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TURBULENT MIXING IN DIFFUSION FLAMES

by

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SUMMARY

Analysis of turbulent diffusion flames depends critically on the correct formulation of the turbulent diffusion terms used. The effects of the strong density gradients must be correctly handled. Libby has used a transformation relating the compressible eddy viscosity to that in incompressible flows. The failure of this analysis for diffusion flames in co-flowing streams has led other workers to propose different models for the compressible eddy viscosity which are neither universally applicable nor physically satisfying. The paper shows that Libby's transformation is probably correct. The discrepancy arises due to the fact that the eddy Reynolds number varies considerably in incompressible flows. New experimental data are presented for variations in eddy Reynolds number in incompressible flows and turbulent diffusion flames.

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Introduction

Turbulent diffusion flames are of interest as they are frequently encountered in areas of great technical importance, for example: industrial furnaces, free burning fires, and high speed propulsion systems. Turbulent flow with chemical reactions is one of the least understood areas of fluid mechanics. In many turbulent diffusion flames the rate of combustion is controlled by the rate of turbulent diffusion or mixing of the fuel and air, the chemical reactions being so fast that they are effectively instantaneous. In these cases the analytical prediction of the flame and flow field development depends on the correct formulation of the turbulent diffusion terms used in the boundary layer equations for momentum, energy, and species transport.

The Prandtl-Boussinesq eddy viscosity, ε, where (in the usual nomenclature)

$$\tau = \rho\,\overline{u'v'} = \rho\varepsilon\frac{\partial u}{\partial y} = \frac{1}{R_T}\,\rho\, b\,(u_{max} - u_{min})\,\frac{\partial u}{\partial y} \qquad \text{---(1)}$$

has been used with good results in incompressible free turbulent mixing problems. (In the above b is an arbitarily chosen length scale for the width of the turbulent flow, and R_T an empirical constant, the "eddy Reynolds number"). Various modifications of the eddy viscosity formulations have been used to account for the effects of density gradients in compressible and non-uniform density flows. In flows without combustion density differences are soon attenuated and it is difficult to choose the correct form of the eddy viscosity expression. Using a transformation derived from work on compressible turbulent boundary layers Libby [1]* has proposed the following relationship of the compressible eddy viscosity ε to the incompressible $\bar{\varepsilon}$ for axisymmetric flows (r is the radial coordinate)

$$\rho^2 r^2 \varepsilon = \rho_o^2\,\bar{\varepsilon}\int_o^r (\rho/\rho_o)\,2r'\,dr'$$

$$= \rho_o^2\,\frac{1}{R_T}\,b\,(u_{max} - u_{min})\int_o^r (\rho/\rho_o)\,2\,r'\,dr' \qquad \text{---(2)}$$

with ρ_o an appropriately chosen reference density in the flow. Using this formulation Libby obtained solutions for the complete range of axisymmetric jet mixing problems including those involving diffusion flames. Agreement with experimental results was reasonably good in all cases except, as was later found, for diffusion flames formed from fuel jets mixing in co-flowing streams of air, where the flame length predictions were far too long, particularly at low values of the initial jet to free-stream velocity ratio. This difficulty has led to other formulations for the compressible eddy viscosity, notably that of Schetz [2]

$$\rho^2\varepsilon = \frac{1}{R_T}\,b\,\rho_o\,\{(\rho u)_{max} - (\rho u)_{min}\} \qquad \text{---(3)}$$

and that of Ferri [3]

$$\rho\varepsilon \propto \frac{1}{R_T}\,b\,(\rho u)_{max} \qquad \text{---(4)}$$

These formulations have had success for diffusion flames over narrow ranges of conditions but are not universally applicable and are also somewhat unsatisfactory from a physical point of view.

Recently Bradbury and Riley [4] have shown for two-dimensional mixing of a plane jet into a co-flowing isothermal air stream that the value of R_T varies considerably with U_c, the ratio of the velocity at jet centre to the velocity in the free-stream. For the fully-developed portion of the turbulent flow field R_T decreases monotonically from R_{T_J} (the value for a jet into still air) at high values of U_c and asymptotes to R_{T_W} (the value for the plane wake) as U_c approaches unity. With $R_{T_J} = 33.4$ and $R_{T_W} = 14.7$ the variation is quite substantial. Townsend [5] has explained the different values of R_T for jets and wakes in terms of a simple model of the structure of free turbulent flow and the use of the turbulence energy equation. Bilger [6] showed that the use of Townsend's model gave good predictions of Bradbury and Riley's results for jets mixing in co-flowing streams. For axisymmetric jets and wakes R_{T_J} is about 44 while values of R_{T_W} derived from the literature range as low as 1.6. (For the values of R_T quoted above b is taken as the

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* Numbers in brackets refer to references listed at the end of this paper.

distance from the centre-line to where the velocity is the mean of the local centre-line velocity and the external stream velocity.) For axisymmetric jets mixing in a co-flowing stream it can be expected that R_T may decrease very strongly as U_c approaches unity.

This relatively strong mixing (low values of R_T) at low values of U_c has prompted the belief that the eddy viscosity formulation as used by Libby may in fact be the correct one. The shortcoming of Libby's results appears to lie in his use of a value of R_T of 40. The use of lower values of R_T where U_c is small would yield predictions of shorter flame lengths in accord with experimental findings. This paper examines this hypothesis.

Experimental Apparatus

Figure 1 shows the test rig used for both the isothermal jet mixing and for the diffusion flame experiments. The central jet is aligned with the axis of the main tunnel which has a cross section of 305mm by 305mm with corner fillets. The roof and the floor of the test section are movable so that a zero pressure gradient can be maintained. Jet nozzle diameter can be varied from 3mm to 20mm, and the jet supplied from the shop air supply, a water vane compressor, or from fuel gas cylinders. The jet may also be heated. The siting of the jet nozzle is such that the boundary layer on the external contour remains attached. Some measure of control of the external boundary layer can be achieved by means of a suction slot at the beginning of the external contour. Internally the flow contracts from the 50mm diameter of the supply pipe to the nozzle diameter in a smooth contour giving a uniform jet exit profile with thin boundary layers. The velocity of the external air flow can be varied up to about 40 m/s. The free-stream turbulence level is 0.1 per cent.

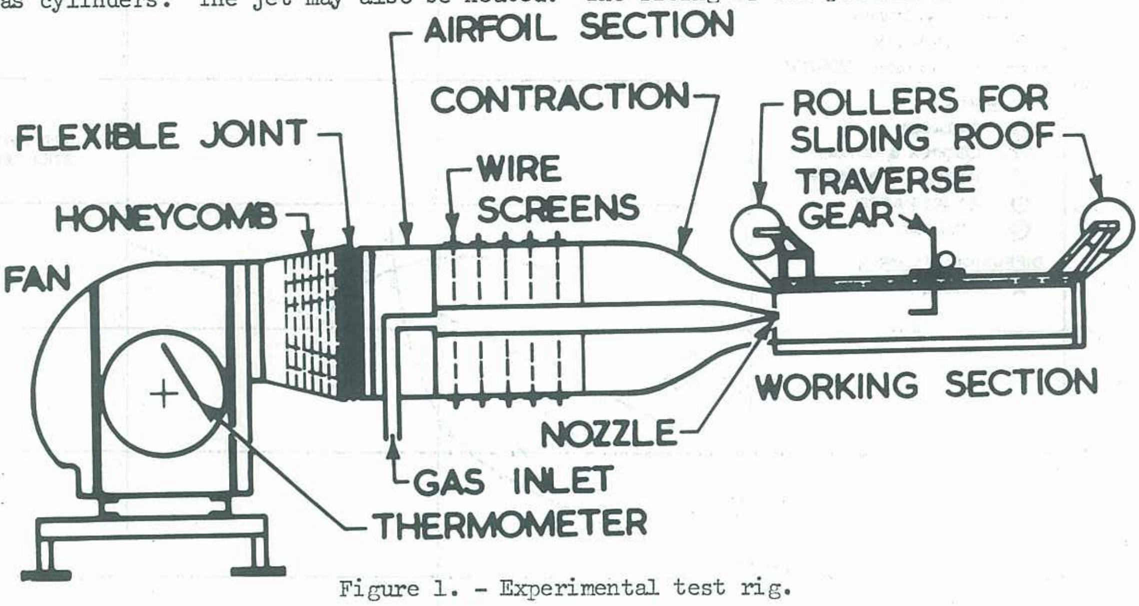

Figure 1. - Experimental test rig.

Vertical and axial traverses of the jet were made through a small slot in a sliding brass sheet panel in the tunnel test section roof. Horizontal transverse traverses were also made to establish the precise location of the jet centre and to test for axi-symmetry. Pitot and static traverses of the flow field were made with pressure differentials recorded with a "Texas Precision Pressure Gauge". In the diffusion flame experiments temperature traverses were made with a non-catalytic coated thermocouple (Kent [7]) and radiation corrections were applied. Lately, provision for iso-kinetic sampling and on-line analysis for composition has been developed.

Isothermal Jet Mixing

Tests have been carried out for a wide range of conditions for isothermal air into air mixing. Velocity profiles were determined from pitot-static traverses at various axial stations up to 1.8m downstream from the jet nozzle. Eddy Reynolds numbers were derived from the use of the half momentum integral equation. Figure 2 shows a typical result for eddy Reynolds number variation with U_c for a nozzle diameter of 5.65mm, a free-stream velocity of 30m/s and an initial jet to free-stream velocity ratio of 2.93. It can be seen that R_T decreases steeply at a value of U_c of about 2. Other results obtained show similar behaviour but there is no direct correlation with U_c, values for different jets having values of R_T which vary by a factor of up to two at any particular value of U_c.

Also shown in Figure 2 are the results of Challen [8] which were made on this rig and which were believed to be for the fully developed turbulent flow, that is, far downstream away from the effects of initial conditions. These results were produced by the virtual origin shift method used by Bradbury and Riley [4]. The present results should asymptote to those of Challen at low values of U_c but in fact do not show a consistent trend to do this. Also shown in Figure 2 are data deduced from the isothermal jet mixing results of Forstall and Shapiro [9] and Maczynski [10] which also were considered to be for the fully developed part of the jet. It is evident that the effects of the initial conditions persist quite strongly and that there is some doubt as to whether a fully turbulent region really exists in axisymmetric jet flows. The wake results shown in Figure 2 show a strong effect of the initial conditions on the values of R_T. Once again the results are for conditions well downstream where the wakes may be expected to be fully developed. The data from Carmody [11], Cooper and Lutzky [12], and Hwang and Baldwin [13] are for wakes from bluff bodies, while Demetriades [14] and Chevray [15] used slender bodies.

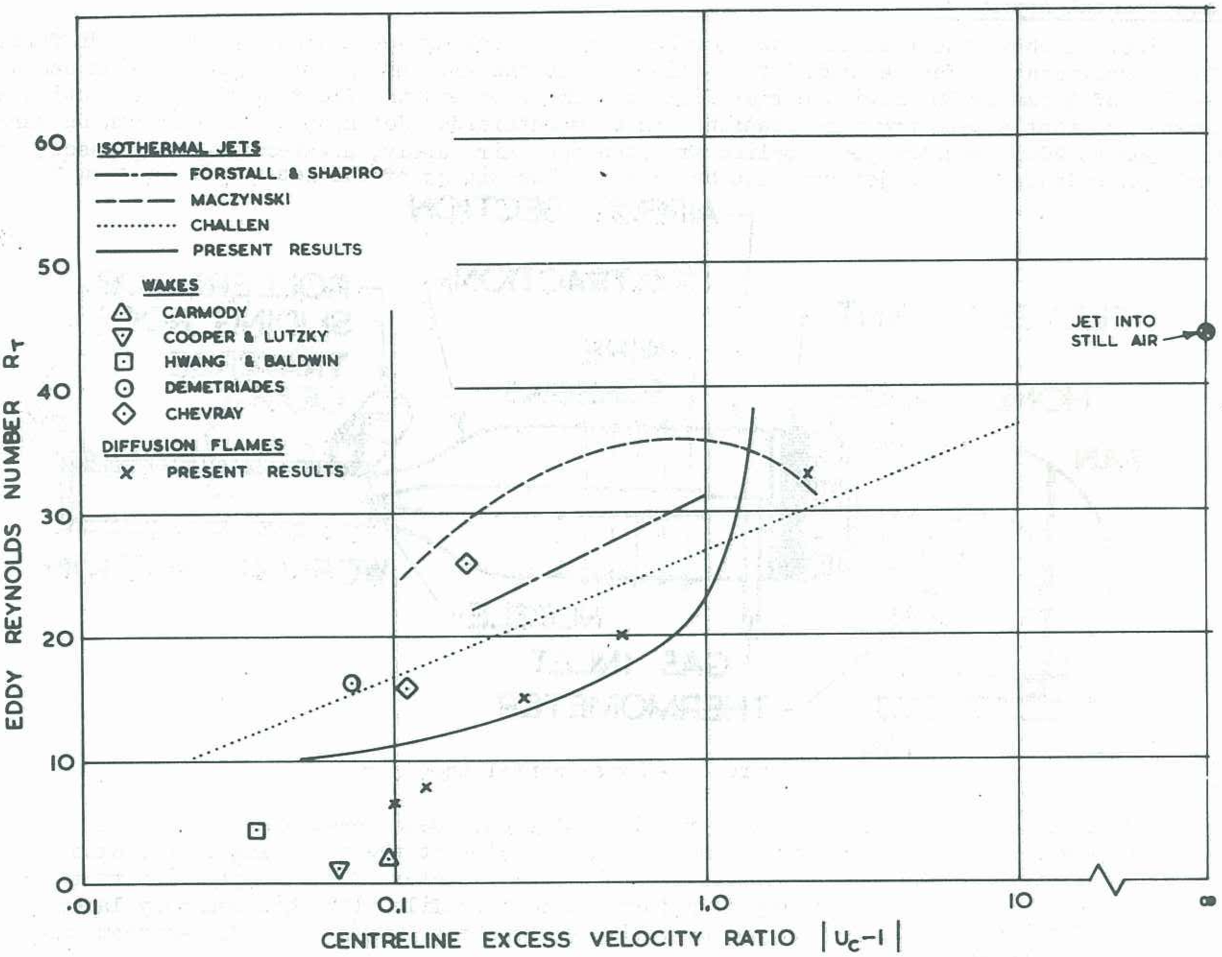

Figure 2. - Eddy Reynolds numbers in axi-symmetric free turbulent flows.

An understanding of why the effects of the initial conditions are so persistent has been gained from an examination of the turbulent energy equation (Bilger [16]). The Townsend analysis, used so successfully for plane wakes and jets (as discussed earlier), breaks down when the advection term in the turbulent energy balance becomes greater than the production term. For a round jet into still air advection is less than production while for round wakes it is greater. When production dominates advection it appears that free turbulent flows soon obliterate the effects of the initial source and that a "fully-developed" flow becomes established. On the other hand, when advection is large compared with production the effects of the initial source persist.

This appears to be the case with round wakes and jets in co-flowing streams with low values of U_c.

Diffusion Flames

The diffusion flame experiments were carried out at jet to stream velocity ratios of 5:1, 2:1, 1.25:1, 0.8:1 and 0.5:1. This allowed both jet and wake flames to be studied. Nozzle sizes ranging from 3mm to 7.6mm were used to gain an insight into the effects of initial boundary layers on the flames. The fuel gas used was made up of two parts hydrogen to one part nitrogen. This composition produced a stable flame on the burner rim without the excessively high temperatures of a pure hydrogen flame.

Flame shapes and lengths were determined by defining the flame surface as the locus of maximum temperature and traversing the field with a thermocouple. This definition coincides with that given by Libby [1], and provides a more accurate method of measurement than visual estimation particularly in the case of these non-luminous flames. Nevertheless difficulty was encountered in accurately determining flame lengths as the temperature gradients at the flame tip were rather small.

From the measured flame lengths the values of R_T that need to be used in Libby's theory were deduced. These are shown in Figure 2. They have been plotted against the mean values of $|U_c-1|$ over the flow field to the flame tip. The deduced values of R_T are also, of course, mean values over this range. These results are summarized in Table 1. (L_f = flame length, 2a = nozzle diameter).

Table 1 : Diffusion Flame Results

Initial Velocity Ratio	5	2	1.25	0.8	0.5
$L_f/2a$, theory : $R_T = 40$	61	119	343	324	100
$L_f/2a$, measured	55	70	78	68	43
Deduced mean R_T	33	20	8.2	6.8	15
Mean U_c	3.0	1.5	1.13	0.9	0.75

It can be seen that the deduced values of R_T are consistent with those found for jets and wakes at low values of U_c. Thus it appears that the basic fault with Libby's theory is that it does not take account of the dependence of eddy Reynolds number on the velocity ratio of the jet. (From what has been said earlier it is also apparent that the theory should also account for the persistence of the effects of the initial conditions. This will of course be a much harder task.)

In Figure 3 measured flames shapes are compared with the theoretically predicted shapes where the theory uses the modified mean values of R_T deduced above. The discrepancies are as would be expected from the use of a mean value of R_T for the whole of the flow field. Near the nozzle R_T would be expected to be higher (higher $|U_c-1|$) which means relatively slower mixing. The flame front is in general at the radius predicted by the theory which suggests that the compressibility effects on the eddy viscosity are correctly handled by the Libby formulation of Equation 2.

Conclusion

The foregoing results support the view that Libby's formulation for the effects of density variations on the eddy viscosity is correct provided the effects of variations in the eddy Reynolds number R_T found in isothermal jet mixing flows are incorporated. However, R_T is not a simple function of U_c; it depends also on the initial conditions at the jet nozzle. The problem remains then to find a way of correctly computing R_T and its variations given the initial conditions at the jet nozzle. The turbulent energy equation should provide the link but it is evident that a model of the turbulence structure different from that used by Townsend will be needed.

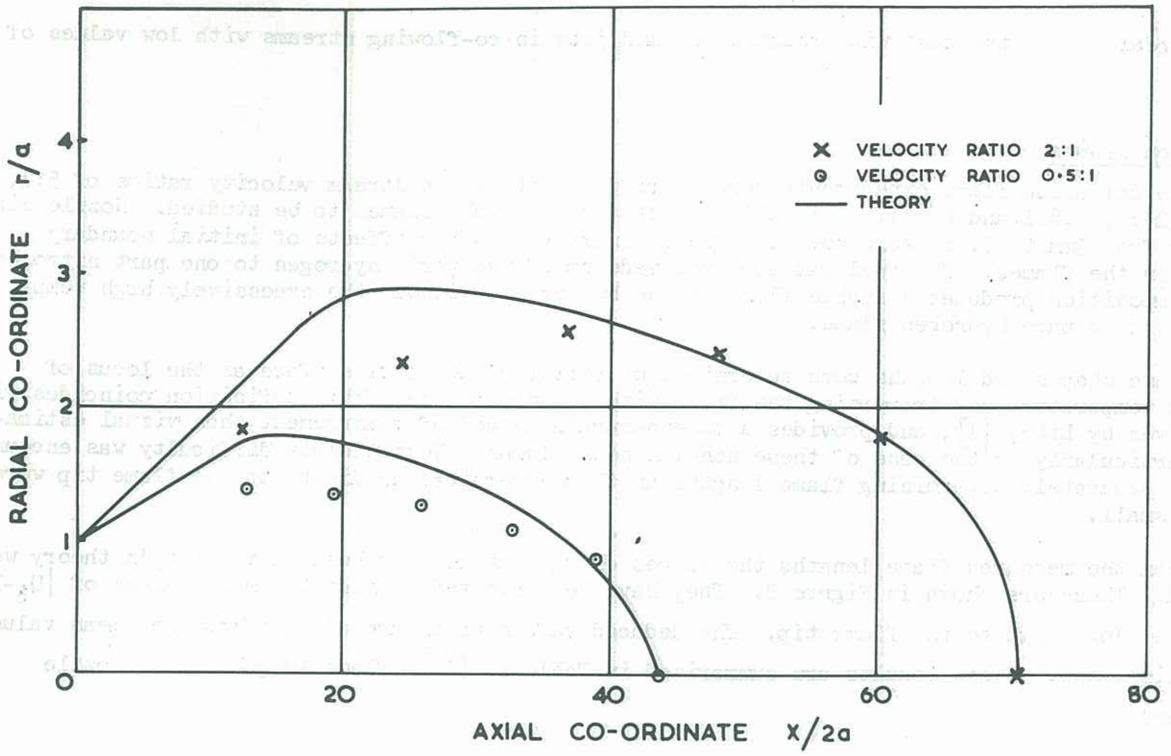

Figure 3. - Diffusion flame shapes

Acknowledgements

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References

1. Libby, P.A., A R S Journ., v. 32, pp 388-396, 1962.

2. Schetz, J.A., Supersonic Flow, Chemical Processes and Radiactive Transfer (Ed.Olfe and Zakkay) Pergamon Press, pp 79-91, 1964.

3. Ferri, A., J. Roy. Aer. Soc., v.68, pp 575-597, 1964.

4. Bradbury, L.J.S., and Riley, J., J. Fluid Mech., v.27, pp 381-394, 1967.

5. Townsend, A.A., J. Fluid Mech., v. 26, pp 689-715, 1966.

6. Bilger, R.W., Third Australasian Conference on Fluid Mechanics and Hydraulics, The Institution of Engineers, Australia, pp 159-162, 1970.

7. Kent, J.H., Combustion and Flame, v. 14, pp 279-282, 1970.

8. Challen, J.M., M.Eng.Sc. Thesis, The University of Sydney, 1968.

9. Forstall, W., and Shapiro, A.H., J. App. Mech. v.72, pp 399-408, 1950.

10. Maczynski, J.F.J., J. Fluid Mech. v. 27, pp 131-144, 1962.

11. Carmody, T., J. Basic Eng'g. v. 86, pp 869-882, 1964.

12. Cooper, R.D., and Lutzky, M., U.S. Navy, David Taylor Model Basin Report 963, 1955.

13. Hwang, N.H.C., and Baldwin, L.U., J. Basic Eng'g., v. 88, pp 261-8, 1966.

14. Demetriades, A., AIAA Journ., v. 6, pp 432-9, 1968.

15. Chevray, R., J. Basic Eng'g., v. 90, pp 275-284, 1968.

16. Bilger, R.W., Tech.Note F-3, Department of Mech.Eng, The University of Sydney, 1969.