FOURTH AUSTRALASIAN CONFERENCE
on
HYDRAULICS AND FLUID MECHANICS
at

Monash University, Melbourne, Australia
1971 November 29 to December 3

REAERATION IN RIVERS

by

D. Proctor*

SUMMARY

A correlation is proposed (with an accuracy of $\pm 10\%$) for the prediction of the mass transfer coefficient of atmospheric oxygen dissolving in deaerated rivers and channels from a knowledge of overall river velocities, depths and y-direction turbulent intensities in the vicinity of the free surface and is based on experimental work with open channels. Such a correlation is necessary in order to control river pollution.

---

* The author is a Research Scientist, Commonwealth Scientific and Industrial Research Organization, Division of Mechanical Engineering, Highett, Victoria, Australia.

INTRODUCTION

It is increasingly necessary to have means of estimating accurately how much effluent a given river will safely accommodate, so that the permissible limits of exploitation of a given river may be forecast and legislated for. If we are to solve the problem of river pollution and prevent rivers from dying biologically we must know how much atmospheric oxygen any part of any given river takes up, to replace that removed by the effluent during the natural self cleansing process taking place in rivers.

Previous workers in Britain and U.S.A. (1-8) have tried to predict how much oxygen a deoxygenated river will take up but with only very limited success. Although their correlations only involved a relationship between reaeration coefficient, k_2 defined as :

$$\frac{\partial C_B}{\partial t} = k_2(C_S - C_B) \qquad \ldots(1)$$

the overall average velocity of the flowing water, U_{av}, and the mean depth of the river, d; sufficient data was available from their work to be able to express all the results in a dimensionless form, as shown in figure (1). The line in figure (1) represents the empirical equation

$$\frac{k_L}{U_{av}} = \left(\frac{D_m \rho}{\mu}\right)^{0.5} \left(\frac{U_{av} d_{HD} \rho}{\mu}\right)^{-0.5} \qquad \ldots(2)$$

where k_L is the mass transfer coefficient defined as :

$$\frac{\partial C_B}{\partial t} = \frac{k_L A}{V}(C_S - C_B) \qquad \ldots(3)$$

The hydraulic depth has been used rather than mean depth, d, since this is customary in open channels and river work. The line in figure (1) is approximately the same for all width to depth ratios above 10. (All points in figure (1) have a width to depth ratio greater than 11 except one point which has a width to depth ratio of 7.35).

From figure (1) it is seen that equation (2) follows the trend of the data but with an average scatter of ± 200% about the line for both British and American rivers. The question which must now be answered is "What caused this scatter in the experimental results?" Was it possibly

(i) errors in measuring the oxygen concentration in water?

(ii) omission of some chemical variable causing alteration in the properties of the water, e.g. due to dissolved materials?

(iii) omission of some physical variable not previously considered?

The answer is omission of some physical variable since those workers who have minimised possibilities (i) and (ii), have results showing the same amount of scatter as those of other workers. Three possible physical variables were considered in the work carried out at the University of Edinburgh. These were turbulent intensity, scale of turbulence and eddy diffusivity in the vicinity of the free surface in the depth or y-direction. (x-axis is along direction of flow and z-axis is across the width of the river.) The rate of oxygen uptake of the river is assumed to be unaffected by any variation in these physical properties in the x and z directions.

Although no mention has been made of the effect of waves on mass transfer rates, these if they are present in rivers can be taken care of by the turbulent intensity, scale of turbulence or eddy diffusivity terms.

The y-direction turbulent intensity was chosen as the parameter for further study being a dimensionless quantity. Use of the other two physical variables would involve the other physical variables previously considered with the net result that the correlation between the equation connecting the new dimensionless group with the three groups in equation (2) and the experimental data might appear better than it actually was. Hence the equation which has to be established for the mass transfer coefficient will be of the form :

$$\frac{k_L}{U_{av}} = A_1 \left(\frac{D_m \rho}{\mu}\right)^{0.5} \left(\frac{U_{av} d_{HD} \rho}{\mu}\right)^{-0.5} \left(\frac{\sqrt{\overline{v}^2}}{U}\right)_{SS}^{\alpha} \quad \ldots(4)$$

Experimental procedure to determine A_1 and α

The experiments to determine A_1 and α were carried out in a 15.24 cm by 15.24 cm open channel 33.66 cm long which formed part of a closed recirculation loop. To eliminate entrance and exit effects in the channel two nozzles were designed to convert closed duct flow into open channel flow and then back to closed duct flow. The design and construction of these nozzles is given in reference (9).

Two sets of experiments were carried out. The first set was to determine the hydrodynamics of open channel flow, with and without submerged obstacles of different height, and to compare the results with similar experiments carried out in a river. The second set were mass transfer experiments involving the uptake of oxygen by deoxygenated water, the atmosphere above the water being either pure oxygen or atmospheric oxygen. The experiments were carried out at different hydraulic depth Reynolds numbers and turbulent intensities.

(a) Hydrodynamic experiments and results

There were two possible ways of measuring the local hydrodynamic properties in the bulk of a flowing liquid which seemed suitable in this case. One was to use a hot film wedge, similar to the hot wire anemometer used for gases, and the other was to observe, by means of photography, foreign matter injected or suspended in the flowing liquid.

Each method has its disadvantages. The second method was chosen since all the disadvantages in this case can be eliminated by suitable choice of foreign matter. The foreign matter used was a solution of pyrogallolic acid, sodium carbonate distilled water and methanol, darkened by exposure to air. This solution had the same density as that of the water used in the open channel but its viscosity was 4% less.

Before meaningful results could be obtained from the cine film, three corrections had to be applied to the observed image on the film. First there was a parallax correction due to three media existing between the dispersing foreign matter and the cine film, viz. water, perspex and air; secondly a velocity correction due to the variation in the overall velocity of the water with depth and lastly a correction to allow for the fact that the film emulsion could not detect and record weak concentrations of the dispersing foreign matter. These corrections are discussed in full in reference (9).

Qualitative experiments were carried out with lycopodium powder spread on the free surface of the open channel and river.

The first main result obtained from the qualitative experiments and the foreign matter dispersions in the vicinity of the free surface when no obstacles are present is shown in Figure (2). It shows the nature of the flow, contrary to what one might have expected (i.e. a set of contra-rotating eddies as postulated by Fortescue and Pearson (10)), as a set of unidirectional roll cells or eddies at the free surface. Although the main body of the fluid flow is in one direction the flow in the roll cells at the free surface is in the opposite direction. The size of these eddies is about 1/10th the channel depth.

The second result is the effect of submerged obstacles on the size and direction of these roll cells. As the height of the submerged obstacle is increased the size of the roll cell decreases until it disappears when the submerged obstacle height is 2/3rds the channel or river depth. As the submerged obstacle height is further increased the roll cells reappear but their rotational direction changes and the size of the cells grows as the obstacle height approaches that of the depth. These results are summarized in figure (3).

The effect of each obstacle on the fluid above it lasts some 8 to 11 cms downstream of the obstacle. By erecting a series of similar obstacles about 6 cms apart it is possible to promote the same hydrodynamic conditions as obtained with one obstacle over the whole length of the open channel. Thus it is possible to alter the y-direction turbulent intensity at the free surface.

(b) Mass transfer experiments and results

The reaeration experiments were carried out with distilled water. The complete removal of dissolved oxygen was achieved by adding hydrazine hydrate. The reaction which takes place is :

$$N_2H_4 \, . \, H_2O + O_2 = N_2 + 3H_2O$$

i. e. the reaction products do not contaminate the water.

Measurement of the dissolved oxygen concentration was by electrochemical means. A polythene covered silver/lead electrode cell was made small enough to leave the flow in the open channel almost unaffected. This cell was several times smaller than the commercially available cells and had a far superior response time. Furthermore it was not velocity sensitive, a property which was advantageous in this work. Two oxygen analysing cells were used in the open channel at the same time (as is shown in the theory for calculating the mass transfer coefficients below).

Consider a section of the channel as shown in figure (4). Taking a mass balance over the control volume :

$$w.\,d.\,dl.\,\frac{\partial \overline{C}_B}{\partial t} = w.\,d.\,U(yz)\left(\overline{C}_B - \frac{\partial \overline{C}_B}{\partial l} \cdot \frac{dl}{2} - \overline{C}_B - \frac{\partial \overline{C}_B}{\partial l} \cdot \frac{dl}{2} \right) + N_A \left(C_S - \overline{C}_B\right) \qquad \ldots (5)$$

where $N_A = k_L \, w \, dl$

Simplifying equation (5)

$$\frac{\partial \overline{C}_B}{\partial t} = - U(yz) \frac{\partial \overline{C}_B}{\partial l} + \frac{k_L}{d} \left(C_S - \overline{C}_B\right) \qquad \ldots (6)$$

Since two oxygen analysing cells cannot be put an infinitesimal distance dl apart, equation (6) has to be converted into a finite difference equation :

$$\frac{\Delta \overline{C}_B}{\Delta t} = - U(yz) \frac{\Delta \overline{C}_B}{\Delta l} + \frac{k_L}{d} \left(C_S - \overline{C}_{Bav}\right) \qquad \ldots (7)$$

(where $\overline{C}_{Bav}$ is $\overline{C}_B$ averaged over l (see reference (9) for the calculation $\overline{C}_{Bav}$ and C_B.)

If there is a 1st order of pseudo 1st order chemical reaction occurring equation (7) becomes

$$\frac{\Delta \overline{C}_B}{\Delta t} = - U(yz) \frac{\Delta \overline{C}_B}{\Delta l} + \frac{k_L}{d} \left(C_S - \overline{C}_{Bav}\right) - k_1 \overline{C}_{Bav} \qquad \ldots (8)$$

Alternatively if a mass balance is taken over the entire apparatus, then :

$$\frac{\partial \overline{C}_{Bav}}{\partial t} = \frac{k_L A}{V} \left(C_S - \overline{C}_{Bav}\right) \qquad \ldots (9)$$

Integration of equation (9) gives

$$\frac{\left(C_S - \overline{C}_{Bav}\right)}{C_S} = e^{\frac{-k_L At}{V}} \qquad \ldots (10)$$

From the measurements made and equations (7), (8) and (10) the following mass transfer coefficients were obtained for various flow rates and y-direction turbulent intensities (see table 1). The results are plotted in figure (5) and (6). Figure (5) is a plot of k_L/U_{av} versus Re_{HD} i.e. equation (2). This figure shows that the mass transfer coefficient for oxygen dissolving in water cannot be correlated with only the average velocity and depth of a river or channel.

Figure (6) is a plot of $\frac{k_L}{U_{av}}(Re_{HD})^{0.5}$ versus $\left(\frac{\sqrt{\bar{v}^2}}{U}\right)_{SS}$ at constant Schmidt number. The discrepancy between equation (4) and the experimental results is 10% which is within the calculated experimental error. It therefore appears likely that the omission by other workers of a physical variable, viz. the y-direction turbulent intensity, accounts for the large discrepancy in figures (1) and (5). From figure (6) A_1 is 2.65 and α is 0.5. Hence equation (4) becomes

$$\frac{k_L}{U_{av}} = 2.65\left(\frac{D_m\rho}{\mu}\right)^{0.5}\left(\frac{U_{av}d_{HD}\rho}{\mu}\right)^{-0.5}\left(\frac{\sqrt{\bar{v}^2}}{U}\right)^{0.5}_{SS} \qquad \ldots(11)$$

or

$$Sh = 2.65 \,.\, Sc^{0.5} \,.\, Re_{HD}^{-0.5} \,.\left(\frac{\sqrt{\bar{v}^2}}{U}\right)^{0.5}_{SS}$$

Discussion and application of the results to the control of river pollution

In order to use equation (11) for predicting how much atmospheric oxygen will re-enter a de-oxygenated river, the value of the y-direction turbulent intensities over the length and width of the river in question must first of all be ascertained. This is by no means an easy task.

The way in which equation (11) would be used to control river pollution is as follows :

(i) The desired dissolved oxygen concentration in the river at some fixed point downstream is set at a specific value by law and it is required that the dissolved oxygen concentration between the effluent discharge point and the fixed point should remain above a certain level.

(ii) The river reach downstream of the effluent discharge point is divided into areas such that the y-direction turbulent intensities are the same in each area.

(iii) From equation (ii) the mass transfer coefficient is calculated for the area at the point of discharge.

(iv) Using this mass transfer coefficient in equation (8) the dissolved oxygen concentration of the water flowing out of the discharge area is found.

(v) Repeat (iii) and (iv) for the next area downstream and so on till the area is reached in which (a) the chemical reaction has ceased, (in which case equation (7) is then used) or (b) the required dissolved oxygen concentration is obtained at the fixed point downstream.

(vi) If the dissolved oxygen concentration falls below the minimum value set for the stretch of river before the fixed point is reached, then the value of k_1, the reaction rate constant used in equation (8) has to be changed by altering the amount and/or quality of the effluent discharged.

(vii) The correct solution is reached when the two conditions in (i) are met.

The significance of figure (3) in the control of river pollution lies in the fact that when the river level rises or falls the height of the submerged obstacles to the river depth changes and as a consequence the mass transfer coefficients change. Hence the control of the quantity and quality of effluent being discharged into a river becomes a very complex problem but not insoluble.

This problem can be partially circumvented by artificially creating high rates of mass transfer with submerged obstacles suspended close to the free surface at a predetermined distance no matter whether the river level rose or fell. Thus it should be possible to increase the effluent load which a river can safely accommodate or at least make it meet its present effluent overload.

TABLE 1

Velocity (cm/sec U_{av})	Obstacle height in terms of depth	$\left(\frac{\sqrt{\overline{v^2}}}{U}\right)_{SS}$	Mass transfer coefficient k_L (cm/sec)	Re_{HD}
8.06	None	0.0518	0.0033 0.0032 0.0032 0.0031	4370
7.94	0.916 d	0.164	0.0062 0.0069	4300
7.45	0.666 d	0.0346	0.0028 0.0027 0.0028	4080
9.46	None	0.0918	0.0061 0.0050	5130
9.23	0.916 d	0.346	0.0089 0.0105	5000
8.74	0.666 d	0.0173	0.0023 0.0023 0.0024 0.0022 0.0029	4730
14.20	None	0.0864	0.0068 0.0075 0.0065 0.0059	7700
13.72	0.916 d	0.354	0.0111 0.0119 0.0111 0.0116	7450
13.02	0.666 d	0.0216	0.0030 0.0031 0.0031 0.0037	7060

Nomenclature

A	free surface area.
A_1	constant.
C_B	dissolved oxygen concentration in bulk of liquid.
$\overline{C}_B$	dissolved oxygen concentration in bulk of liquid averaged over d.
$\overline{C}_{Bav}$	dissolved oxygen concentration in bulk of liquid averaged over d and l.
C_S	saturation dissolved oxygen concentration in liquid.
D_m	molecular diffusivity of oxygen in water.
d	depth.

d_{HD}	hydraulic depth.
k_1	reaction rate constant.
k_2	reaeration coefficient.
k_L	mass transfer coefficient.
l	length measured in the x-direction.
N_A	mass flux of oxygen through the surface area of control volume.
t	time.
U	time-averaged velocity at free surface in x-direction.
U_{av}	average overall velocity in x-direction.
V	volume of water being aerated.
$\sqrt{\overline{v}^2}$	root mean square local velocity in y-direction.
w	channel width.
α	constant.
μ	viscosity of water.
ρ	density of water.

References

(1) Gameson, A. L. H. & Truesdale, C. A.; T. Inst. Water Eng., 13, No. 2 (1959).

(2) Gameson, A. L. H., Truesdale, C. A. & Downing, A. L.; ibid, 9, 571 (1955).

(3) Gameson, A. L. H., Truesdale, C. A. & Varley, R. A.; The Water and San. Eng., Aug. (1956).

(4) Ogden, C. G., Gibbs, J. W. & Gameson, A. L. H.; The Water and Waste Treatment Jr., Oct. (1959).

(5) Owens, M., Edwards, R. W. & Gibbs, J. W.; Int. J. of Air-Water Pollution, 8, 469 (1964).

(6) O'Connor, D. J. & Dobbins, W. E.; Proc. Amer. Soc. Civ. Eng., 82, 546 (1956).

(7) Churchill, Elmore & Buckingham; Int. J. of Air-Water Pollution, 6, 467 (1962).

(8) W. P. R. L., "Surface Aeration", Report (1965).

(9) Proctor, D., Ph. D. Thesis, University of Edinburgh (1968).

(10) Fortescue, G. E. & Pearson, J. A. R.; Chem. Eng. Sci., 22, (1967).

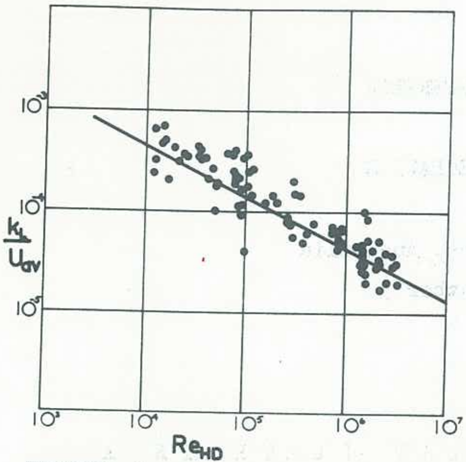

FIGURE 1
Results obtained by other workers.

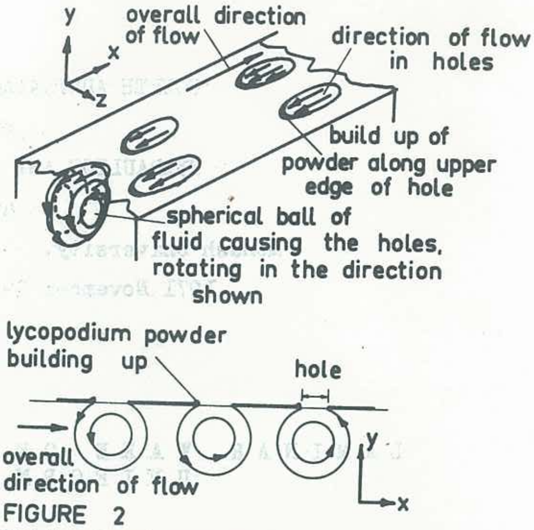

FIGURE 2
Unidirectional roll cells at the free surface.

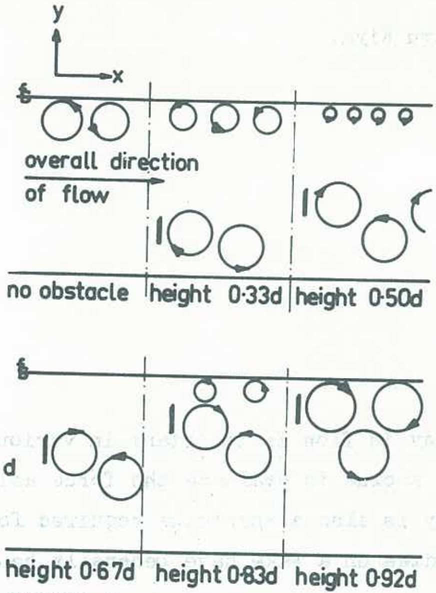

FIGURE 3
Effect of obstacles on roll cell size and direction of rotation.

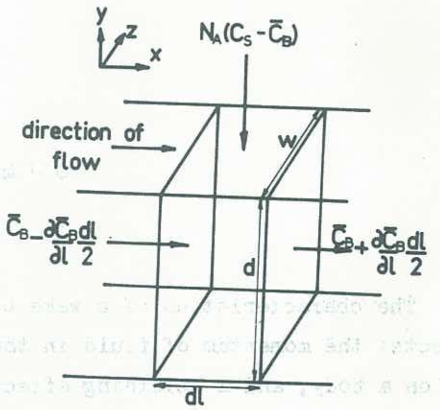

FIGURE 4
Control volume.

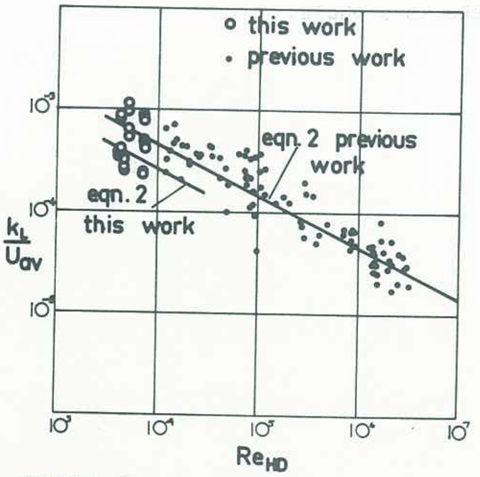

FIGURE 5
Comparison of this work with previous work.

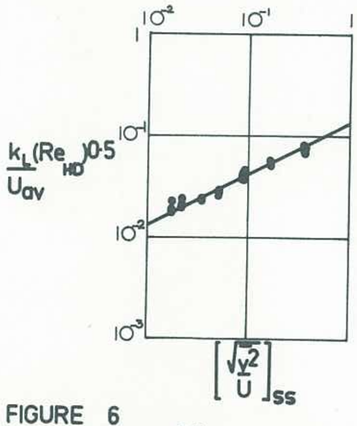

FIGURE 6
Plot of Sh. $Re_{HD}^{0.5}$ vs. turbulent intensity in the y-direction in the sub-surface layers of fluid.