FOURTH AUSTRALASIAN CONFERENCE
on
HYDRAULICS AND FLUID MECHANICS
at
Monash University, Melbourne, Australia

1971 November 29 to December 3

LOSS CHARACTERISTICS OF LONG ORIFICES

by

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SUMMARY

A theoretical solution is presented for determining the loss coefficients for flow through long orifices. The theory is applicable for small ratios of orifice diameter to pipe diameter as prevalent in practice for long orifices. Entrance flow model is used to determine the pressure drop across the orifice while the continuity and the momentum equations are used to determine the pressure recovery immediately downstream of the orifice. Experiments have been conducted for long orifices with β ratio of 0.2 and the results are compared with the theoretical solution to find the range of validity of the theory.

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GLOSSARY OF TERMS

- A - cross sectional area of pipe
- d - diameter of orifice
- g - acceleration due to gravity
- h - pressure head
- h_d - excess loss due to the disturbance
- k - excess pressure drop factor in entrance flow
- K - loss coefficient
- l - length of orifice
- m - constant in Eq. 2
- p - pressure
- R_d - orifice Reynolds number, $\bar{u}_d\, d/\nu$
- $\bar{u}$ - mean axial velocity through the pipe
- $\bar{u}_d$ - mean axial velocity through the orifice
- α - energy coefficient
- α_m - momentum coefficient
- β - ratio of orifice diameter to pipe diameter
- γ - specific weight
- ρ - density
- ν - kinematic viscosity

INTRODUCTION

In metering problems, flow through long orifices have been treated as an entrance flow case to obtain theoretical estimates of the coefficient of discharge. As long orifices are generally used with low β ratios, such a model gives useful results at low and moderate Reynolds numbers. Rivas and Shapiro (1) determined the coefficient of discharge for rounded entrance nozzle type flow meters, using the results of Shapiro et al(2) for excess loss in the entrance region. The effect of rounded entrance was included by adding an equivalent length to the orifice length. Lichtarowicz et al(3) used the entrance flow model for long orifices with square edge, taking the excess loss in the entrance region from Langhaar's theoretical solution(4). But the theoretical coefficient of discharge agreed with the experimental values only for very low Reynolds numbers as no account was taken of the fact that flow may be only partially developed in a long orifice. Dickerson and Rice(5) used Langhaar's entrance flow solution for square-edged long orifices with $\beta < 0.1$ and obtained good agreement with experimental results for the coefficient of discharge. In the present paper, a theoretical estimate is obtained for the loss coefficient of long orifices using entrance flow model to determine the pressure drop across the orifice and the momentum equation to estimate the pressure recovery downstream of the orifice. The entrance flow pressure drop is estimated from Hornbeck's numerical solution(6) as it is known to agree very well with experimental results and other successful analytical solutions(7).

THEORETICAL DEVELOPMENT

The loss coefficient K is defined by the equation

$$h_d = K\,\frac{\bar{u}^2}{2g} \qquad \ldots (1)$$

Referring to Fig.1, the theoretical model is based on the assumption that the velocity distribution at the entrance of the long orifice is uniform and that the flow in the long orifice develops as in the entrance region of pipes. The entrance loss is determined based on Hornbeck's entrance flow solution in laminar flow. Downstream of the orifice, the pressure recovery from sections

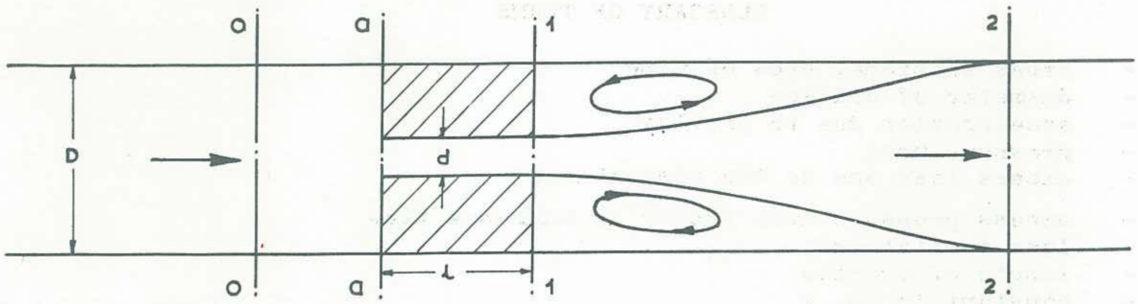

FIG.1.- DEFINITION SKETCH FOR THEORETICAL MODEL

11 to 22 is obtained from the momentum equation and the flow development losses downstream of section 22 are ignored. Those assumptions may be expected to give useful results for low β ratios as prevalent in practice for long orifices and for sufficiently long orifices, as long as the flow inside the orifice does not become turbulent. For a rounded entrance, Rivas and Shapiro(1) used the results of flow past a flat plate for determining the onset of turbulence, but in the present case, the origin of turbulence should be expected at a lower Reynolds number because of the contraction and consequent expansion of the jet at the square entrance. Hence the range of validity of the theoretical model will be determined based on comparison with experimental results.

The energy equation for sections 00 and 11 (Fig.1) may be written as

$$\frac{p_o}{\gamma} + \alpha_o \frac{\bar{u}^2}{2g} = \frac{p_1}{\gamma} + \alpha_1 \frac{\bar{u}^2}{2g} + \frac{64}{R_d}\frac{1}{d}\frac{\bar{u}_d^2}{2g} + m\frac{\bar{u}_d^2}{2g} \quad \ldots (2)$$

The third term on the right side of Eq. 2 represents the normal laminar friction loss for flow through the orifice and the last term accounts for the excess viscous dissipation due to flow development. For small β ratios, as in the present case, the friction loss between sections 00 and aa may be ignored. The energy equation for sections aa and 11 is given by

$$\frac{p_a}{\gamma} + \frac{\bar{u}_d^2}{2g} = \frac{p_1}{\gamma} + \alpha_1\frac{\bar{u}_d^2}{2g} + \frac{64}{R_d}\frac{1}{d}\frac{\bar{u}_d^2}{2g} + m\frac{\bar{u}_d^2}{2g} \quad \ldots (3)$$

where the assumption of uniform velocity distribution at section aa is used. In entrance flow, the excess pressure drop factor due to flow development is defined by

$$\frac{(p_a-p_1)/\gamma}{\bar{u}_d^2/2g} = \frac{64}{R_d}\frac{1}{d} + k \quad \ldots (4)$$

where the excess pressure drop factor k is a function of $\frac{1/d}{R_d}$. From Eqs.3 and 4,

$$k = \alpha_1 + m - 1 \quad \ldots (5)$$

The continuity equation gives

$$\bar{u}_d = \frac{\bar{u}}{\beta^2} \quad \ldots (6)$$

Using Eqs. 5 and 6, and writing the pressure head p/γ as h, Eq. 2 simplifies to

$$\frac{h_o-h_1}{\bar{u}^2/2g} = \frac{1}{\beta^4}\left[k + 1 + \frac{64}{R_d}\frac{1}{d} - \alpha_o\beta^4\right] \quad \ldots (7)$$

The momentum equation for the fluid between sections 11 and 22 is given by

$$\rho A \bar{u} [\alpha_{m1} \bar{u}_d - \alpha_{m2} \bar{u}] = (p_2 - p_1) A \qquad \ldots (8)$$

Dividing both sides by $\rho A \bar{u}^2$ and using Eq.6, Eq.8 simplifies to

$$\frac{h_2 - h_1}{\bar{u}^2/2g} = \frac{2(\alpha_{m1} - \alpha_{m2} \beta^2)}{\beta^2} \qquad \ldots (9)$$

Ignoring the flow development losses downstream of section 22,

$$K = \frac{h_o - h_1}{\bar{u}^2/2g} - \frac{h_2 - h_1}{\bar{u}^2/2g} \qquad \ldots (10)$$

Using Eqs. 7 and 9, Eq. 10 becomes

$$K = \frac{1}{\beta^4} [k + 1 + \frac{64}{R_d} \frac{l}{d} - \alpha_o \beta^4 - 2 \alpha_{m1} \beta^2 + 2 \alpha_{m2} \beta^4] \qquad \ldots (11)$$

For laminar approach flow for which the present solution holds good, $\alpha_o = 2$. Downstream of the orifice the flow becomes turbulent at a critical Reynolds number, the value of which is generally small for small β ratios. The momentum coefficient is between 1 and 1.33; it will be closer to 1 for Reynolds number above the critical value and for Reynolds number below the critical value it will be closer to 1.33. However, for small β ratios, an error in α_{m2} will not lead to significant errors in K as seen from Eq. 11. Hence α_{m2} is assumed unity at all Reynolds numbers. With $\alpha_{m2} = 1$ and $\alpha_o = 2$, Eq. 11 reduces to

$$K = \frac{1}{\beta^4} [k + 1 + \frac{64}{R_d} \frac{l}{d} - 2 \alpha_{m1} \beta^2] \qquad \ldots (12)$$

Eq. 12 is the theoretical expression for the loss coefficient for long orifices. α_{m1} and k in Eq. 12 are functions of $(l/d)/R_d$ and may be obtained from Hornbeck's numerical solution for entrance flow(6).

COMPARISON BETWEEN THEORY AND EXPERIMENTS

The theoretical solution may be expected to give satisfactory results provided the length of the orifice is long enough to satisfy the assumptions involved in the theoretical model. A comparison with experimental results shows that the theoretical method may be used for $l/d \geqslant 2$ for low and moderate Reynolds numbers. Experiments were conducted in an oil recirculation system for long orifices with a nominal β ratio of 0.2. The pressure field was measured for sufficiently long distances upstream and downstream of the orifice and the loss coefficient was determined taking care to include the flow development losses besides the losses in the immediate vicinity of the orifice.

The loss coefficients given by theory (Eq.12) are compared with experimental results in Fig.2 for the nominal β value of 0.2 for 3 long orifices with nominal l/d values of 2, 4 and 10. It may be seen from the figure, that theory gives very satisfactory results for $R_d / \frac{l}{d} \leqslant 300$. For higher $R_d / \frac{l}{d}$ values, the data starts gradually deviating from the theoretical line, but for $l/d = 4$, there is seen to be good agreement even up to $R_d / \frac{l}{d} = 700$. It may be seen from Fig. 2 that for low Reynolds numbers, the relationship between K and R_d tends to become linear, with K being inversely proportional to R_d. This may be seen directly from Eq. 12 where for low values of $R_d / \frac{l}{d}$, except the third term, the other terms are relatively small and hence K is inversely proportional to Reynolds number.

CONCLUSION

A theoretical solution is given for the determination of the loss coefficient for long orifices using entrance flow model to determine the

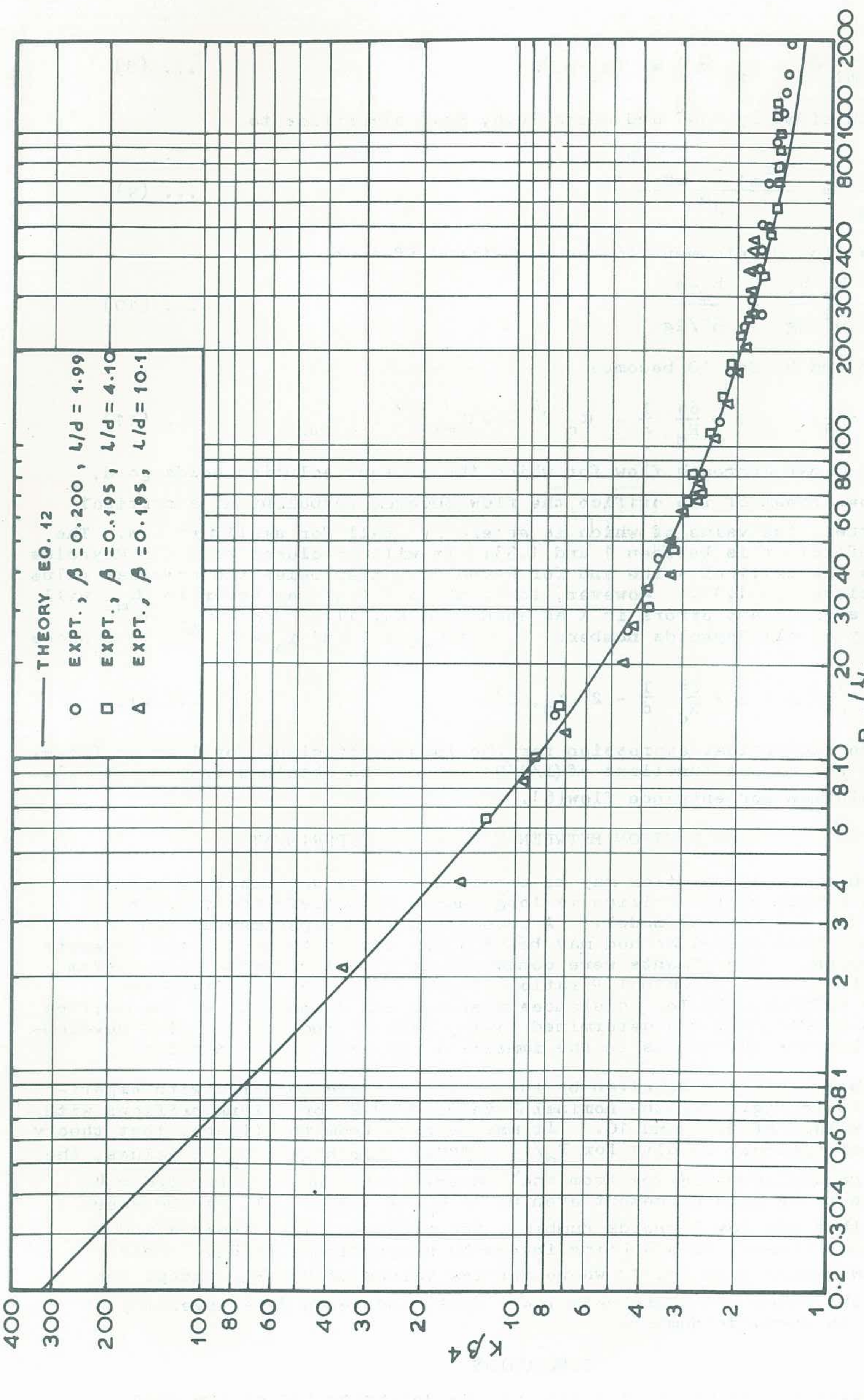

FIG. 2 - COMPARISON BETWEEN THEORY AND EXPERIMENTS

pressure drop across the orifice and the momentum equation to obtain the pressure recovery downstream of the orifice. The theory is valid for small ratios as prevalent in practice for long orifices. A comparison with experimental results shows that the theoretical solution is valid for $l/d \geqslant 2$ and $R_d \leqslant 300\ l/d$.

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