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THE RELATIONSHIP BETWEEN THE COHERENCE LENGTH OF A TWO PHASE  
MIXTURE AND THE RESPONSE OF A CYLINDRICAL CANTILEVER

by

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S U M M A R Y

A preliminary examination of the relationship between the coherence or mixing length of a flowing air-water mixture and the response of a solid cylindrical cantilever in axial air-water flow shows that the r.m.s. strain level at the base of the cantilever increased as the void fraction was increased and decreased as  $\lambda/L$  was decreased\*. There was no obvious dependence of the r.m.s. strain on the values of  $\lambda$  used in these experiments and this result is consistent with theoretical predictions.

\*  $\lambda$  is the coherence length of the two phase flow and L is the length of the cantilever.

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1. INTRODUCTION

The interaction between a flowing fluid and a mechanical structure has been intensively studied for the case of a single phase fluid flowing parallel to the axis of symmetry of plain circular rods. Burgreen, Byrnes and Benforado [1] examined the vibration of rods induced by water in parallel flow, whilst Basile, Faure and Ohlmer [2] studied various nuclear fuel rod models, again using a parallel flow of water. Many attempts have been made to produce correlations which can predict mechanical vibration amplitudes in parallel turbulent flow; Reavis's work [3] is typical of this line of approach.

The usual aim in current work on flow induced vibration is to produce data which will enable nuclear fuel element assemblies to be designed to withstand the turbulent forces they will endure in operation. However, if the number of reactors which have some boiling generation in their cores increases, the extension of single phase studies to the case of two phase flow seems likely. Some preliminary work in simple geometry using a single cylindrical cantilever in axial air-water flow has been completed by Harris and Holland [4]. They suggest that the major driving force arises from momentum fluctuations in the fluid caused by the presence or absence of bubbles. Their results were correlated by a temporally and spatially averaged value of the void fraction, defined as the fractional content of undissolved air in the mixture; no attempt was made to characterise the nature of the two phase flow.

This paper attempts to extend the work of Harris and Holland by using a simple characteristic of the two phase system as a normalising parameter. Hubbard and Duckler [5], using a flush mounted wall pressure transducer, used salient features of the power spectral density of the pressure fluctuations to characterise various flow regimes in axial air-water flow. More recently Carrard and Ledwidge [6] have developed a simple sensor to measure fluctuations in void fractions at two axially spaced locations. They have also shown that for bubble flow the statistics of the void fluctuations can be determined using cross correlation analysis between the spaced detectors to yield a coherence or mixing length. This coherence length is defined as the length over which the cross correlation coefficient falls to  $1/e$  of its value when the separation between the probes is zero.

2. THEORY

No complete theory exists to date for flow-induced vibration in a two-phase fluid. However, many of the concepts for a single-phase fluid can be extended provided one considers the dominant excitation force to correspond to momentum transfer via bubble contact with the solid body. Since we are dealing with a random process, a statistical approach is necessary, and an expression for the mean-square deflection at the free end of a cantilever can be written [3] as;

$$\langle y^2(t) \rangle = 1/(M^2\omega_0^4) \int_0^\infty df |H(f)|^2 \int_0^L \int_0^L S_{xx'}(f) \phi(x) \phi(x') dx dx', \quad \dots(1)$$

where  $H(f)$  is the transfer function of the system, in terms of the frequency  $f$ , using a simple single degree of freedom model,  $S_{xx'}(f)$  is the cross-spectral density,  $\phi(x)$  is the eigenfunction for the cantilever,  $M$  is the mass weighted by the eigenfunction,  $\omega_0$  is the angular frequency for the fundamental flexural resonance and the co-ordinates  $x$  and  $x'$  are as shown in figure 2.

The problem is now one of choosing the expression for the cross-spectral density, and also the form of the variation of the damping factor in  $H(f)$  as the length of a cantilever is varied. It has been shown that the bubble statistics in a two-phase fluid approximates to a Markoff process [6], and thus we can put

$$S_{xx'}(f) = A^2 \exp(-|x-x'|/\lambda), \quad \dots(2)$$

where  $\lambda$  is a coherence length and is related to the mean-square momentum transfer due to bubble contact (which is approximately proportional to (void fraction)<sup>2</sup> for low voidage [4]).

Since damping forces mainly arise from the fluid surrounding a cantilever, they will not change dramatically for void fraction up to 25%, [4] and thus we can write for the transfer function (where  $\omega = 2\pi f$ );

$$H(i\omega) = 1/\left(1 - \omega^2/\omega_0^2 + 2i\zeta \omega/\omega_0\right), \quad \dots(3)$$

where  $\zeta = \text{constant}/(\text{mass of cantilever})$  and  $i = (-1)^{\frac{1}{2}}$ .

An estimate of the relationship between void fraction and mean-square stress level was made using the well-known relationships between the deflection and stress at the root of a cantilever.

The results are shown in figure 1 for a fixed 15% void fraction.

### 3. EXPERIMENTAL ARRANGEMENT

The experimental assembly shown in figure 2 consisted essentially of a cylindrical aluminium cantilever of diameter 0.953 cm and length 50.8 cm (initially), mounted in a shaped support to minimise entrance effects. The cantilever was situated inside a 91 cm length of 2.54 cm i.d. perspex tube. Water into which air could be introduced was passed at 54.5 litres/minute through the test section. Strain gauges were attached near the fixed end of the cantilever and a crossed-wire conductivity probe was placed upstream of the test section. The air and water flow rates were measured using rotameters and a turbine flowmeter respectively. The length of the cantilever was varied by decrements of 5.08 cm to 30.48 cm.

Signals from the strain gauges were analysed using a Hewlett Packard correlator to obtain an estimate of the variance. Signals from the crossed-wire probe were also analysed using the same correlator yielding two autocorrelograms and one crosscorrelogram. These correlograms allowed calculation of the coherence length in the air-water mixture.

### 4. RESULTS AND DISCUSSION

The results for the r.m.s. strain as a function of  $\lambda/L$ , where  $\lambda$  is the characteristic mixing length and  $L$  the length of the cantilever, are given in figure 3. The measurements at each value of voidage are grouped together. It is apparent that in the range  $0 < \lambda/L < 0.2$  the only dependence on  $\lambda/L$  is via the length and void fraction.

Figure 1 compares the r.m.s. strain as a function of  $\lambda/L$  for a fixed voidage (15%) with the theoretical trends calculated using equation (1). The evaluation of equation (1) showed that for the range of parameters being studied, the results did not depend to any extent on  $\lambda$ . The increase in the r.m.s. strain which occurred at a cantilever length of 35.56 cm for all void fractions has no obvious explanation.

### 5. CONCLUSIONS

The flow-induced vibration for a cantilever in a flowing air-water mixture shows an increasing characteristic for increasing void fraction, and a generally decreasing trend for decreasing length of cantilever (or increasing  $\lambda/L$ ). The coherence length is apparently not significant for values of  $\lambda/L$  up to 0.2.

Further experiments are necessary to examine the response of a simple cantilever to air-water flow for values of  $\lambda/L > 1.0$ .

### 6. ACKNOWLEDGEMENTS

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### REFERENCES

- [1] Burgreen, D., Byrnes, J. J. and Benforado, D. M., "Vibration of Rods Induced by Water in Parallel Flow", Trans. ASME 80, 991 (1958).
- [2] Basile, D., Faure, J. and Ohlmer, E., "Experimental Study on the Vibrations of Various Fuel Rod Models in Parallel Flow", Nuclear Engineering and Design 7,517 (1968).
- [3] Reavis, J. R., "Vibration Correlation for Maximum Fuel-Element Displacement in Parallel Turbulent Flow", Nuclear Science and Engineering, 38,63 (1969).
- [4] Harris, R. W. and Holland, P. G., "Response of a Cylindrical Cantilever to Axial Air Water Flow", Conference on Structural Mechanics in Reactor Technology, Berlin, 1971.
- [5] Hubbard and Duckler "The Characterisation of Flow Regimes for Horizontal Two-phase flow", Proceedings of 1966 Heat Transfer and Fluid Mechanics Conference, University of Santa Clara, California.
- [6] Carrard, G. and Ledwidge, T. J. "The Measurement of Slip Distribution and Void Fraction in Air-water Flow", International Symposium on Two-Phase Systems, Haifa, Israel, 1971.

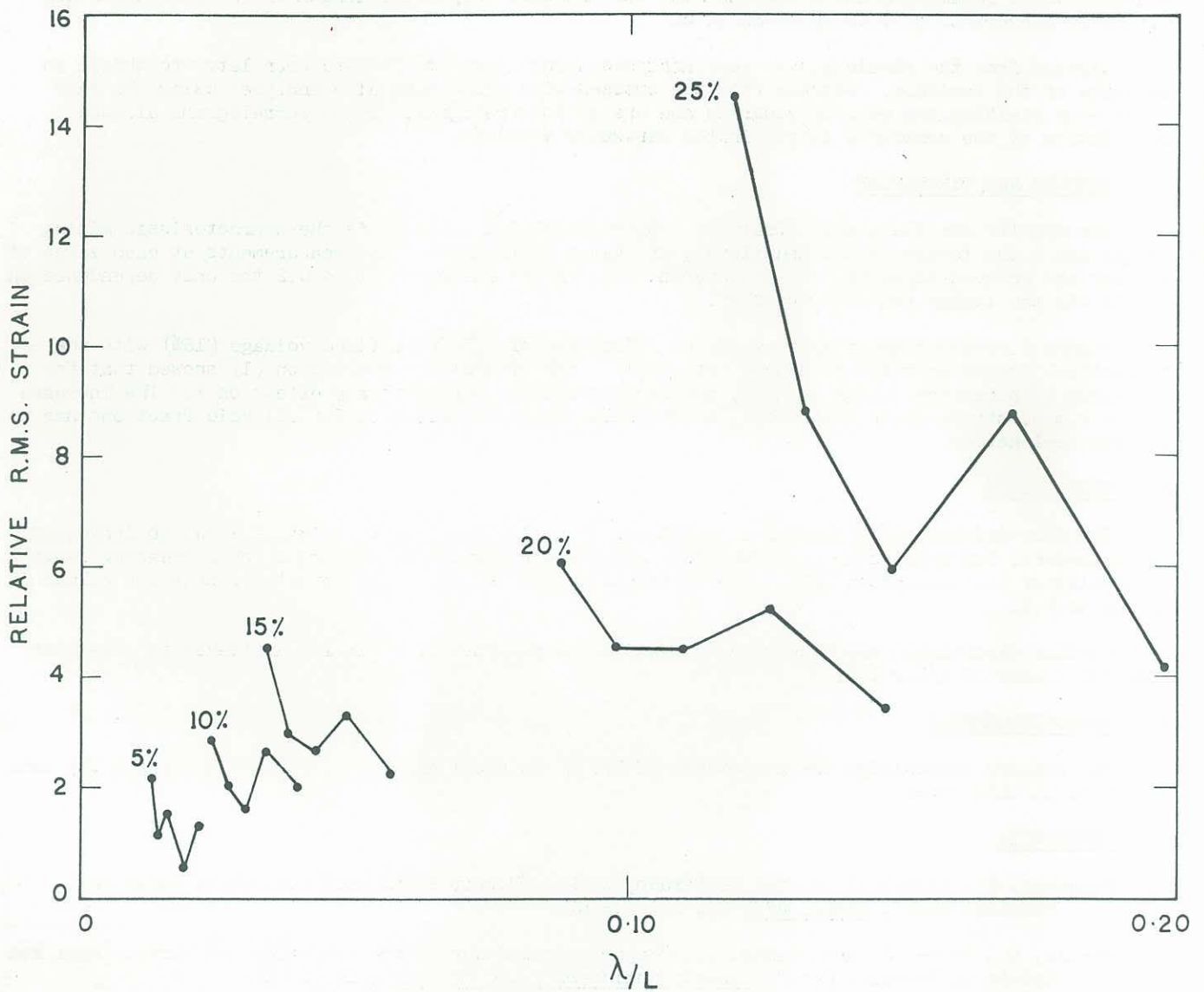


FIGURE 3. EXPERIMENTAL R.M.S. STRAIN AS A FUNCTION OF  $\lambda/L$  FOR VARIOUS VOIDAGES

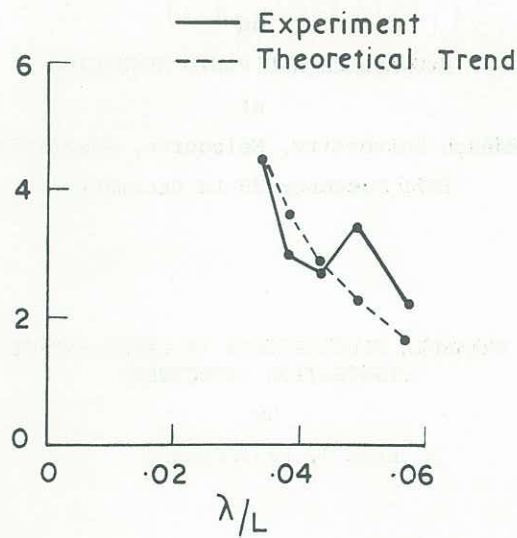


FIGURE 1. EFFECT OF VARYING  $\lambda/L$  ON THE RELATIVE R.M.S. STRAIN FOR FIXED 15% VOIDAGE

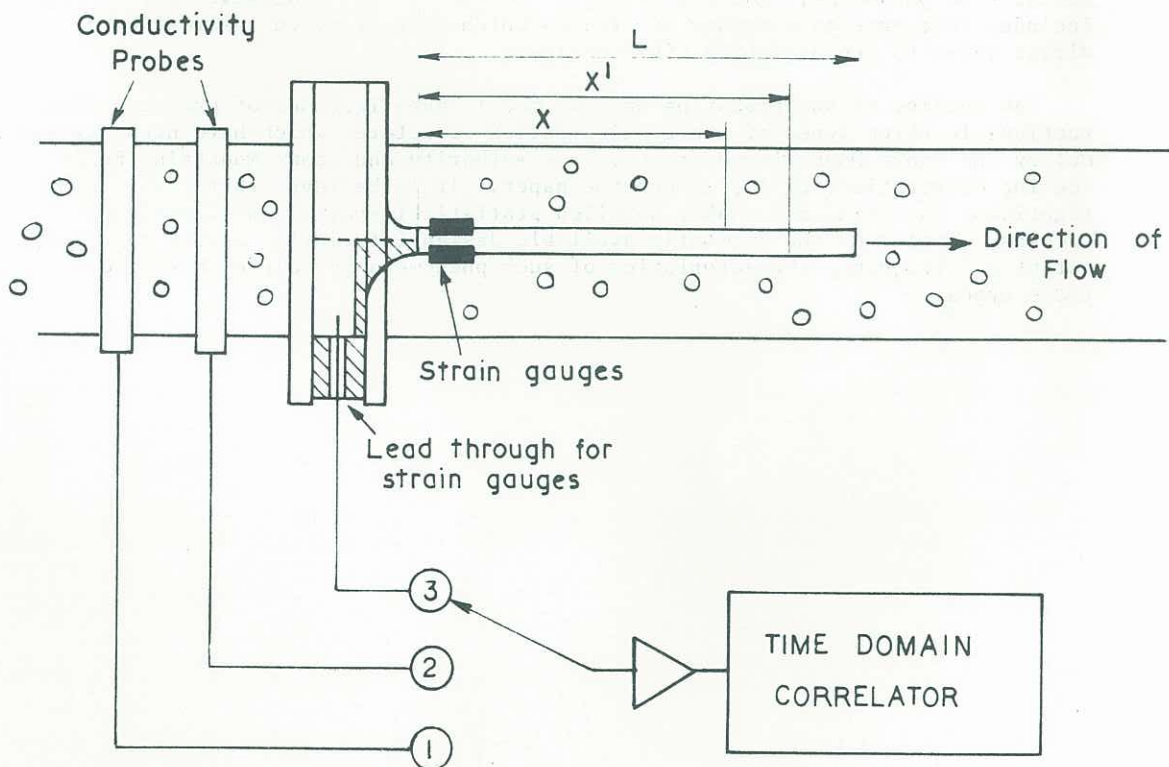


FIGURE 2. SCHEMATIC SHOWING CO-ORDINATE SYSTEM AND MEASUREMENT POINTS