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SELECTIVE WITHDRAWAL FROM DENSITY STRATIFIED FLUIDS IN RESERVOIRS

by

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SUMMARY

The technique of selectively withdrawing from density stratified water in reservoirs has become widely used as a method of controlling the quality of water released.

In this paper, the effect of different boundary shapes on the ability to selectively withdraw a single layer from a multi-layered fluid, is examined. The two basic geometrical shapes investigated are the withdrawal through a contraction with the crest of a broad-crested weir at its minimum width and the axisymmetric withdrawal over a weir. The theories used in this investigation have been verified experimentally.

The results indicate that the selective withdrawal process becomes more sensitive to boundary geometry as the density difference between the flowing layer and the denser stationary layer beneath it, decreases.

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INTRODUCTION

Density stratification of the water in a reservoir has a large effect on the movement of that water within the reservoir. As the quality of water contained in reservoirs, cooling ponds etc. is often interrelated to density (temperature, dissolved salts, suspended sediment, dissolved oxygen etc.) the management of the quality of water released depends to a large extent on how well the water movements can be predicted and controlled. The selective withdrawal technique, whereby the water released from a reservoir can be chosen knowing the density stratification conditions and given outlets at various levels, has become widely used to this end. (A.S.C.E. Task Committee on Outlet Works, (1)).

In this paper, two cases of the flow of a multilayered fluid are examined:

(i) Three-dimensional flow from a reservoir through a smoothly contracting channel with a definite minimum width and with the crest of a broad-crested weir at the point of minimum width.

(ii) Axisymmetric flow from a reservoir over a broad crested weir at the exit.

For both flow cases, the pressure distributions within flowing layers are assumed to be hydrostatic. This assumption is an extension of the one-dimensional approach used in open channel hydraulics (Henderson, (2)) and is valid provided that streamline curvature is small. The fluid is assumed to be inviscid and the flow is considered to be steady. This latter restriction is of minor importance provided the reservoir is sufficiently large so that the time for a particle to travel through the contraction is short compared with the time for streamline patterns to change due to the withdrawal of fluid from the reservoir.

In the flow situations examined the reservoir will be considered to contain three stable layers, the uppermost of which always remains stationary.

The conditions under which the weir prevents withdrawal from the lowest layer with a single layer being withdrawn from the reservoir will be examined for each case (see Figure 1).

The effect of different boundary shapes on the selective withdrawal process is examined using the theories derived.

THE SELECTIVE WITHDRAWAL FROM A SINGLE LAYER

Case (i): Withdrawal from a reservoir through a contraction with the crest of a broad-crested weir at its minimum width.

Let the shape of the weir be given by $h = h(x)$ and that of the contraction by $b = b(x)$ and suppose that the total depth behind the weir $(Y_1 + Y_2)$ is known. However, the individual values of the layer depths (Y_1 and Y_2) are not known. It is then required to determine the minimum depth of layer 1 (Y_1) such that there is no withdrawal from the lower layer.

It can be shown that critical depth must occur at the point of minimum width and weir crest for the flow of a single layer (Henderson 1966). Thus the discharge from layer 1 is given by

$$Q_1^{\,2} = \frac{2}{3} \frac{\Delta\rho_1}{\rho_1} g\, Y_t\, b_m^{\,2} \left(\frac{2}{3} Y_t\right)^2 \qquad (1)$$

where
Q_1 = the volumetric discharge from layer 1
ρ_1 = the density of layer 1
$\Delta\rho_1$ = the density difference across interface 0-1
g = gravitational acceleration
b_m = the minimum width of the contraction
Y_t = the height of the interface 0-1 above the weir crest

which for a given weir and contraction depends only on the conditions in the reservoir.

Assuming that velocities in the reservoir are negligible, the energy equation along the interface 0-1 upstream of the point of contact, (A), is

$$\tfrac{1}{2}\,\mathbb{F}_1^{\,2}\;y_1 + y_1 + y_2 + h \;=\; (Y_1 + Y_2) \tag{2}$$

where

$$\mathbb{F}_1^{\,2} = \frac{\rho_1}{\Delta\rho_1 g}\left(\frac{Q_1}{b y_1}\right)^2 \cdot \frac{1}{y_1} \tag{3}$$

and
$\mathbb{F}_1$ = a densimetric Froude Number of layer 1 at any point
y_1 = the depth of layer 1 at any point
y_2 = the depth of layer 2 at any point
h = the height of the weir at any point
b = the width of the contraction at any point

Upstream of the point of contact we also have the condition of hydrostatic pressure along the x-axis -

$$\alpha y_1 + (1+\alpha)(y_2+h) = \alpha Y_1 + (1+\alpha)Y_2 \tag{4}$$

where $\alpha = \dfrac{\Delta\rho_1}{\Delta\rho_2}$

$\Delta\rho_2$ = the density difference across interface 1-2

Now Equations (2) and (4) must hold simultaneously at the point of contact, thus by eliminating the term $(y_2 + h)$ from (2) and (4) we get the equation

$$\tfrac{1}{2}(1+\alpha)\,\mathbb{F}_1^{\,2}\;y_1 + y_1 \;=\; Y_1 \tag{5}$$

which must hold at the point of contact of the interface 1-2 with the weir. By varying the position of the point of contact x_c, and solving for y_1 at this point using equation (2), a plot of Y_1 versus x_c can be obtained and the minimum value of Y_1 (= Y_1') selected.

It can also be shown that at the point of contact

$$(1+\alpha)\,\mathbb{F}_1^{\,2} < 1 \tag{6}$$

and thus the search for the minimum value of Y_1 should only start after this requirement is satisfied.

Case (ii): Axisymmetric withdrawal from a reservoir over a broad-crested weir.

The axisymmetric flow of a single layer from a reservoir over a broad-crested weir differs markedly from the flow case just considered.

Consider the situation illustrated in Figures 1(b) and 1(c).

The energy equation along the interface 0-1 is

$$\frac{1}{2}\,\frac{\rho_1}{\Delta\rho_1 g}\left(\frac{Q_1}{2\pi x y_1}\right)^2 + y_1 + h \;=\; (Y_1 + Y_2) \tag{7}$$

where x = the radial distance to a point

or

$$\tfrac{1}{2}\,\mathbb{F}_1^{\,2}\,y_1 + y_1 + h = (Y_1 + Y_2) \tag{8}$$

where

$$\mathbb{F}_1^{\,2} = \frac{\rho_1}{\Delta\rho_1 g}\left(\frac{Q_1}{2\pi x y_1}\right)^2 \frac{1}{y_1} \tag{9}$$

Differentiating (8) gives

$$\frac{dy_1}{dx} = \frac{\frac{y_1}{x}\,\mathbb{F}_1^{\,2} - \frac{dh}{dx}}{(1 - \mathbb{F}_1^{\,2})} \tag{10}$$

Now impose the condition that $\frac{dy_1}{dx}$ must remain finite (i.e. there can be no discontinuities in depth along the interface).

Thus, when $\mathbb{F}_1^{\,2} = 1$ (critical flow)

$$\frac{y_1}{x}\,\mathbb{F}_1^{\,2} - \frac{dh}{dx} = 0 \tag{11}$$

otherwise $\frac{dy_1}{dx}$ goes to infinity.

Therefore at the control we have

$$y_1 = x\,\frac{dh}{dx} \tag{12}$$

Substituting $\mathbb{F}_1^{\,2} = 1$ into Equation 8

$$y_1 = \frac{2}{3}\left[(Y_1 + Y_2) - h\right] \tag{13}$$

Solving (11) and (12) simultaneously, gives

$$x\,\frac{dh}{dx} = \frac{2}{3}\left[(Y_1 + Y_2) - h\right] \tag{14}$$

Given the shape of the weir $h = h(x)$, equation (14) can be solved for x - the position of the control - and by substituting into (11) and (12) the depth at the control and the discharge from layer 1 are found.

From equation (14) it can be seen that the position of the control for a given weir is a function of the height to the interface 0 - 1 (i.e. $Y_1 + Y_2$), and the geometry of the weir. Also, as the term $[(Y_1 + Y_2) - h]$ must be positive, then at the control $\frac{dh}{dx}$ must be positive (as $x > 0$ always). Thus the control for the axisymmetric flow of a single layer cannot occur at the crest of the weir but must occur downstream of the crest.

Now having determined the behaviour of a single layer, the minimum depth of layer 1 for no withdrawal from layer 2 is determined in the same manner as for Case (i) using equations (5) and (6), the only difference being that for the axisymmetric flow case

$$\mathbb{F}_1^{\,2} = \frac{\rho_1}{g\,\Delta\rho_1}\left(\frac{Q_1}{2\pi x y_1}\right)^2 \cdot \frac{1}{y_1} \tag{15}$$

Experimental results obtained for both cases have agreed closely with the predictions of the theories. It has been observed that for a given value of Y_t layer 2 begins to flow at a slightly lower value of Y_1 than the theory predicts. This would be due to viscous effects which are not accounted for in the theory.

DISCUSSION OF THE RESULTS.

The effect of the value of α $(= \Delta\rho_1/\Delta\rho_2)$ is illustrated in Figures 2, 3 and 4. For a given geometry and value of Y_t the minimum depth of layer 1 at which there is no withdrawal from layer 2 (Y_1') increases as the value of α increases (i.e. as $\Delta\rho_2$ decreases). This is to be expected as withdrawal occurs from the entire depth of the reservoir when $\Delta\rho_2$ approaches 0 (i.e. α approaches infinity).

In addition, the sensitivity of the value of Y_1' to changes in geometry is significantly dependent upon the value of α. Figures 2 and 3 show that a change in geometry (e.g. by changing the shape of the weir profile from W1 to W2) produces a significant change in the value of Y_1 when α = 100, but has virtually no effect when α = 1.

This effect may be explained by considering the energy equation along the interface 1-2:

$$z = \frac{\rho_1}{\Delta\rho_2} \frac{V_1^2}{2g} \qquad (16)$$

where z = the elevation of a point on the interface above the level of the interface in the reservoir

V_1 = the velocity of the fluid in layer 1 at the point

From Equation (16) it can be deduced that a change in V_1 at a point (due to a change in geometry) will have a greater effect when $\Delta\rho_2$ is small than when $\Delta\rho_2$ is large.

In comparing the withdrawal through a contraction, which has the crest of a broad-crested weir at the minimum width (case (i)), with the axisymmetric withdrawal over a broad-crested weir (case (ii)) it should be noted that the length of the weir crest in case(i) (b_m) has been made equal to the length of the crest of the axisymmetric weir (= $2\pi r_c$, where r_c = the radius to the weir crest). Figure 4 indicates that, for a given value of Y_t, the value of Y_1' is greater for case (i) than for case (ii) when the weir profile is the same in each case. The difference between the two cases may be explained as follows. In case (i) the control for the discharge from layer 1 is fixed at the weir crest. However, for the axi-symmetric flow of a single layer over a weir, the control occurs downstream of the crest and its location is a function of the shape of the weir and the level of interface 0-1 in the reservoir (as implied by Equation (14)). This means that in the axisymmetric case the value of the densimetric Froude Number of layer 1 ($\mathbb{F}_1$) at the crest of the weir is considerably less than the critical value of unity, which is the value of $\mathbb{F}_1$ at the crest of the weir in the case of the withdrawal through a contraction. Equation (6) then implies that (for a given weir profile, a given value of Y_t and a fixed value of α) the point of contact of interface 1-2 with the weir for case (i) will occur further upstream, and hence at a lower elevation, than for case (ii). Furthermore, this implies that the value of Y_1' will be greater in case (i) than in case (ii).

It has also been noted that the control for the axisymmetric flow of a single layer over a weir moves closer to the crest of the weir with the following changes in geometry:-

(a) Changing the shape of the weir profile from W1 to W2 (i.e. increasing the steepness of the faces of the weir).

(b) Increasing the radius to the crest of the weir.

As indicated in Figure 4, the effect of the control in case (ii) moving closer to the weir crest (the location of the control in case (i)) is to reduce the difference between the values of Y_1' in each case (for given values of Y_t and α and a fixed weir profile).

CONCLUSION

The investigation described has demonstrated the striking effect of boundary geometry on the ability to selectively withdraw from a stable multi-layered fluid in a reservoir..

It has been show that the selective withdrawal process becomes more sensitive to boundary geometry as the density difference between the flowing layer and the denser stationary layer decreases.

REFERENCES

(1) A.S.C.E. Task Committee on Outlet Works, Committee on Hydraulic Structures. Register of Selective Withdrawal Works in the U.S.A. Proceedings of A.S.C.E. Vol.96 HY9, September 1970.

(2) Henderson, F.M. Open Channel Flow, Macmillan and Co., New York.

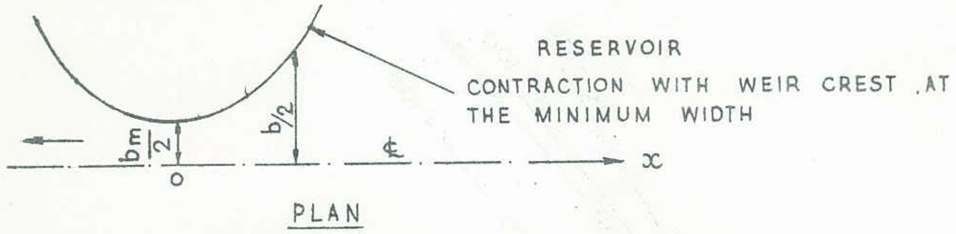


FIGURE 1a : WITHDRAWAL THROUGH A CONTRACTION

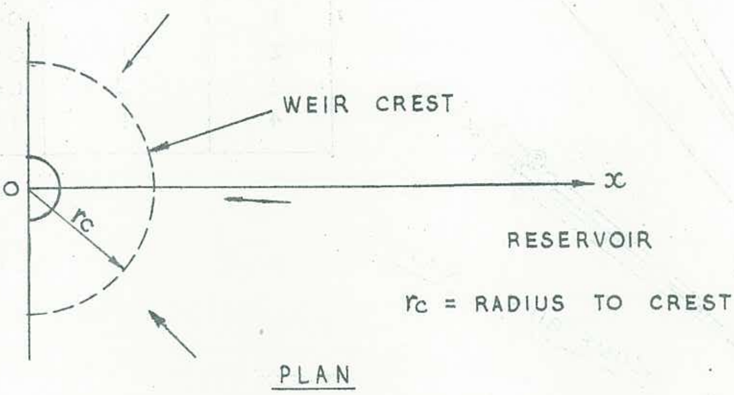


FIGURE 1b : AXISYMMETRIC WITHDRAWAL OVER A WEIR

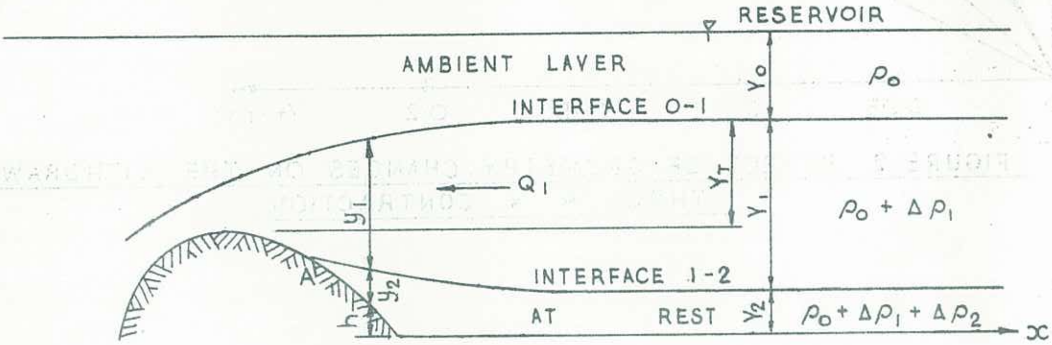
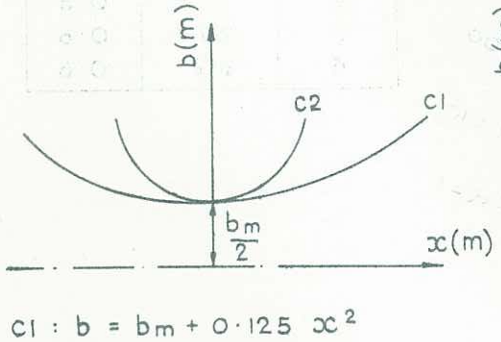
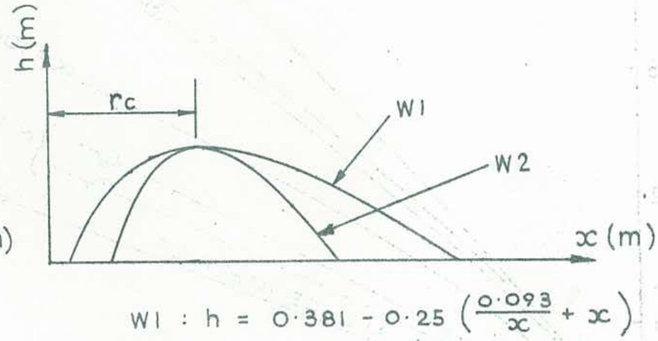


FIGURE 1c : SECTION ALONG - X - AXIS OF FIGURES 1a AND 1b



$$C1 : b = b_m + 0{\cdot}125\ x^2$$
$$C2 : b = b_m + 0{\cdot}5\ x^2$$

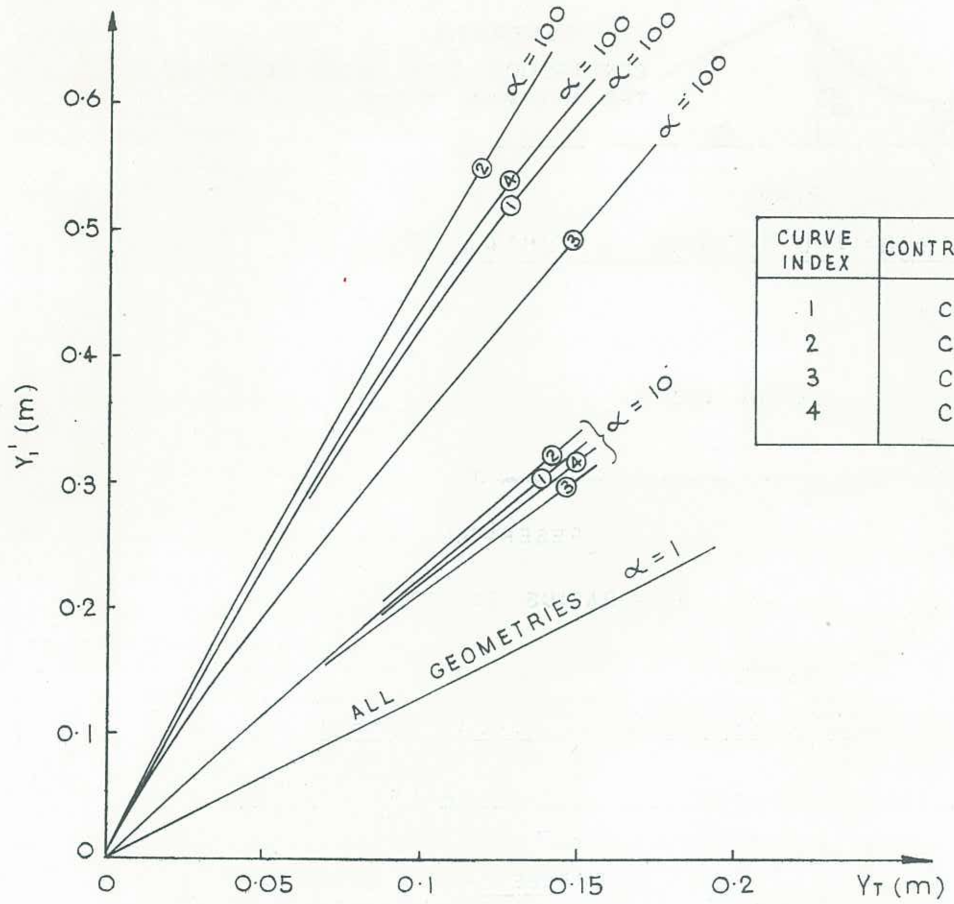
FIGURE 1d : GEOMETRY OF CONTRACTIONS



$$W1 : h = 0{\cdot}381 - 0{\cdot}25 \left(\frac{0{\cdot}093}{x} + x\right)$$
$$W2 : h = 0{\cdot}838 - \left(\frac{0{\cdot}093}{x} + x\right)$$

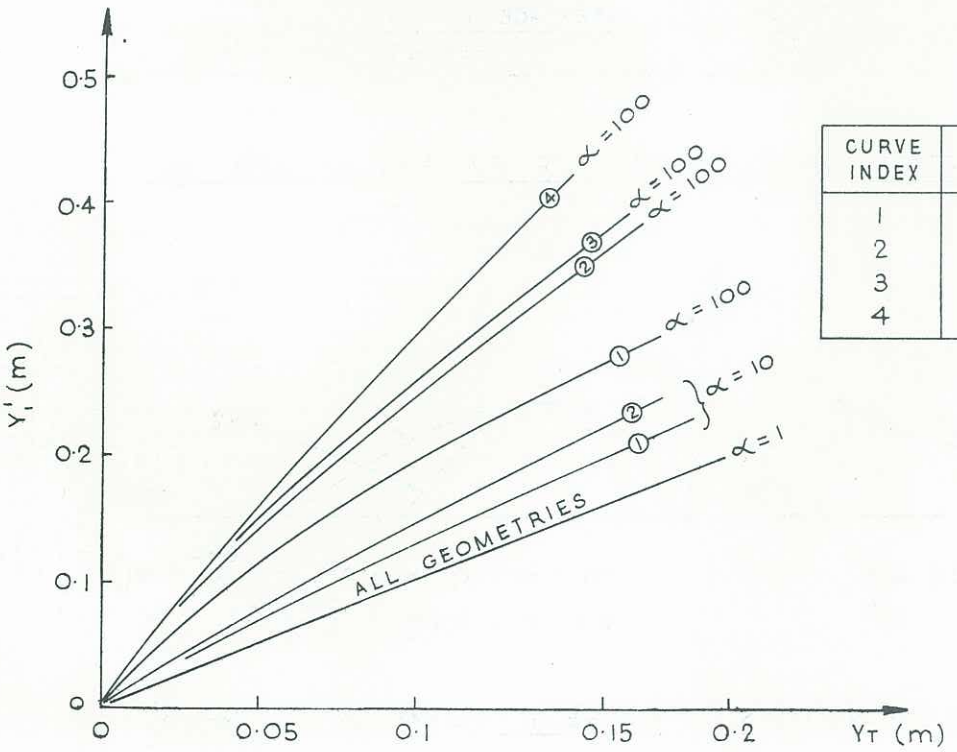
FIGURE 1e : GEOMETRY OF WEIRS

FIGURE 1



CURVE INDEX	CONTRACTION	bm (m)	WEIR
1	C1	0·6 π	W1
2	C1	0·6 π	W2
3	C2	0·6 π	W1
4	C1	1·2 π	W1

FIGURE 2 : EFFECT OF GEOMETRY CHANGES ON THE WITHDRAWAL THROUGH A CONTRACTION



CURVE INDEX	WEIR	rc (m)
1	W1	0·3
2	W2	0·3
3	W1	0·6
4	W2	0·6

FIGURE 3 : EFFECT OF GEOMETRY CHANGES ON AXISYMMETRIC WITHDRAWALS

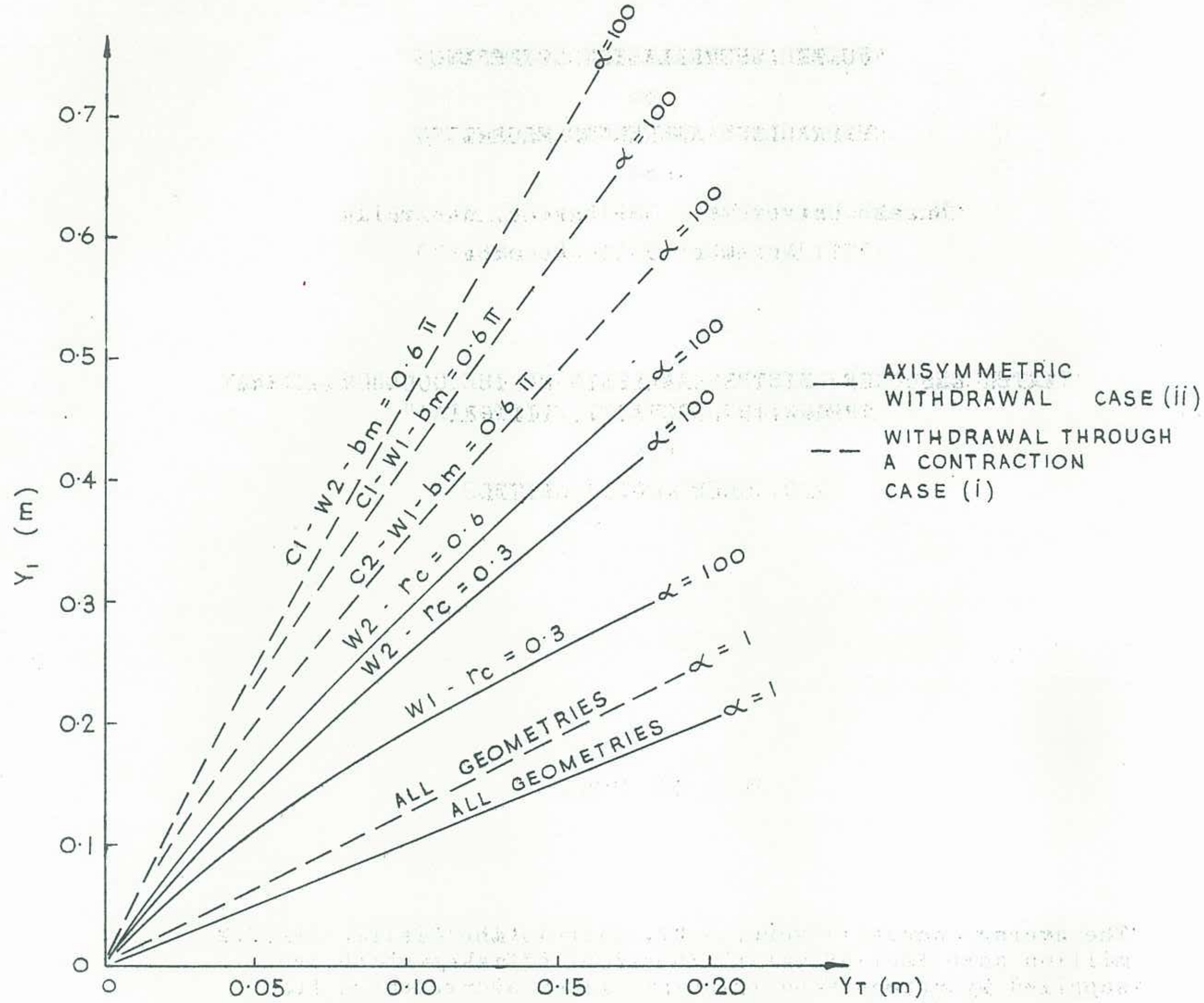


FIGURE 4 : COMPARISON OF AXISYMMETRIC WITHDRAWAL AND THE WITHDRAWAL THROUGH A CONTRACTION

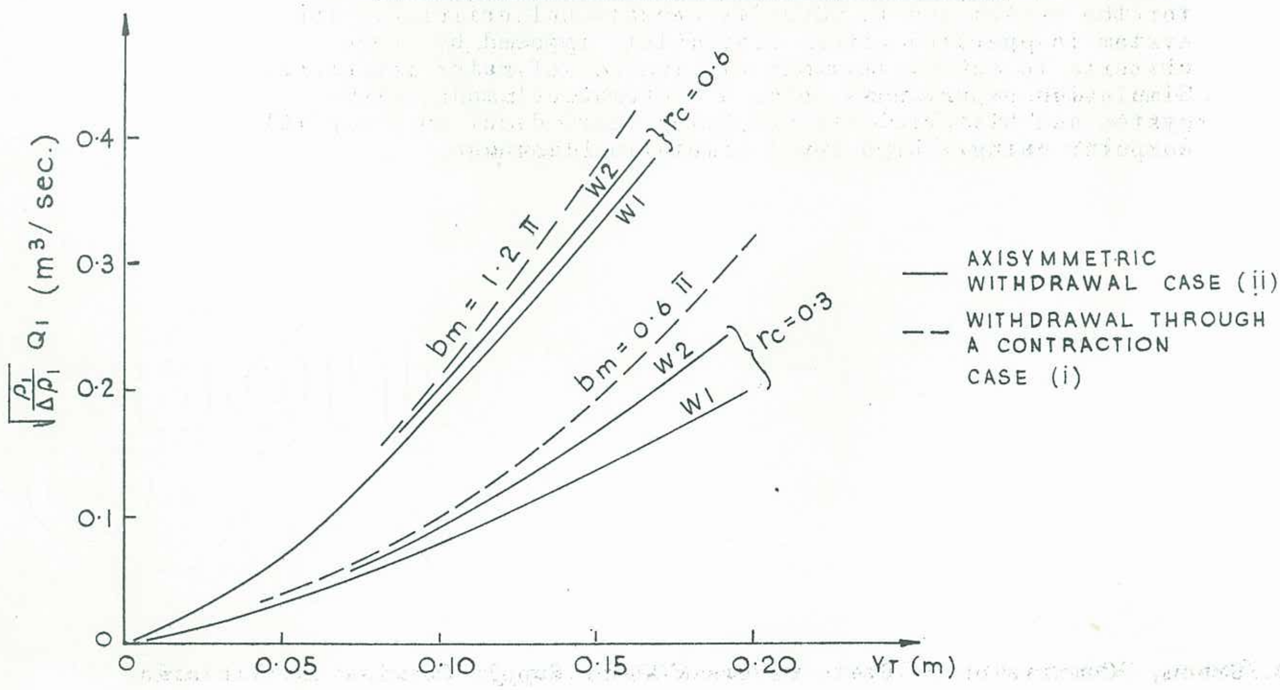


FIGURE 5 : DISCHARGE CURVES