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SEPARATION OF SOLID PARTICLES IN A SINK-VORTEX FLOW

by
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SUMMARY

An analytical solution to the equation of particle motion in a sink-vortex flow is obtained for very small particles. The solution is used to show that the concentration of pulverised brown coal particles suspended in a gas stream can be reduced in one part of the stream to the point where that part can be economically discarded. The means by which it is proposed to do this in practice are briefly indicated.

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GLOSSARY OF TERMS

SI units are used throughout

A bar over a symbol denotes a vector quantity

c_i	(i = 1,2,3) numerical constant
C	$\rho_F d^2/18\mu$ (units of time)
d	particle diameter (diameter of sphere of equal volume)
g	gravitational acceleration
K	vortex strength (circulation/2π)
M	sink strength (volume flow per unit time/4π)
p	pressure
q	speed
Q	rate of flow of fluid volume through separator
S	separation, fraction of volume flow clear of particles
t	time
U	uniform axial fluid velocity in cylindrical separator
v	swirl component of fluid velocity in separator exit plane
z,r,θ	axes of cylindrical polar co-ordinates (see Figure 1)
R,ϕ,θ	axes of spherical polar co-ordinates (see Figure 1)
ρ	density
μ	viscosity
∇	vector differential operator $(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z})$

Subscripts

o,1	inner and outer surfaces of separator
i,e	entry and exit planes of separator
s	outer boundary of cleared stream in exit plane of separator
F,P	fluid and particle
z,r,θ,R,ϕ	component in that direction

INTRODUCTION

Raw brown coal contains up to 70% moisture which is partially removed prior to combustion in steam-raising boilers by milling in the presence of hot, recycled combustion gases. Considerable economies are possible if all the pulverised fuel can be concentrated in part of the gas, as a fuel stream for transport to the furnace, and the balance of the inert, vapour-laden drying gas discarded to waste. However, a separating efficiency greater than that attainable in cyclone-type devices is needed to clean the vapour stream of all but the finest coal particles. The analysis of particle motion in a sink-vortex flow reported in this paper was carried out to determine whether the required separation is theoretically possible with practical swirl velocities.

The sink-vortex flow was chosen not merely for its mathematical simplicity, but because there is good reason to believe that it can be approximated in practice. Furthermore, whereas the diffusive effects of turbulence, which will oppose the establishment of particle concentration gradients, are the *sine qua non* of turbulent shear flow, they are incidental to potential flow, encouraging the expectation that theoretical predictions will be realistic.

The parameter characterising the performance of the proposed sink-vortex separator is the separation with respect to a certain particle size, the fraction of flow completely cleared of all particles of that size. Since all larger particles will have experienced a greater lateral movement, it is only necessary to compute the separation with respect to a critical size defined such that the cleared stream contains no more than a specified small fraction of the solid burden. The separation desired between mill gas and pulverised fuel, say 1% fuel in the vapour stream, implies a critical particle size of about 10 μm. For such fine particles the equation of motion can be extensively simplified and, because they will diverge relatively slowly from the fluid streamlines, a first order approximation to the velocity of the particle relative to the fluid is adequate, enabling an analytical solution to be obtained.

EQUATIONS OF MOTION

The co-ordinate systems used are shown in Figure 1.

With a point sink located at the origin, and the axis of a rectilinear vortex coincident with the z-axis, the fluid velocity is

$$\overline{q_F} = \frac{a\,M}{R^2} + \frac{b\,K}{r}$$

where a and b are unit vectors in the R and θ directions.

It is assumed, but justified numerically *a posteriori*, that the flow of fluid relative to the particle is Stokesian. The equation of motion for a single spherical particle whose position vector is $\bar{r}$ is then (1)

$$\rho_P \frac{\pi}{6} d^3 \frac{d^2\bar{r}}{dt^2} = 3\pi\,\mu\,d\,(\overline{q_P} - \overline{q_F}) + 3\pi\,\mu\,d\,\left(\frac{d}{2}\sqrt{\frac{\rho_F}{\pi\,\mu}}\int_{t_o}^{t}\frac{\frac{d\overline{q_P}}{dt'} - \frac{d\overline{q_F}}{dt'}}{\sqrt{t - t'}}\,dt'\right) +$$

$$\frac{1}{2}\frac{\pi}{6} d^3 \rho_F \left(\frac{d\overline{q_P}}{dt} - \frac{d\overline{q_F}}{dt}\right) + \bar{g}\frac{\pi}{6} d^3 (\rho_P - \rho_F) + \frac{\pi}{6} d^3\,\nabla p$$

Because the ratio ρ_P/ρ_F is large in a solid-gas system, the "added mass" term (third on the right) can certainly be neglected, as can the buoyancy component $\bar{g}\,\pi\,d^3\,\rho_F/6$ of the gravitational term. The forces on the particle due to gravity and the fluid pressure gradient are also neglected, on the grounds that they are proportional to d^3 and therefore, when d is very small, much smaller than the drag force. The second term on the right represents the effects of acceleration on the viscous drag; it is neglected in the first instance intuitively but, as with the other terms neglected, this can be ultimately justified numerically (2,3).

The simplified equation of motion is

$$C\,\frac{d^2\bar{r}}{dt^2} = \overline{q_F} - \frac{d\bar{r}}{dt}$$

It is considered to be a good approximation for dilute systems of small particles of irregular shape.

SOLUTION OF THE EQUATION OF PARTICLE MOTION

The method of solution depends on the fact that for very small particles the parameter C is much less than unity. For example, for a 10 μm particle of pulverised brown coal (ρ_P = 641 kg/m^3) in typical mill gas (ρ_F = 0.77 kg/m^3), C = 1.814 x 10^{-4} sec. It is assumed, but again justified numerically, that terms of $O(C^2)$ or higher are neglected in power series expansions of the derivatives.

The equation of motion in the θ direction is

$$\frac{C}{r}\left[\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)\right] = \frac{K}{r} - r\frac{d\theta}{dt}.$$

a linear equation in $r^2\frac{d\theta}{dt}$ whose solution is

$$r^2\frac{d\theta}{dt} = K + c_1 \exp(-t/C)$$

If the particle is injected with a velocity $(q_P)_\theta$ equal to the fluid velocity $(q_F)_\theta$ at that point then $c_1 = 0$, and the particle continues to move through each point in its trajectory with this component of its velocity equal to that of the fluid at the same point; that is, its angular momentum is conserved. When C is small this condition is quickly reached whatever the initial particle velocity.

In the z direction

$$-C\frac{d^2z}{dt^2} = \frac{M\cos^3\phi}{z^2} + \frac{dz}{dt}$$

and, using $r\frac{d\theta}{dt} = \frac{K}{r}$, in the ϕ direction,

$$C\left[\frac{d^2\phi}{dt^2} + 2\frac{dR}{dt}\frac{d\phi}{dt} - \frac{K^2\cos\phi}{R^3\sin^3\phi}\right] = -R\frac{d\phi}{dt}$$

It is to be expected that small particles will diverge slowly from the fluid streamlines, so that it is likely that the second derivatives will be small compared with the first derivatives. We therefore take as first approximations

$$\frac{dz}{dt} = -\frac{M\cos^3\phi}{z^2}$$

$$R^2\frac{d\phi}{dt} = \frac{CK^2\cos^3\phi}{z^2\sin^3\phi}$$

and by repeated differentiation and back-substitution obtain

$$\frac{dz}{dt} = -\frac{M\cos^3\phi}{z^2}\left[1 - 2\left(\frac{CM}{R^3}\right) + 3\left(\frac{CK}{r^2}\sin\phi\right)^2 \;\ldots\ldots\right]$$

$$\frac{d\phi}{dt} = \frac{CK^2\cos^5\phi}{z^4\sin^3\phi}\left[1 - 2\left(\frac{CM}{R^3}\right) + 3\left(\frac{CK}{r^2}\right)^2 \;\ldots\ldots\right]$$

Finally, to $O(C)$

$$\frac{d\phi}{dz} = -\frac{CK^2\cos^2\phi}{Mz^2\sin^3\phi}$$

Integrating along the particle trajectory originating from a point on the cone $\phi = \phi_o$ in the plane $z = z_i$ yields

$$\sec\phi + \cos\phi = \frac{CK^2}{M}\left(\frac{1}{z} - \frac{1}{z_i}\right) + \sec\phi_o + \cos\phi_o$$

of which the appropriate solution is

$$\cos\phi = \frac{1}{2}\left[\frac{CK^2}{M}\left(\frac{1}{z} - \frac{1}{z_i}\right) + \sec\phi_o + \cos\phi_o\right] - \left\{\frac{1}{4}\left[\frac{CK^2}{M}\left(\frac{1}{z} - \frac{1}{z_i}\right) + \sec\phi_o + \cos\phi_o\right]^2 - 1\right\}^{\frac{1}{2}}$$

Subsequent evaluation in cases of interest has shown that the terms $2\,(\frac{CM}{R^3})$ and $3\,(\frac{CK}{r^2})$ are both $O(10^{-3})$. Note that to the order of accuracy applied the particle moves relative to the fluid only in the r direction, with a velocity

$$\frac{dr}{dt} = -\frac{M \sin\phi}{R^2} + \frac{CK^2}{r^3} .$$

The particle Reynolds number is thus

$$\frac{d\,\rho_F}{\mu}\,|\overline{q_P} - \overline{q_F}\,| = \frac{d\,\rho_F}{\mu}\frac{CK^2}{r^3}$$

and has a maximum value of about 0.5, strictly outside the regime of true Stokes flow ($Re < 0.1$), but sufficiently close to ensure that the error is small.

SEPARATION IN THE SINK-VORTEX FLOW

A streamline of the sink-vortex flow is defined by

$$\frac{dR}{(q_F)_R} = \frac{R \sin\phi\, d\theta}{(q_F)_\theta} = \frac{R\, d\phi}{(q_F)_\phi}$$

from which, since $(q_F)_\phi = 0$, ϕ = constant; that is, the streamlines lie on the surfaces of cones having apices at the origin. It is therefore possible to select two such cones, with semi angles ϕ_o and ϕ_1, as the inner and outer physical boundaries.

The total volume flow is

$$Q = \int_{z_i \tan\phi_o}^{z_i \tan\phi_1} - (q_F)_z\, 2\pi\, r\, dr$$

$$= 2\pi M\,(\cos\phi_o - \cos\phi_1)$$

Consider a particle which traverses the flow from ϕ_o to ϕ_s between z_i and z_e. Then the volume flow $2\pi M\,(\cos\phi_o - \cos\phi_s)$ through the plane $z = z_e$ contains only particles smaller than this particle. The separation between z_i and z_e with respect to this particular particle size is

$$S = \frac{\cos\phi_o - \cos\phi_s}{\cos\phi_o - \cos\phi_1}, \text{ where}$$

$$\cos\phi_s = \frac{1}{2}\left[\frac{CK^2}{M}\left(\frac{1}{z_e} - \frac{1}{z_i}\right) + \sec\phi_o + \cos\phi_o\right]$$

$$- \left\{\frac{1}{4}\left[\frac{CK^2}{M}\left(\frac{1}{z_e} - \frac{1}{z_i}\right) + \sec\phi_o + \cos\phi_o\right]^2 - 1\right\}^{\frac{1}{2}}$$

Note that in geometrically similar separators with equal fluid velocities S increases as the linear scale decreases, in qualitative accord with the well known fact that under the same conditions small cyclones are more effective than large ones. Conversely, for constant separation (geometrically similar particle paths) the swirl component of fluid velocity is proportional to the square root of the linear scale. A similar result would hold for fine particles in any comparable swirling flow since the leading term of the particle velocity relative to the fluid will always be $C|(q_F)_\theta|^2/r$; however, any swirling flow other than the potential vortex is maintained by turbulent processes, and these will themselves be subject to the influence of scale through the curvature of the flow.

The separation equation has been extensively investigated. It has been shown that to achieve a given separation, with practical limitations on the swirl component of the fluid velocity and on the axial length of the separator, the cone angles must be small. It is not yet known to what extent this compromises the aerodynamic purpose of the sink flow, namely, to keep the boundary layers thin by ensuring that the flow speed on the walls is everywhere increasing in the direction of motion of the gas. Putting aside this consideration, it is convenient to use for illustrative purposes the limiting case of a cylindrical vortex separator ($\phi = 0$) in which the axial velocity U is constant. The separation equation is

$$S = \frac{r_s^2 - r_o^2}{r_1^2 - r_o^2}$$

$$\text{where } r_s = \left[\frac{4CK^2}{U} (z_i - z_e) + r_o^4 \right]^{\frac{1}{4}} .$$

Typical design data are

$$S = 0.6$$
$$Q = 47.2 \text{ m}^3/\text{s}$$
$$C = 1.814 \times 10^{-4} \text{ s}$$
$$z_i - z_e = 6.1 \text{ m}$$
$$U = 24.4 \text{ m/s}$$

Figure 2 shows how the required swirl velocities vary with the inner cylinder radius r_o. The mean velocity v_e in the exit plane $z = z_e$ is defined by

$$\pi v_e^2 (r_1^2 - r_o^2) = \int_{r_o}^{r_1} (\frac{K}{r})^2 \, 2\pi \, r \, dr \quad ;$$

it is desirable that v_e should be minimised since, at best, only part of the swirl head can be recovered. Obviously, in the case shown this implies r_o>0.3 m. In general, it may not be possible to recover any of the fuel stream (between r_s and r_1) velocity head; in fact, in one projected application of the vortex separator it is proposed to deliberately dissipate this energy in the furnace to promote combustion. However, since it is likely that only a modest efficiency of head recovery can be achieved with straightening vanes in the vapour stream, the need to keep the inner swirl velocities down is probably paramount from this point of view. On the other hand, the velocity v_1 on the outer wall, where the solid particles are concentrated, increases with r_o; because the rate of abrasive wear on the wall is proportional to the square, or possibly a higher power, of the velocity, an upper limit on r_o is implied. When r_o = 0.5 m then v_1 = 45 m/s, v_e = 60 m/s and v_o = 85 m/s, which are considered to be practical.

THE SINK-VORTEX SEPARATOR

Thus far it has been assumed that a three-dimensional velocity distribution appropriate to the sink-vortex flow exists in the entry plane $z = z_i$. In cyclones the necessary swirl is induced by a tangential jet or jets, in other devices by a cascade of blades. In either case most of the pressure drop across the swirlers is dissipated in downstream turbulence, which will tend to redistribute solid particles "centrifuged" into the outer flow. In the sink-vortex separator it is proposed to induce the radial velocity distribution in a curved contracting duct designed by the Helmholtz-Kirchhoff method such that the velocity on both inner and outer walls is either constant or increasing in the direction of motion. It has been verified experimentally that the specified flow in similar devices designed by this method is closely approximated in practice.

The equation of a streamline in the sink-vortex flow is

$$R = -\frac{M \sin^2 \phi}{K} \theta + c_3, \; \phi = \text{constant}$$

By tracing back through an angle $\theta = -2\pi$ from points $R_i = z_i \tan \phi$ ($\phi_o \leq \phi \leq \phi_1$) on a line θ = constant in the plane $z = z_i$, the shape of a surface which can be chosen to close the top of the separator is determined. Figure 3 is an impression of the separator and inlet arrangement. Note that because the cone angles to be used are small it is judged sufficient to use a two-dimensional contracting inlet design.

The exit arrangements depend somewhat on how the separator is to be used. As already mentioned, one possibility is for the swirling fuel stream to be discharged into the furnace from a unit attached to the furnace wall. The vapour stream can then be conveniently returned inside the centre body.

As has been noted the potential character of the flow, the means by which it is induced and the acceleration in the direction of motion are all features of the separator which have been chosen to minimise turbulence. Nonetheless, turbulence will be present in the flow through an actual separator, and the question arises as to whether the theoretical model is realistic. This doubt can only be resolved finally by experiment, but it is appropriate here to mention that since turbulent mixing is enfeebled near convex surfaces (4,5) the tendency to oppose separation will be minimised in the inner flow. Conversely, the tendency is strengthened near a concave surface, but this is of little consequence in the outer flow, where, in any case, it is expected that the high concentration of large particles will tend to damp the turbulent motion.

CONCLUSION

The analysis presented demonstrates that it is possible in theory to separate all but a very small fraction of a suspended pulverised brown coal burden from a substantial portion of its carrier gas, in units of practical size and with practical fluid velocities. Furthermore, there are very good grounds for believing that the postulated sink-vortex flow in which this separation is achieved will be satisfactorily approximated in practice. Experiments are now in progress to investigate the aerodynamics of the proposed sink-vortex separator, and, ultimately, to verify the theory.

The authors are confident that the work reported will be the basis of a complete design method for a cyclone-type device of unequalled efficiency. Obviously, the potential applications of the concept are not limited to the fuel-vapour separation problem which motivated its development. For example, consideration is being given to a sink-vortex precipitator in which spent boiler gas would be stripped of its ash burden and discharged as clean stack gas within the limits prescribed by legislation (6).

Further details can be obtained from Reference 3, of which this paper is a summary. An application for patent rights to devices embodying the sink-vortex concept has been made.

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FIG. 1.

CO-ORDINATE SYSTEMS.

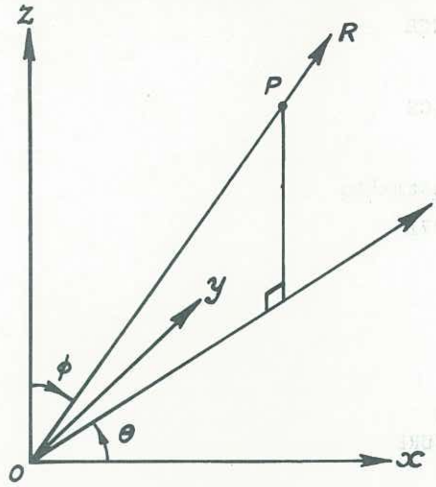
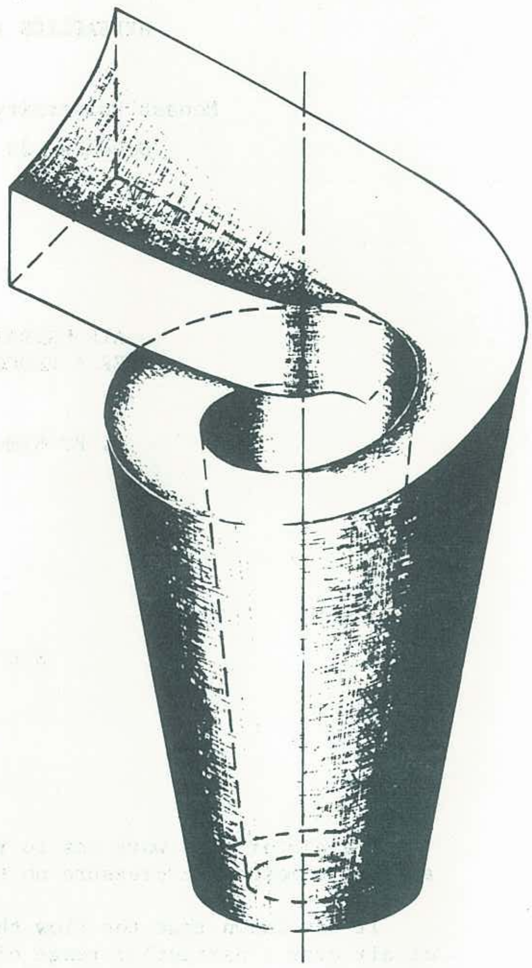


FIG. 3.

SINK-VORTEX SEPARATOR WITH CONTRACTING INLET.



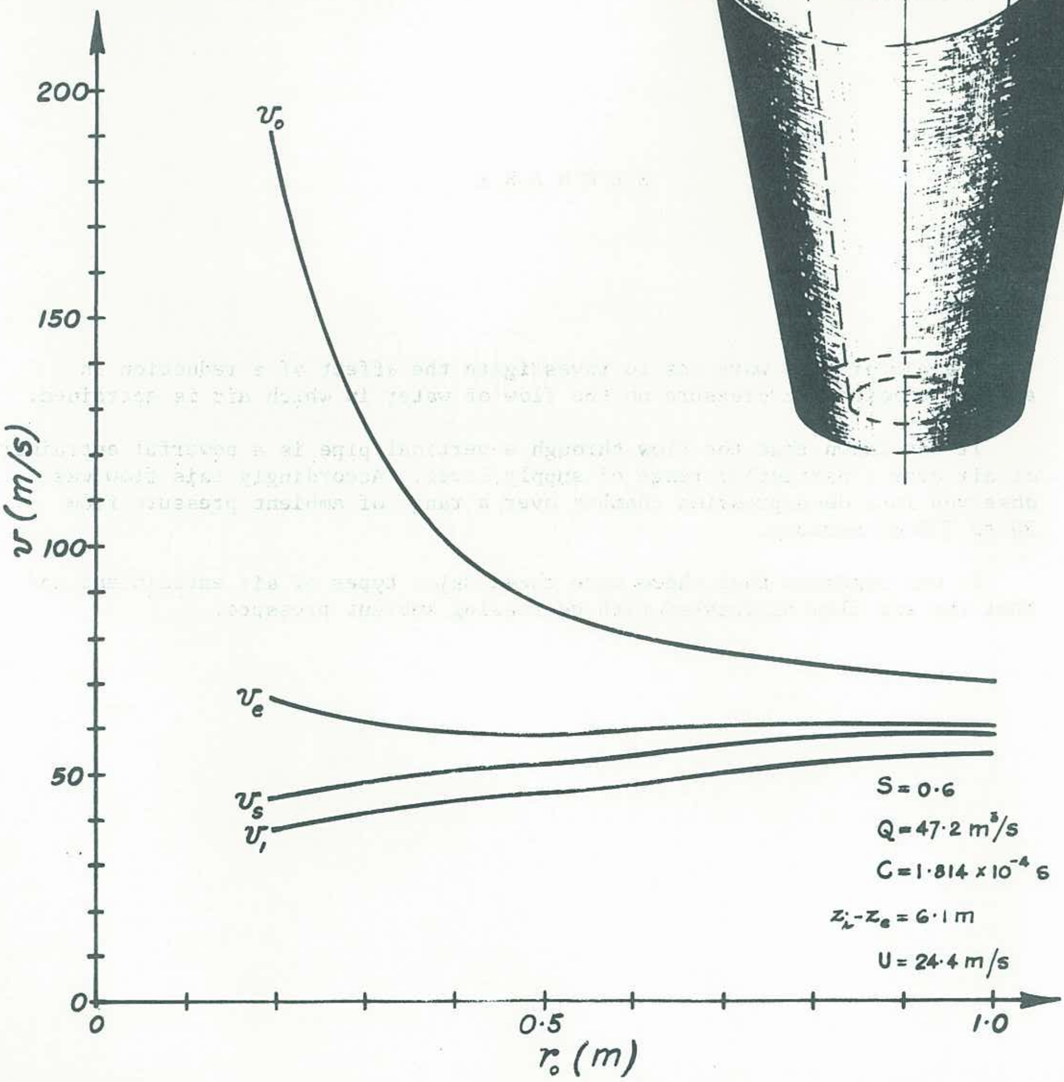


FIG. 2. SWIRL COMPONENTS OF FLUID VELOCITY IN THE EXIT PLANE ($Z = Z_e$) OF CYLINDRICAL SEPARATORS.