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SOME MEAN FLOW PROPERTIES OF A BOUNDED TURBULENT WATER JET

by

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SUMMARY

The paper begins with a description of the methods of defining and measuring mixing, especially those developed in the chemical engineering literature. The response of the system to a tracer fed into the input stream can be interpreted on various levels, but the most meaningful interpretation is closely related to the fluid mechanics, or flow structure, of the system. Based on mixing measurements made by one of the authors using simple geometrical systems, a first approximation to the flow structure had been shown to be promising in explaining the observations. To develop a more refined model the present study of the mean velocity field was undertaken. The findings were as follows -

(a) The radial velocity profiles are similar, for all cross sections other than near the inlet, and follow the'error curve'.

(b) The axial variation of centre-line velocity departs significantly from the free-jet behaviour.

(c) The rate of spread of the jet appears to be somewhat greater than for a free jet.

(d) The entrainment characteristic of the confined jet, as manifested by the axial variation of mass flow rate, shows little difference from the free-jet, up to about 20 jet diameters from the inlet. Thereafter the rate of increase of mass-rate decreases.

(e) The extent of the end-zone of recirculation is relatively small.

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1. Introduction

This work is a small part of a larger project devoted to a study of the influence of mixing in reacting flow systems. (The term 'mixing' is not intended to include systems involving mechanical stirring devices). Situations which reduce, in essence, to the mixing of two or more fluid streams are many and varied; they range from meterological and oceanographical phenomena down, in scale, to industrial and laboratory phenomena. Of particular current interest are the problems of dispersing noxions effluents into the environment, while problems of industrial interest include the design of jet mixers, combustion chambers, and chemical reactors.

A large body of information exists in this broad area, and a good degree of success has been achieved in describing the phenomena in terms of basic transport equations. Further progress largely depends on a better understanding of turbulence; until then, the less sophisticated models based on eddy transport parameters must be used. It is interesting to speculate why in chemical engineering, in particular, the approach to measuring and defining mixing in flow systems has proceeded along rather unique lines. Concepts involving the various 'age' distributions, segregation, and micro mixing, were pioneered by Danckwerts (1) and have since been extended by many others (2) (3). One of the reasons, no doubt, is the complex nature of a reacting system which, allied to restrictive boundary conditions, is not easily described by transport equations.

The measurement of mixing is made in terms of the time-response of a flow system to a tracer introduced into the inlet stream or streams (3). The least-sophisticated method of interpreting the response involves modelling the flow system as a network of interconnected regions, each region being idealized to be either plug-flow, perfectly stirred, or dead-space. The total flow is split into a stream which may short-circuit the network and a stream which flows through the network. This technique, of course, allows any response to be accurately modelled, since there are a large number of degrees of freedom in devising the network and the flow distribution. At best, if the model bears some resemblance to the observed flow structure, the method can be quite useful; at worst, it can be nothing more than expedient curve fitting.

2. The Present Work

In a previous study (4), the author has used the tracer response method to measure the mixing in several geometrically simple flow systems. A typical flow system is shown in Figure 1 : not only is the flow axisymmetric, but its structure, as revealed by visualisation studies, is very simple. In essence there are two distinct regimes : a turbulent jet-region and a reverse-flow, seemingly laminar, region, with exchange between them due to entrainment from the surroundings into the jet. With a considerable body of information available on free and confined jets there seemed to be some hope of describing the mixing in terms of transport equations.

The mixing was measured by filling a vessel with a dilute NaCl solution and measuring the NaCl displaced from the vessel by an inlet stream of pure water. The fraction of NaCl displaced, as a function of time, was found to be given by -

$$\ln(1-\psi) = K.\,t/\tau \qquad \text{..........(1)}$$

where ψ is fraction tracer displaced
K is mixing parameter
t is time
τ is ratio vessel volume/feed rate

The parameter K is related to the geometry of the vessel and the inlet diameter. A simple analysis of the problem was attempted by using the mass transport equations for the tracer -

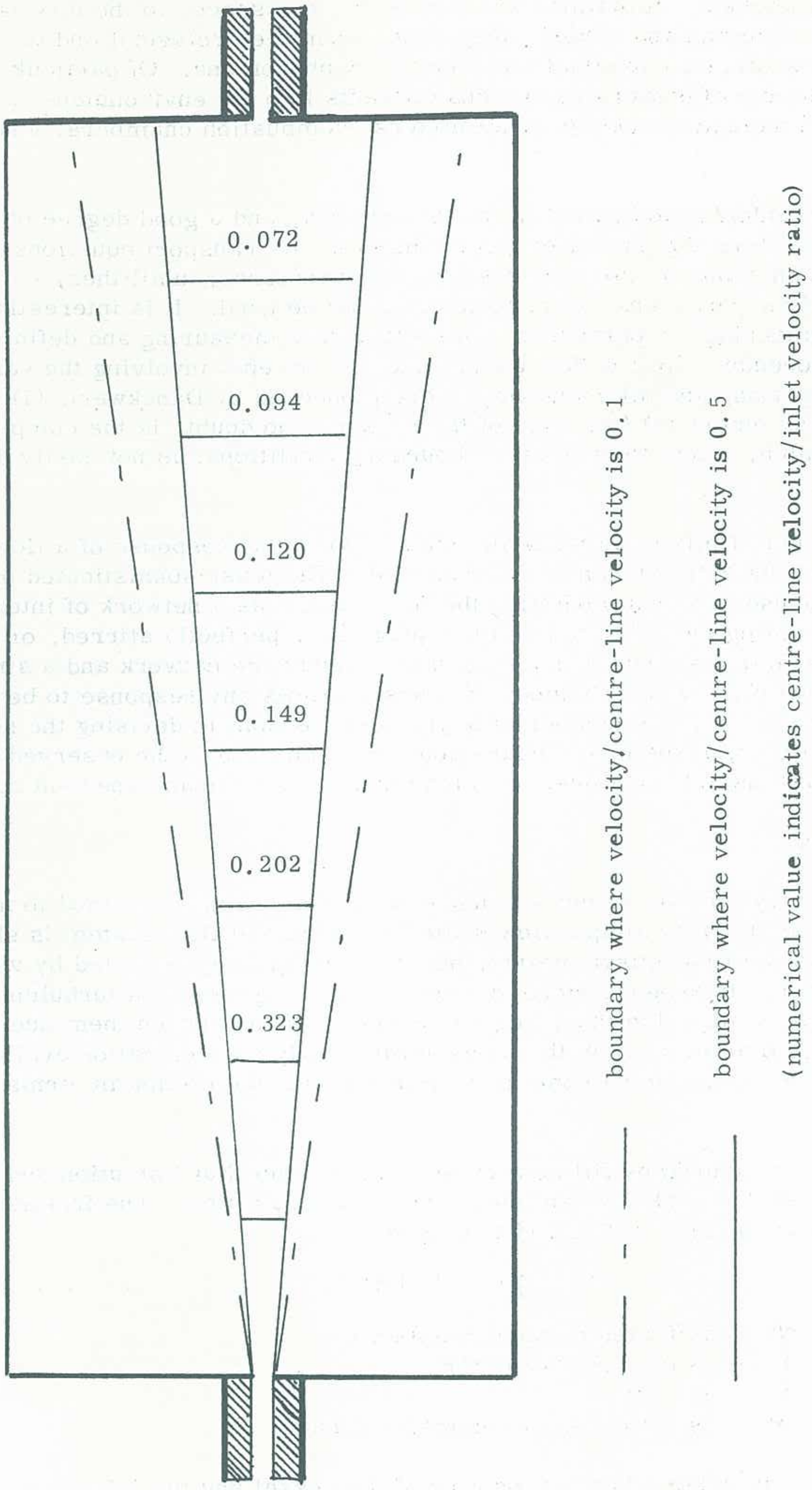


FIGURE 1 8" vessel with 1/8" nozzle, showing main features of flow field

$$\frac{\partial c}{\partial t} + \frac{u.\partial c}{\partial x} + \frac{E}{a}.c = \frac{E}{a}.c'$$

$$\frac{\partial c'}{\partial x} + \frac{u'.\partial c'}{\partial x} = 0$$

where

c	is tracer concentration in jet
c'	is tracer concentration in surrounds
a	is cross sectional area of jet
E	is entrainment rate per unit length from surrounds into jet
u	is velocity in jet region
u'	is velocity in surroundings
x	is axial distance from inlet
t	is time

In solving the equations, the free-jet relationships for u, a, and E as functions of x were assumed. A further simplification was to assume 'top-hat' velocity and concentration profiles in both the jet region and the surrounds. From the solution for exit NaCl concentration as a function of time, the experimental relationship shown in equation (1) was deduced. The parameter K was shown to relate to the vessel geometry as follows -

$$K = (1 + r/L.\cot\theta)^{-1} \qquad \ldots\ldots\ldots\ldots(2)$$

where

K	is mixing parameter
r_o	is inlet jet radius
L	is vessel length
θ	is free-jet half-angle

The jet half-angle was deduced from the experimental measurements of K and shown to average 6.3°, in fair agreement with reported values. Vessels with length/diameter ratios of up to 2 were studied and, noticeably, the larger the ratio, the less consistent was the value of . The explanation is clearly related to the extent to which the jet boundary approaches the wall of the vessel : the closer the approach, the greater the velocity of reverse flow in the surroundings, and the less valid is the free-jet assumption.

To refine the existing model, the answers to the following questions were then sought in the present work :

- what is the mean-velocity structure of the flow?
- are the axial and radial profiles similar?
- what is the extent of the exit region, where the entrained fluid is recycled?

3. Experimental

The two vessels studied were perspex cylinders, $3\frac{1}{2}$ in (82.6 mm) internal diameter, with lengths of 4 in (102 mm) and 8 in (203 mm). Each vessel could be fitted with either of two screw-in nozzles, in (3.18 mm) and in (4.76 mm) diameter, respectively. The exit nozzles were of the same diameter as the inlet in all experiments, although previous experiments had shown that the exit diameter had little influence on the mixing. Velocity measurements were taken using an impact tube in conjunction with an accurate two-fluid manometer, using water-benzyl chloride for the lower velocity range and water-carbon tetrachloride for the high velocity range. On the basis of turbulent intensity results obtained by Rosler and Bankoff (5) for a water jet, the question of correcting impact tube measurements was considered to be not worth while at this stage of the project - although, at a later time, greater accuracy may be required. Further experimental details can be found in the thesis by Phillip (6).

4. Results

A series of radial traverses were taken at $\frac{1}{2}$ in intervals along each vessel, but the sensitivity of the manometer system limited velocity measurements to 100 mm/sec and above. Each traverse was found to follow the 'error - curve' universal profile, with the exception of the region adjacent to the inlet -

$$u/u_a = \exp -0.693\,(r/r^*)^2 \qquad \ldots\ldots\ldots\ldots(3)$$

where u is velocity at radius r
u_a is centre-line velocity
$r*$ is radius where velocity is $0.5\ u_a$

A graph showing this relationship is shown in Figure 2 for typical profiles taken from each vessel.

The axial decay of centre-line velocity was found to be universal when plotted on a non-dimensional basis, as shown in Figure 3. Also shown is the relationship obtained by Rosler and Bankoff (5) for a free turbulent water jet -

$$u_o/u_a = A.\frac{(x + B)}{d_o} \qquad \ldots\ldots\ldots\ldots(4)$$

where u_a is centre-line velocity
u_o is inlet centre-line velocity
x is axial distance from inlet
d_o is inlet jet diameter
A, B are constants

The centre-line velocity at the inlet nozzle is, itself, the maximum velocity at that plane : the precise profile at the inlet could not be measured, of course, but a value of $n = 4.5$ seemed to fit the observed ratio of mean/maximum velocity, assuming a profile of the form -

$$u / u_o = (1 - r/r_o)^n$$

Using the 'half-radius' as a characteristic measure of the spread of the jet, a further linear and universal relationship for this variable was found to be -

$$r^*/r_o = D.\frac{(x + B)}{d_o} \qquad \ldots\ldots\ldots\ldots(5)$$

where B, D are constants

Figure 4 shows the 'half-radius' relationship, together with Rosler and Bankoff's results.

5. Discussion

The universal 'error-curve' relationship, found by many workers to be valid for free jets, appears to be also valid for the present work, over the range of velocities covered. As mentioned previously, velocities less than about 100 mm/sec could not be accurately measured with the manometer system used. Attempts to measure reverse-flow velocity traverses in the surroundings were completely unsuccessful, so that the best that can be deduced about this region is that velocities are very small indeed. Previous studies involving liquid jets are very few, and are confined to the free-jet case (5) (7) (8). Two of the studies confirm the universal nature of the 'error-curve' velocity profile given in equation (3), although, at larger values of r from the axis the velocities are smaller and subject to considerable error, so that each study expresses doubt about the profiles for $r^* > 2$. Confined jet studies for air, but not water, have been made and again confirm the universal nature of the velocity profiles - at least, beyond a certain small initial region. Becker, Hottel, and Williams (9), following on the

FIGURE 2

Radial Velocity Profile

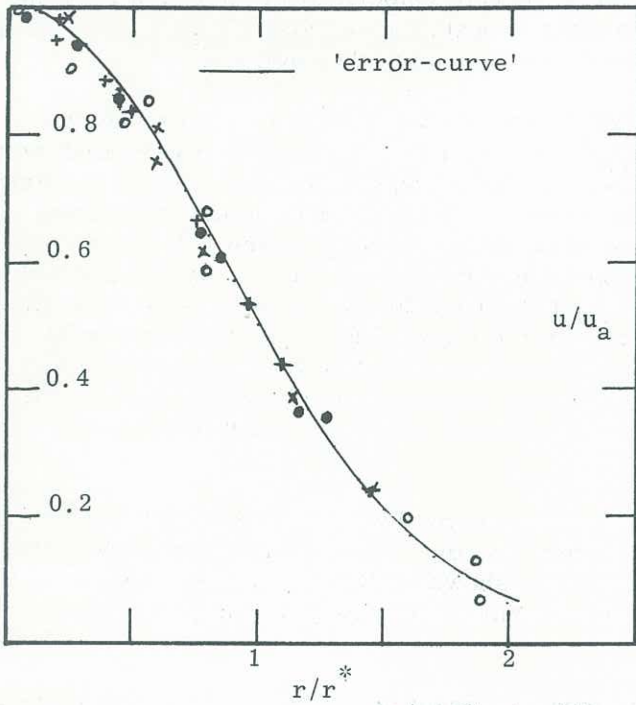
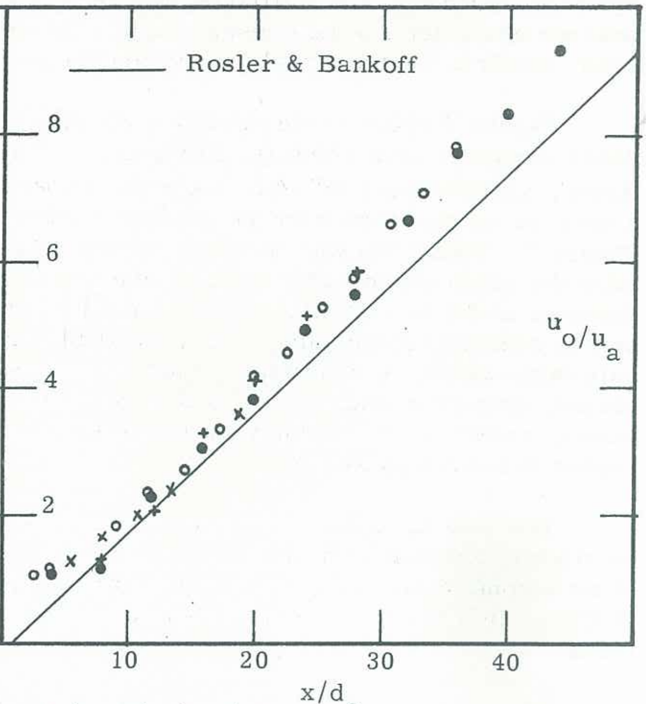


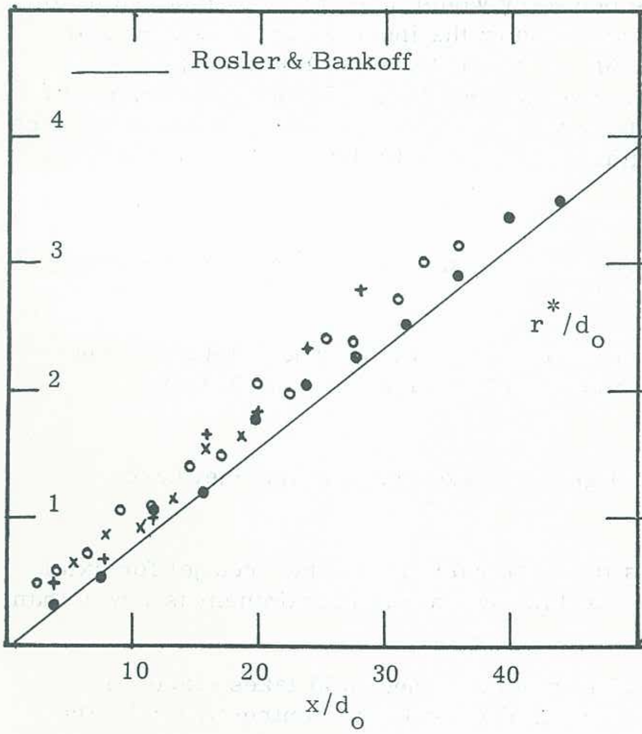
FIGURE 3

Centre-Line Velocity Relationship



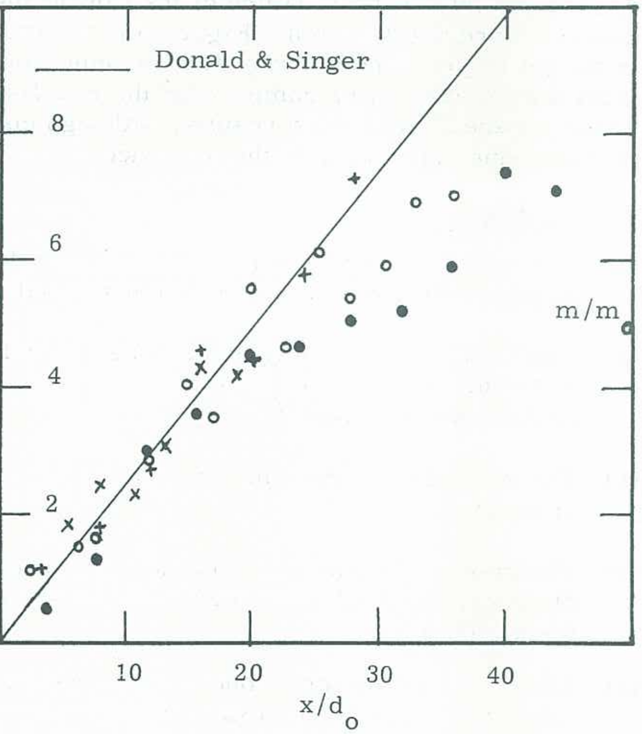
● L: 8" d_o: 1/8" × L: 4" d_o: 1/4"

○ L: 8" d_o: 3/16" + L: 4" d_o: 3/16"



Jet Spread Relationship

FIGURE 4



Entrainment Relationship

FIGURE 5

earlier studies of Craya and Curtet (10) (11), and Thring and Newby (12), give an extensive analysis of the turbulent jet issuing into a confined cocurrent stream. The system shown in Figure 1 is, of course, rather different; the lateral boundaries at the inlet and exit planes, particularly the latter, are likely to affect significantly the flow structure. This possible effect was really the main reason why the study reported here was undertaken. In the study by Becker, Hottel, and Williams, the case of zero cocurrent velocity at the inlet plane is the nearest equivalent to the present study : interestingly enough, the authors, for this particular case, confirm the universal profile for the entire cross section of the duct.

Figure 3 shows quite clearly a significant departure from the free-jet results for the decay of centre-line velocity, particularly at larger values of x. The results given in Becker, Hottel, and Williams (9) show a similar departure, but their graphs are not sufficient to allow a detailed comparison with the present work to be made. Two significant points arise from Figure 2 : first, the way in which results from each of the four flow systems fall on essentially the same curve, and, second, the way in which the velocity decreases to within a small distance of the exit nozzle. In the 8 in (203 mm) vessel, the velocity 1 in (24.5 mm) from the exit is still decreasing and is only about 10 per cent of the inlet value. The recovery to the exit value which, by continuity, must be almost equal to the inlet value (depending on profile shape), therefore takes place over a very short distance indeed. This observation, to some extent, confirms the assumption made by Wood (4) that the extent of the recirculation end region is relatively small.

The rate of spread of the jet region, as characterized by the 'half-radius' r^*, seems to be slightly greater than that of a free water jet (Figure 4); the scatter of the results does not allow a confidential judgement to be made. The results shown by Becker, Hottel, and Williams indicate a more rapid rate of spread for a confined jet, with a distinct non-linear trend.

The entrainment characteristics of the confined jet cannot be completely determined from the experimental measurements. The boundary of the jet region is the radius at which the velocity becomes zero; because of the limited accuracy of the impact tube and manometer system the precise boundary cannot be located. In any case, even with a much more sensitive system the intermittent nature of the flow at the boundary would have made it impossible to detect its precise location. Figure 5 is an attempt to show the increase in mass flow rate in the jet region: the velocity profile, equation (3), is integrated to a distance equal to $2.5r^*$ from the jet axis. For comparison the results of Donald and Singer (7) for a free turbulent water jet are shown. The results, although considerably scattered, seem to confirm the free jet value due, no doubt, to the presence of a significant velocity in the reverse flow region.

6. Conclusions

(a) The radial velocity profiles are similar when plotted on the basis of the 'error-curve', equation (3), up to a radial distance equal to $2r^*$.

(b) The centre-line velocity decay is similar for all flow geometries when plotted on the non-dimensional basis shown in Figure 2. Significant departures from free-jet behaviour are apparent.

(c) The rate of spread of the jet appears to be slightly greater than the free-jet case (Figure 4).

(d) The rate of entrainment into the jet appears to be the same as for the free jet for axial distances from the exit of 20 diameters; above this the rate of entrainment is lower than for a free-jet.

(e) The extent of the 'end-zone', where recirculation of entrained fluid takes place, is relatively small. For example, in the 8 in (203 mm) vessel, the centre-line velocity is still decreasing 1 in (25.4 mm) from the exit.

References

1. Danckwerts, P.V., Chem. Engng. Sci. (1953), 2, 1

2. Zwietering, Th. N., Chem. Engng. Sci. (1959), 11, 1

3. Levenspiel, O., 'Chemical Reaction Engineering'. Wiley 1962

4. Wood, T., Chem. Engng. Sci., (1968), 23, 783

5. Rosler, R. S. and Bankoff, S. G., A.I.Ch.E. Journal, (1963), 9, 672

6. Phillip, R. G., M. Eng. Sc. Thesis. University of Sydney, 1966

7. Donald, M. B., and Singer, H., Trans. Instn. Chem. Engrs., (1959), 37, 255

8. Forstall, W., and Gaylord, E. W., J. Appl. Mech., (1962), 12, 241

9. Becker, H. A., Hottel, H. C., and Williams, G. C., Ninth Symposium (International) on Combustion, 1962.

10. Craya, A., and Curtet, R., Compt. rend. (1955), 241, 261

11. Curtet, R., Combustion and Flame, (1958), 2, 383

12. Thring, M. W., and Newby, M. P., Fourth Symposium (International) on Combustion, p. 783, 1953.