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DETERMINATION OF THE PERIOD OF A FLUIDIC MULTIVIBRATOR

by
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SUMMARY

A fluidic multivibrator consists of a fluidic device with appropriate connections from the outputs to the control nozzles. These feedback connections may be symmetrical or not and lead to symmetrical or unsymmetrical output waveforms as desired. Only the symmetrical case is considered here; the algebra is simpler and the modifications needed to treat the unsymmetrical case are obvious.

The half period of oscillation of a symmetrical multivibrator may be divided into the mutually exclusive intervals:

(i) the attachment time, during which the power stream flows 'along' one wall;

(ii) the switching time, during which the power stream transfers from one wall to the other.

The period is computed from a mathematical model and agrees well with the period measured on a pneumatic multivibrator.

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LIST OF PRINCIPAL SYMBOLS

a	Area of cross-section
c	Local velocity of sound
C	Fluid capacitance {defined as $\frac{V}{c^2\rho}$}
$C' = \frac{C\rho c}{a_m t_1}$	Non-dimensional fluid capacitance
L	Length
L_w	Separation bubble wall length (maximum)
p	Pressure
Q	Separation bubble, leakage volume flow rate
R	Flow resistance per unit of cross-sectional area

t	Time
t_1	Acoustic delay
t_2	Attachment time
t_3	Half period
$t' = \frac{t}{t_1}$	Non-dimensional time
u	Flow velocity
$u' = \frac{u}{c}$	Non-dimensional velocity
V	Volume
x	Attachment line at attachment and separation
ρ	Fluid density

Subscripts

a	Region on free side of power stream
b	Separation bubble region
c	Capacitor
f	Control line

m	Power stream
n	Control nozzle
o	Active output channel

Other symbols are defined at appropriate places in the text.

1. INTRODUCTION

The object of this investigation was to derive a means for predicting the period of oscillation of a wall attachment fluidic multivibrator in terms of the properties of the power stream, separation bubble and control nozzle of the fluidic device, and of a delay line, fluid resistor and fluid capacitor included in the feedback paths. In this paper the influence of a fluid capacitor in the feedback path is investigated for a range of values of capacitance and of subsonic power stream velocity.

2. THE FLUIDIC MULTIVIBRATOR

2.1 The Wall Attachment Bistable Element

A bistable element is shown schematically in figure 2.1. A power source (P) and nozzle (N) produce a turbulent, submerged power stream that can be made to attach to one of the two walls (1) and (2). Pressure recovery in the active output channel may be used as an output signal. Switching of the power stream from one wall to the other may be effected by a control flow issuing from nozzle (N1) or (N2).

2.2 The Multivibrator

Because of its bistable behaviour, the wall attachment element is suitable for use as a multivibrator. This is achieved by using a control line and capacitor as a positive feedback path from each output channel to the control nozzle on the same side of the element, as shown schematically in figure 2.2. Thus, after a time lapse, the pressure recovered in the output channel produces a control flow causing switching of the power stream. This change-over is then repetitive, giving the device an oscillatory output in both channels (1) and (2). The approximately rectangular output pressure waveforms for a symmetrical device are shown in figure 2.3.

The half period of oscillation may be divided into two parts (a) and (b):

(a) the attachment time during which the power stream flows along one wall; and

(b) the switching time during which the power stream transfers from one wall to the other.

2.3 Attachment

The mechanism of attachment of a fluid stream to a wall is not well understood in detail, but in general terms a stable flow condition occurs in which both momentum and continuity conditions are satisfied. The tendency for a turbulent submerged stream to deflect towards and become attached to a nearby wall is called the Coanda effect [1]. After leaving the nozzle the power stream becomes turbulent, entraining the fluid surrounding the stream. If the stream is sufficiently close to a wall on one side so that entrainment from that side is restricted, the pressure (p_b) on that side becomes lower than the pressure (p_a) on the unrestricted or free side (A), and also lower than the pressure (p_c) in the capacitor. The pressure differential (p_a-p_b) curves the stream further towards the wall until eventually the stream attaches to the wall, thereby enclosing a volume of fluid (the separation bubble) between the stream and the wall. The pressure differential (p_c-p_b) produces a flow through the control nozzle into the separation bubble region. A line across the wall is established (the attachment line) at which the power stream flow divides, most proceeding downstream to the output channel, but some returning back along the wall into the separation bubble. The difference between entrained bubble flow and returned flow may be regarded as a leakage from the bubble.

During the transient conditions within the separation bubble immediately following attachment the leakage flow (Q) exceeds the inflow from the control nozzle, such that the resulting pressure within the bubble increases the pressure differential across the power stream, causing the attachment line to move upstream. Steady state bubble conditions occur if equilibrium is reached between control and leakage flows.

In this device attachment commences when the power stream has approached a wall sufficiently to form an attachment line at the end (x) of the wall.

2.4 Separation

When control flow into the separation bubble exceeds the leakage flow, the resulting pressure within the bubble decreases the pressure differential across the power stream, causing the attachment line to move downstream. In this device separation from the wall occurs when the attachment line reaches the end of the wall.

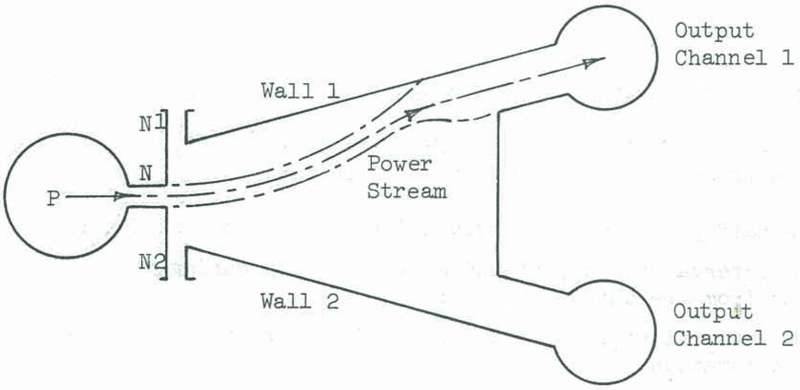


Figure 2.1

THE WALL ATTACHMENT BISTABLE ELEMENT

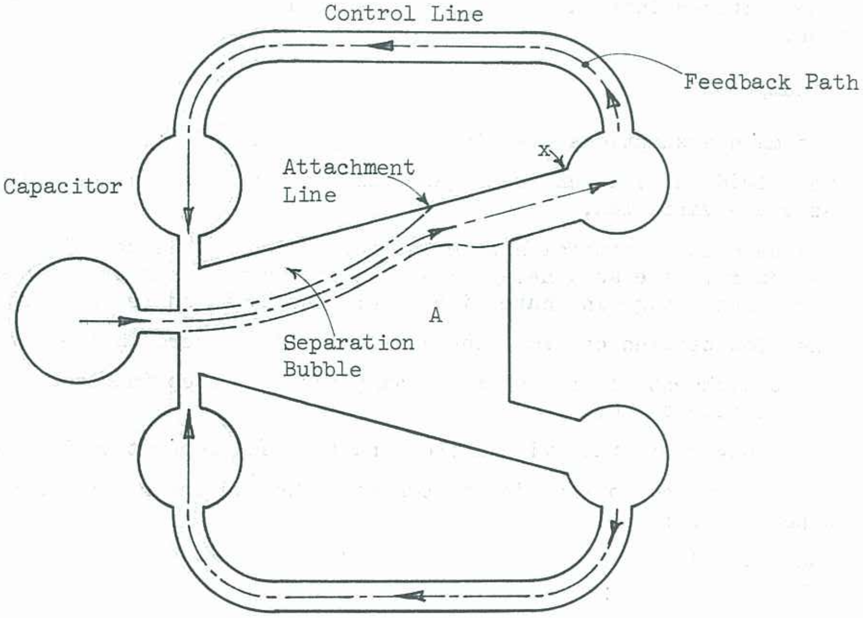


Figure 2.2

THE MULTIVIBRATOR

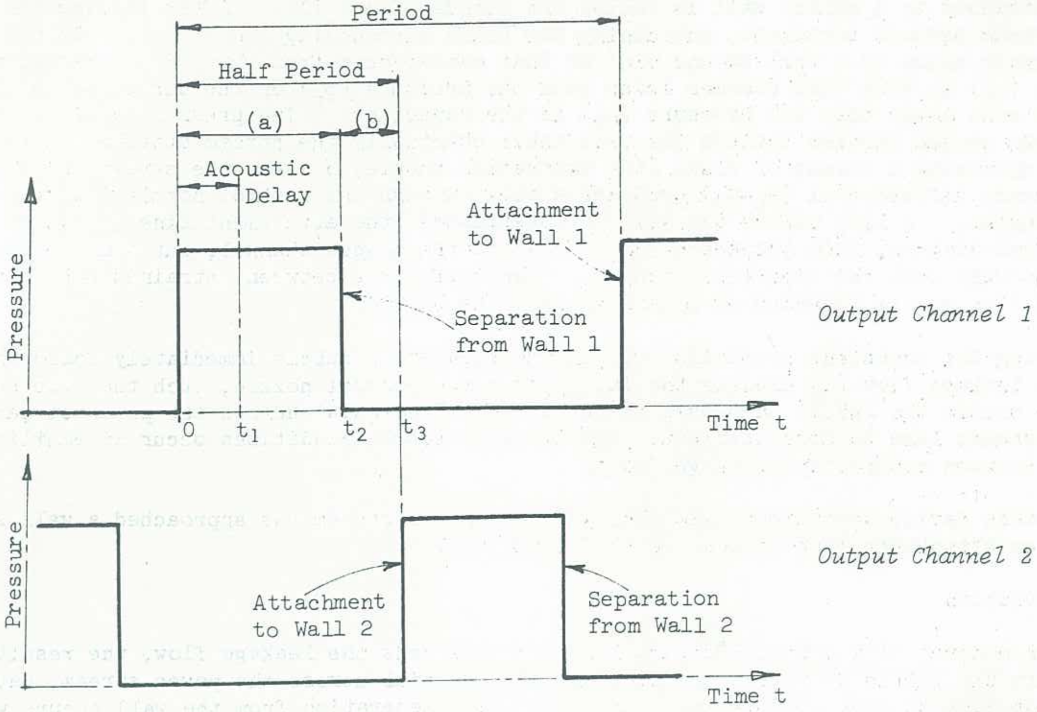


Figure 2.3 OUTPUT PRESSURE WAVEFORMS

3. THE MATHEMATICAL MODEL AND COMPUTATION OF PERIOD

Since the multivibrator is symmetrical it is only necessary to describe its behaviour during the half period $(0, t_3)$ of its oscillation.

The half period may be divided into three disjoint intervals:

(1) The interval $(0, t_1)$ of the acoustic delay during which no disturbance has yet reached the capacitor from the output channel;

(2) The interval (t_1, t_2) from the time of arrival of this disturbance at the capacitor to the time of separation;

(3) The interval (t_2, t_3) during which the power stream transfers from one wall to the other.

The combined interval $(0, t_2)$ is the attachment time; the interval (t_2, t_3) is the switching time.

3.1 Assumptions

The main assumptions used in the analysis are:

(1) The fluid (air) is an ideal gas, and the mean temperature and pressure of the system may be used as state variables;

(2) The capacitor, control nozzle and separation bubble may be considered as lumped elements, and the control line as a delay line that attenuates pressure disturbances with distance along it but without change in shape with time; the delay is referred to as the acoustic delay;

(3) The flow between control line and capacitor is zero in the interval of the acoustic delay;

(4) The attachment of the power stream produces a step increase in the associated output pressure at time $t = 0$;

(5) The changes of state within the capacitor and separation bubble occur isentropically;

(6) The resistance to flow in the control line and the control nozzle may be described by lumped parameters;

(7) The switching time is the time for a particle in the power stream to travel from the power nozzle to the end of the wall.

3.2 The Mathematical Model in the Interval $(0, t_2)$

The model is constructed using:

(1) The continuity equation with respect to the capacitor and separation bubble;

(2) The momentum equation with respect to the control line, the control nozzle and the power stream;

(3) The measured relationships (determined from steady flow tests on the bistable element and control lines) between:

 (i) Output pressure (p_o) and power stream velocity (u_m),
 (ii) Separation bubble flow rate (Q) and pressure (p_b),
 (iii) Control line resistance (R_f) and velocity (u_f),
 (iv) Control nozzle resistance (R_n) and velocity (u_n);

(4) A relationship between separation bubble volume (V_b) and pressure (p_b) computed using a geometric description of the bubble similar to that used by Olsen and Stoeffler [2].

For convenience in digital computation the relationships (3)(ii), (3)(iii), (3)(iv) and (4) were expressed in analytical form by curve fitting. All other quantities are either physical constants or constants of the multivibrator.

The following equations were obtained:

For the control line

$$t_1 = \frac{L_f}{c} \tag{3.1}$$

$$u_f = 0 \; ; \quad 0 \leqslant t \leqslant t_1 \tag{3.2}$$

$$\frac{du_f}{dt} = \frac{p_o - p_c - R_f}{\rho L_f} \; ; \quad t_1 \leqslant t \leqslant t_2 \tag{3.3}$$

$$R_f = F_f(u_f) \tag{3.4}$$

For the capacitor

$$\frac{dp_c}{dt} = \frac{- a_n u_n}{C_c} \; ; \quad 0 \leqslant t \leqslant t_1 \tag{3.5}$$

$$\frac{dp_c}{dt} = \frac{a_f u_f - a_n u_n}{C_c} \; ; \quad t_1 \leqslant t \leqslant t_2 \tag{3.6}$$

For the control nozzle $0 \leqslant t \leqslant t_2$

$$\frac{du_n}{dt} = \frac{p_c - p_b - R_n}{\rho L_n} \tag{3.7}$$

$$R_n = F_n(u_n) \tag{3.8}$$

For the separation bubble $0 \leqslant t \leqslant t_2$

$$\frac{dp_b}{dt} = \frac{a_n u_n - Q}{C_b} \tag{3.9}$$

$$Q = \bar{F}_b(p_b, u_m) \tag{3.10}$$

$$V_b = F_b(p_b, u_m) \tag{3.11}$$

For the output channel $0 \leqslant t \leqslant t_2$

$$p_o = F_o(u_m) = \text{Constant.}^{*} \tag{3.12}$$

* For a fixed value of u_m belonging to a selected set of u_m.

3.3 The Mathematical Model in the Interval (t_2, t_3).

From assumption (7), $(t_3 - t_2) = \dfrac{L_w}{u_m}$ (3.13)

3.4 Input at $t = 0$

From assumption (4), attachment produces a step change in output pressure. The separation bubble pressure changes from its value prior to attachment to a critical value obtained from the analysis of the separation bubble.

3.5 Boundary Conditions

(a) Interval (1):

(i) $t = 0$.

The initial values of u_n, p_c, and p_b, i.e., their values prior to attachment, were determined from steady flow tests.

From assumption (3), $u_f = 0$.

(ii) $t = t_1$.

Interval (1) terminates at $t = t_1$, the end of the acoustic delay given by equation (3.1).

(b) Interval (2):

(i) $t = t_1$.

The initial values of u_n, p_c and p_b for this stage are the terminal values for Interval (1).

From assumption (3), $u_f = 0$.

(ii) $t = t_2$.

Attachment terminates with separation of the power stream at $t = t_2$ when p_b reaches its critical value. If this critical value is never reached then $t_2 \to \infty$.

(c) Interval (3):

(i) $t = t_2$.

Switching commences with separation of the power stream from one wall at $t = t_2$.

(ii) Switching terminates with attachment of the power stream to the opposite wall at $t = t_3$.

If $t_2 \to \infty$, switching does not occur.

3.6 Substitution and Simplification

Substitution of the relationships (3.4), (3.8), (3.10), (3.11), (3.12) and all constants enables equations (3.2), (3.3), (3.5), (3.6), (3.7), (3.9), (3.13) to be written:

Interval (1):

$$\begin{aligned} u_f &= 0 \\ \dot{u}_n &= f_1(u_n, p_c, p_b) \; ; \; u_n(0) \text{ from tests,} \\ \dot{p}_c &= f_2(u_n, C_c) \; ; \; p_c(0) \text{ from tests,} \\ \dot{p}_b &= f_3(p_o, u_n, p_b) \; ; \; p_b(0) \text{ from tests,} \\ p_o(t) &= \text{Step.} \end{aligned}$$

Interval (2):

$$\begin{aligned} \dot{u}_f &= f_4(p_o, p_c, u_f) \; ; \; u_f(t_1) = 0, \\ \dot{u}_n &= f_5(u_n, p_c, p_b) \; ; \; u_n(t_1) \text{ from Interval (1),} \\ \dot{p}_c &= f_6(u_n, u_f, C_c) \; ; \; p_c(t_1) \text{ from Interval (1),} \\ \dot{p}_b &= f_7(p_o, u_n, p_b) \; ; \; p_b(t_1) \text{ from Interval (1),} \\ p_o(t) &= \text{Step.} \end{aligned}$$

Interval (3):

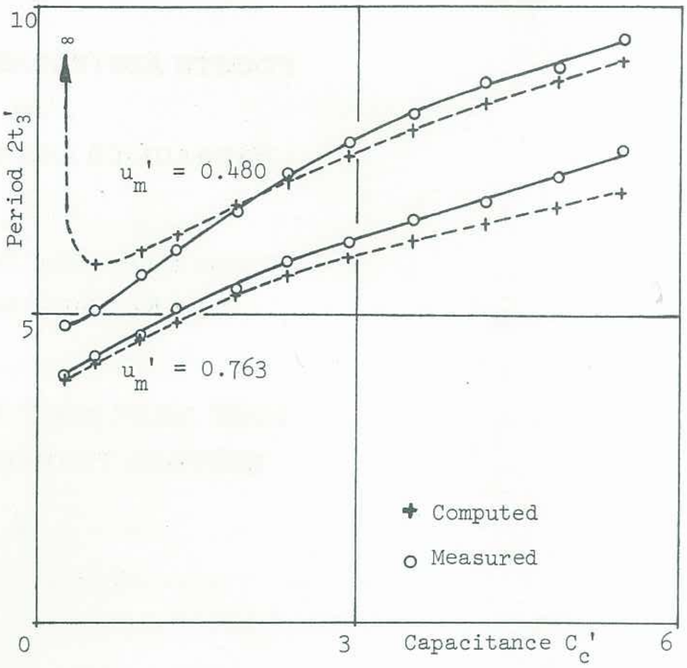
$$t = f_8(u_m) \; ; \; t_2 \text{ from Interval (2).}$$

3.7 Solutions

Solutions were obtained, using digital computation, for a range of C_c values for each p_o, (i.e., for each u_m belonging to the selected set of u_m). Computed period - capacitance relationships are shown graphically in non-dimensional form in figure 3.1.

Figure 3.1

RELATION BETWEEN PERIOD AND CAPACITANCE



4. PERIOD MEASUREMENT

The relation between period and capacitance was measured over the same set of u_m used in the computations. Periods were recorded on a digital counter triggered by signals from a pressure transducer installed in an output channel of the multivibrator. Measured period-capacitance relationships are compared with computed relationships in figure 3.1.

5. DISCUSSION OF RESULTS

For both values (high and low) of the power stream velocity, the computed and measured periods shown in figure 3.1 are in good agreement over most of the range of capacitance (C_c) investigated.

In the low velocity case the computed attachment time (and hence period) became infinite at the lowest C_c investigated; separation did not occur because the bubble pressure remained less than the critical value. The tendency is for infinite periods below certain combinations of u_m (or p_o) and C_c, thus suggesting a lower limit of operation as a multivibrator.

The deviation of computed from measured periods is due partly to errors in experimental data, but mainly to inaccuracies in mathematical modelling.

6. CONCLUSIONS

The effect of fluid capacitance on the period of oscillation of a pneumatic multivibrator has been determined for a range of power stream velocity.

The agreement between computed and measured relationships demonstrates:

(i) That the period may be predicted to acceptable accuracy, and the procedure is therefore applicable to the design of fluidic multivibrators;

(ii) That capacitance is an effective means for the regulation of the period.

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2. Olsen, R. E., and Stoeffler, R. C. "A Study of Factors Affecting the Time Response of Bistable Fluid Amplifiers." A.S.M.E. Symposium on Fully Separated Flows, May 1964, page 73.