\partial\alpha$ for significant changes in α.

In this expression the $\dot{y}$ part will behave as aerodynamic damping, but the $U\Theta$ compliment will behave as an 'aerodynamic stiffness', while the freestream velocity U is a constant.

As $-F_y$ is in the opposite sense to the motion, like the inertia and stiffness terms, the homogeneous equation of motion without the structural damping is

$$M\ddot{y} - F_y + K_2 y = 0 \qquad (7)$$

where M is the mass of the building segment, the acceleration in the transverse direction is $\ddot{y}$ and K_2 is the stiffness of the building for the segment.

It is worth emphasising at this stage that the equations and criteria developed here are only valid for a segment, unless the mean velocity U, the building stiffness, mass and the rotational moment of inertia about the vertical centre of gravity axis (see later), are considered over the whole structure with regard to variation with height and the building vibrational mode shape.

If the single degree of freedom case of transverse oscillation is considered and eq. 6b is substituted in eq. 7, then instability will occur if any force term in this equation increases in the same direction as the motion. This is equivalent to any term being negative and so instability will occur if

$$\frac{\partial C_L}{\partial \alpha} < -C_D \qquad (8)$$

which is equivalent to the condition determined by Den Hartog (2).

3. TORSIONAL AERODYNAMIC MOMENT

The aerodynamic moment on a body is normally determined from

$$M_g = C_m \tfrac{1}{2}\rho U_{rel}^2 S\ell \qquad (9)$$

in which the aerodynamic moment about the centre of gravity axis is given by M_g, C_m is the moment coefficient and, S may again be represented by b multiplied by the sectional height, and the characteristic length ℓ by the streamwise depth d. Therefore using eq. 1 and adopting the same approach that generated eq. 6a,

$$M_g = \tfrac{1}{2}\rho USd\frac{\partial C_m}{\partial \alpha}(\dot{y} - U\Theta) \qquad (10)$$

where $\partial C_m/\partial\alpha$ is the gradient associated with the angle of attack about which the oscillation is occurring.

Similar to eq. 7, $-M_g$ is in the opposite sense to the motion and the homogeneous equation of motion, for the rotational degree of freedom, without the structural damping, is

$$I\ddot{\Theta} - M_g + K_3\Theta = 0 \qquad (11)$$

I is the rotational moment of inertia of the building segment about the vertical centre of gravity axis of the building, which is considered in the centre of the rectangular section, and K_3 is the building stiffness in the Θ mode for the segment.

4. BUILDING INSTABILITY CONDITIONS

This analysis will be restricted to small amplitudes of oscillation where it is mathematically conventional to assume the structural damping is velocity dependent. As this is a stability analysis, it is not necessary to consider any steady-state forcing functions on the segment and so the homogeneous equations of motion from eqs. 6, 7, 10 and 11, with the addition of the structural damping terms, are

$$M\ddot{y} - \tfrac{1}{2}\rho US\frac{\partial C_y}{\partial \alpha}(\dot{y} - U\Theta) + C_2\dot{y} + K_2 y = 0 \qquad (12)$$

and $$I\ddot{\Theta} - \tfrac{1}{2}\rho\, USd \frac{\partial C_m}{\partial \alpha}(\dot{y} - U\Theta) + c_3\dot{\Theta} + K_3\Theta = 0 \qquad (13)$$

where c_2 and c_3 are the equivalent viscous structural damping coefficients in the degrees of freedom 2 and 3 respectively. Motion in the streamwise direction is not important for stability considerations.

If eq. 12 is divided by M and eq. 13 by I and noting that $\omega_2^2 = K_2/M$, $\omega_3^2 = K_3/I$, $c_2/M = 2\zeta_2\omega_2$ and $c_3/M = 2\zeta_3\omega_3$, (pp. 56-4, 11), where ω is the undamped natural frequency and ζ is the fraction of critical damping, for each mode of oscillation, then rearranging gives

$$\ddot{y} + \left(2\zeta_2\omega_2 - \rho\frac{US}{2M}\frac{\partial C_y}{\partial \alpha}\right)\dot{y} + \omega_2^2 y + \rho\frac{U^2S}{2M}\frac{\partial C_y}{\partial \alpha}\Theta = 0 \qquad (14)$$

and

$$\ddot{\Theta} + 2\zeta_3\omega_3\dot{\Theta} + \left(\omega_3^2 + \rho\frac{U^2Sd}{2I}\frac{\partial C_m}{\partial \alpha}\right)\Theta - \rho\frac{USd}{2I}\frac{\partial C_m}{\partial \alpha}\dot{y} = 0 \qquad (15)$$

Solutions of equations of this type are exponential and in the case of y, for example, would be of the typical form

$$y = Y_1 e^{\lambda_1 t} + Y_2 e^{\lambda_2 t} + \ldots \qquad (16)$$

where Y_1, Y_2..., are the amplitudes at time $t = 0$ and e is the base of natural logarithms. λ may be either real or complex, with the complex ones always occurring in conjugate pairs, such as $\lambda = r \pm i\,\omega$.

So terms may appear in the form

$$Y_1 e^{rt} \qquad (17a)$$

which will be stable if r is negative or unstable if r is positive, with the amplitude increasing with time, which by convention may be referred as 'static instability'.

If λ is complex the corresponding term will be of the form

$$Y_1 e^{(r + i\omega)t} + Y_2 e^{(r - i\omega)}$$

which may be rewritten as

$$e^{rt}(A_1 \cos \omega t + A_2 \sin \omega t) \qquad (17b)$$

where $A_1 = Y_1 + Y_2 = i(Y_1 - Y_2)$. As A_1 and A_2 must always be real, this implies that Y_1 and Y_2 must be a complex conjugate pair. In this case the motion corresponding to λ is oscillatory. If r is negative, the motion is a damped convergent oscillation, but if it is positive, an unstable divergent oscillation occurs, which may be termed a 'dynamic instability'.

If it is assumed that eqs. 14 and 15 have transient solutions of the form

$$y = Y_1 e^{\lambda t}$$

$$\Theta = \Theta_1 e^{\lambda t}$$

Substituting these solutions into eqs. 14 and 15 and cancelling the common exponential term of $e^{\lambda t}$ since the degenerate case when $e^{\lambda t} = 0$ is not of interest, then

$$\left[\lambda^2 + \left(2\zeta_2\omega_2 - \rho\frac{US}{2M}\frac{\partial C_y}{\partial \alpha}\right)\lambda + \omega_2^2\right]Y_1 + \rho\frac{U^2S}{2M}\frac{\partial C_y}{\partial \alpha}\Theta_1 = 0 \qquad (18)$$

$$-\rho\frac{USd}{2I}\frac{\partial C_m}{\partial \alpha}\lambda\, Y_1 + \left[\lambda^2 + 2\zeta_3\omega_3\lambda + \left(\omega_3^2 + \rho\frac{U^2Sd}{2I}\frac{\partial C_m}{\partial \alpha}\right)\right]\Theta_1 = 0 \qquad (19)$$

No solutions exist for the unknowns Y_1 and Θ_1 in these types of simultaneous equations containing the parameter λ, unless the determinant of the coefficient matrix is zero. This stability determinant when expanded and set equal to zero, produces the 'characteristic' equation of the dynamic system, (pp.193,9;11 and others), which is

$$\lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3\lambda + a_4 = 0 \tag{20}$$

where

$$a_1 = 2(\zeta_2 \omega_2 + \zeta_3 \omega_3) - \rho\frac{US}{2M}\frac{\partial C_y}{\partial\alpha})$$

$$a_2 = \omega_2^2 + \omega_3^2 + \rho\frac{U^2Sd}{2I}\frac{\partial C_m}{\partial\alpha} + 2\zeta_3\omega_3 (2\zeta_2\omega_2 - \rho\frac{US}{2M}\frac{\partial C_y}{\partial\alpha})$$

$$a_3 = 2\omega_2\omega_3 (\zeta_3\omega_2 + \zeta_2\omega_3) + \rho\frac{US}{2} (2\zeta_2 \omega_2 \frac{Ud}{I}\frac{\partial C_m}{\partial\alpha} - \frac{\omega_3^2}{M}\frac{\partial C_y}{\partial\alpha})$$

$$a_4 = \omega_2^2 (\omega_3^2 + \rho\frac{U^2Sd}{2I})\frac{\partial C_m}{\partial\alpha}$$

The conditions for there to be no unstable modes of vibration in the solution of the equations of motion may be determined from the characteristic equation, by the use of Routh's Criteria (9, 10, 11 and others). These stability conditions are determined by the use of various 'test' functions, formed from the coefficients of the characteristic equation, which must all be positive if all modes are to be stable. For a quartic such as eq. 20, the fourth of the four possible test functions (pp.194, 9) is

$$F_4 = a_1 a_4 F_3 \tag{21}$$

where F_3 is the third test function of the form

$$F_3 = a_1 (a_2 a_3 - a_1 a_4) - a_3^2 \tag{22}$$

which is commonly known as "Routh's discriminant".

The necessary and sufficient conditions for all the test functions to be positive are that a_1, a_3 and a_4 are positive and that F_3 is also positive, which together, infer that a_2 must also be positive (9, pp.195). Duncan (12) has shown that

(a) if only $a_4<0$, then one real root of eq. 20 will be positive and so one divergence will appear in the solution;

(b) if only $F_3<0$, then the real part of one pair of a complex pair of roots will be positive and so a divergent oscillation will occur in the solution;

and that these criteria define, in the case of (a), the boundary between stability and static instability, and in (b) the boundary between a stable and a divergent oscillation.

Firstly, considering case (a) from these conditions and eq. 20, static instability will occur in the torsional mode if

$$\frac{\partial C_m}{\partial\alpha} < -\frac{2I\omega_3^2}{\rho U^2 Sd} \tag{23}$$

Noting that $\omega_3^2 = K_3/I$ and eq. 9, eq. 23 becomes

$$\frac{\partial M_g}{\partial\alpha} < -K_3 \tag{24}$$

which is apparent from inspection.

To obtain an approximate value of the torsional instability threshold of $\partial C_m/\partial\alpha$ for square buildings in uniform flow, a unit segment at the top of the B.H.P. building under construction in Melbourne was considered. This structure has a height of 500 ft. (150 m), is 126 ft. square (37.6 m), with an effective mass of 1.1 x 10^6 kg) giving an effective rotational moment of inertia about the elastic axis of 1.5 x 10^9 slug. ft^2 (2 x 10^9 kg.m^2), the velocity at the top of the building was taken as 150 ft/sec (45 m.s^{-1}). The term 'effective' is used because an assumption of the torsional mode shape being a cyclicly rotated equivalent to the transverse mode shape, in which the deflected shape was approximated by a straight line pivoted about the base, divided the mass and rotational moment of inertia by three. There is no reliable estimate for the torsional period, but recent reports on reasonably similar buildings (13, 14) suggest that the torsional period, within a factor of two, is the same as the mean sway period, and so the period was taken as 5 sec. Substituting this data into eq. 23 gave $\partial C_m/\partial\alpha$ = -11 per rad.

From preliminary static tests by the author on a pressure tapped, prismatic model of a square building with a 5 to 1 aspect ratio (height/breadth) immersed in a simulated suburban profile in a 1 metre square wind tunnel, the $\partial C_m/\partial\alpha$ curve was found to be a stable positive slope from 0 to about 15 deg. The Reynolds Number was 65,000 at the top of the model and the model was immersed to a depth of 50% of the boundary layer height. Between a plus and minus 5 degrees, angle of attack $\partial C_m/\partial\alpha$ was found to be about 0.2 per rad.

So at this stage it appears unlikely from the sign and magnitude of these values of $\partial C_m/\partial\alpha$ that any similar structure under these conditions would become torsionally unstable.

Secondly from case (b), a divergent oscillation will occur if, from

$$a_1(a_2\, a_3 - a_1\, a_4) - a_3^2 < 0 \qquad (25)$$

In order to simplify this condition, the data for the B.H.P. building mentioned previously was substituted into eq. 25 with the addition of the following data. The fraction of critical damping taken as 0.01, the period changed to 2Π sec., and the depth and breadth rounded off to 100 ft. (30m). $\partial C_m/\partial\alpha$ was taken to be of the potentially unfavourable sign, -0.2 per rad., and similarly, from Fig. 2, for $\alpha = 0$, $\partial C_y/\partial\alpha = 2$ per rad, was substituted.

To determine the threshold values of either $\partial C_y/\partial\alpha$ or $\partial C_m/\partial\alpha$ that will define the boundary of a transverse or a torsional divergent oscillation, it was found that by changing the magnitude of a_1 or a_3 by about 1% after using the above data, provided that the transverse and torsional natural frequencies are approximately unity, eq. 25 could be reduced to

$$a_i\, a_j\, (a_2 - a_4 - 1) < 0 \qquad (26)$$

where $i, j = 1$ or 3.

If $i = j = 1$ or 3 then only $a_2 - a_4 - 1 < 0$ can produce a divergent oscillation.

If $i = 1$ and $j = 3$ then,

(i) $a_1 < 0$ generates the condition that a divergent oscillation will occur if

$$\frac{\partial C_y}{\partial\alpha} > \frac{4M\,(\zeta_2\,\omega_2 + \zeta_3\,\omega_3)}{\rho\, US} \qquad (27)$$

which is equal to about 5 per rad. for this data. If $\zeta_3\,\omega_3 = 0$ then this condition reduces to that found by Den Hartog (1) for the galloping of transmission lines and also by Parkinson (5) for a square section in a two dimensional uniform flow oscillating in the transverse mode.

(ii) $a_3 < 0$ generates the condition that a divergent oscillation will occur if

$$\frac{\partial C_y}{\partial\alpha} > \frac{4M}{\rho US}\,\frac{\omega_2}{\omega_3}\,(\zeta_3\,\omega_2 + \zeta_2\,\omega_3) \qquad (28)$$

when the $\partial C_m/\partial\alpha$ term is eliminated from a_3 which changes the magnitude of a_3 by about 0.5% for this data. $\partial C_y/\partial\alpha$ will again be about 5.

(iii) For the condition $a_2 - a_4 - 1 < 0$, substituting for a_2 and a_4, this inequality becomes

$$\omega_2^2 + \omega_3^2 - \omega_2^2\,\omega_3^2 - 1 + 4\,\zeta_2\,\omega_2\,\zeta_3\,\omega_3 - \rho\frac{US}{M}\,\zeta_3\,\omega_3\,\frac{\partial C_y}{\partial\alpha} + (1-\omega_2^2)\,\rho\frac{U^2 Sd}{2I}\,\frac{\partial C_m}{\partial\alpha} < 0 \qquad (29)$$

If a similar building to that being considered is designed so that the transverse and torsional periods are the same and equal to 2Π seconds, i.e. $\omega_2 = \omega_3 = 1$, then the $\partial C_m/\partial\alpha$ term plus the first four terms in eq. 29 vanish and so a transverse divergent oscillation will occur if

$$\frac{\partial C_y}{\partial\alpha} > \frac{4M}{\rho US}\,\zeta_2\,\omega_2 \qquad (30)$$

which is the same as eq. 27 with $\zeta_3\,\omega_3 = 0$, which is that found by Den Hartog and Parkinson and has the value 2.5 for the building data being used.

If however there is a 10% shift in the periods away from 2Π seconds, the contribution of the $\partial C_y/\partial\alpha$ term in eq. 29 will be less than 1%.

So, if $\partial C_y/\partial\alpha$ is eliminated, eq. 29 becomes, after factorising the first four terms,

$$(\omega_3^2 - 1)(1 - \omega_2^2) + 4\,\zeta_2\,\omega_2\,\zeta_3\,\omega_3 + (1 - \omega_2^2)\,\rho\,\frac{U^2 Sd}{2I}\,\frac{\partial C_m}{\partial\alpha} < 0 \qquad (31)$$

So a divergent oscillation in the torsional mode may occur if

$$\frac{\partial C_m}{\partial\alpha} < 2\,[(1 - \omega_3^2) - \frac{4\zeta_2\,\omega_2\,\zeta_3\,\omega_3}{(1-\omega_2^2)}]\,\frac{I}{\rho U^2\,Sd} \qquad (32)$$

and using eq. 9,

$$\frac{\partial M_g}{\partial\alpha} < [(1 - \omega_3^2) - \frac{4\,\zeta_2\,\omega_2\,\zeta_3\,\omega_3}{(1 - \omega_2^2)}]\,I \qquad (33)$$

when the building parameters are similar to those mentioned.

For $\omega_3 = 1.1$ rad/sec and $\omega_2 = \pm\,0.9$ rad/sec $\partial C_m/\partial\alpha = -\,2$, but for $\omega_3 = 0.9$ rad/sec and $\omega_2 = \pm\,1.1$ rad/sec, the gradient is 2.

4. CONCLUSIONS

The boundary criterion for static instability is given by eq. 23. The boundary condition for divergent oscillation is given by Routh's discriminant being less than zero, eq. 25. However, assuming that the torsional and transverse periods are near 2Π seconds, together with small damping and typical data for a tall, slender rectangular building, the boundary conditions for divergent oscillation may be any one of the simplified forms presented in eqs. 27, 28, and 29.

If wind tunnel tests of a model building are being considered, the necessity to model the torsional rotation appears to depend primarily on the magnitude and sign of $\partial C_m/\partial\alpha$. This requirement can be assessed from values of $\partial C_m/\partial\alpha$ determined from a pressure tapped model.

5. ACKNOWLEDGEMENTS

The author is indebted to Dr. W.H. Melbourne, Senior Lecturer, for his guidance, encouragement and many comments regarding this work, and Professor J.D.C. Crisp, Engineering Dynamics for his help and advice; both of the Department of Mechanical Engineering, Faculty of Engineering, Monash University. The writer would like to also thank Mr. J. Fowler, Partner in Irwin, Johnston and partners P/L, Melbourne for the B.H.P. building information.

6. REFERENCES

1. Den Hartog, Transactions, AIEE, Vol. 49, 1930, pp. 444.

2. Den Hartog, "Mechanical Vibrations", McGraw-Hill Book Co., N.Y., 4th ed., 1956.

3. Scruton, C., "The Use of Wind Tunnels in Industrial Aerodynamic Research" Advisory Group for Aeronautical Research and Development Report 309, Oct., 1960.

4. Richardson, A.S., Martuchelli, J.R., and Prise, W.S., Research Study on Galloping of Electric Power Transmission Lines, Sympsium on Wind Effects on Buildings and Structures, National Physical Laboratory, Teddington, England, 1963, pp. 612-686.

5. Parkinson, G.V., Smith, J.D., "The Square Prism as an Aeroelastic Non-Linear Oscillator", Quarterly, Journal of Mechanics & Applied Mathematics, Oxford University Press, Ely House, London, England, Vol. XVII Pt. 2, pp225.

6. Parkinson, G.V., Brooks, N.P.H., "On the Aeroelastic Instability of Bluff Cylinders, "Transactions, Journal of Applied Mechanics, ASME, Vol. 83, 1961 pp. 252-8.

7. Davenport, A.G., "The Treatment of Wind Loading on Tall Buildings, "Proceedings of the Symposium on Tall Buildings", Southampton, England, April, 1966.

8. Robertson, A.D., & Chen, C.F., "Application to Design of Research on Wind Effects on Buildings and Structures., Ottawa, Sept, 1967.

9. Etkins, B., "Dynamicsof Flight", Wiley, N.Y., 1959.

10. Scanlan, R.J., & Rosenbaum, R., "Introduction to the Study of Aircraft Virbration and Flutter", Macmillan, N.Y., 1951.

11. Flugge, W., editor "Handbook of Engineering Mechanics". McGraw-Hill Book Co., 1962, Chapter 57, pp. 19.

12. Duncan, W.J. "Control and Stability of Aircraft". Cambridge University Press, Cambridge 1952.

13. Nielson, N.N. "Vibration Tests of a Nine Story Steel Frame Building". Journal of Engineering Mechanics Div., Proc. Amer. Soc. of Civil Engrs. Feb., 1966.