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A Semi-Numerical Approach for Determining the Hydraulic Conductivity of Unsaturated Porous Materials

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Summary.—A method is presented for determining the hydraulic conductivity (K)-water content (θ) relationship of an unsaturated porous material using a simple experimental arrangement and a computer solution of the differential flow equation. The experimental measurements involve the saturated hydraulic conductivity of the material, the discharge-time relationship of a draining column of the material and the pressure head-water content relationship as determined by sectioning the column at equilibrium. Trial K-θ curves are then used in the computer solution to obtain a discharge-time curve matching the measured curve. The method was used with two sands and found to be sufficiently sensitive and quite satisfactory.

1.—INTRODUCTION

The relationship between the hydraulic conductivity (K) of an unsaturated porous material and its volumetric moisture content (θ) is a basic input requirement in analytical and numerical studies concerned with the movement of water in such materials. As recently noted by Watson (Ref. 1) there is a paucity of reliable experimental data on the relationship in the literature and this reflects the experimental difficulties inherent in its determination. Several laboratory methods have been available for some years; these include those of Childs and Collis-George (Ref. 2), Day and Luthin (Ref. 3) and Richards and Weeks (Ref. 4). One difficulty in the past has been the non-destructive measurement of the water content but this is now possible in experiments with rigid materials by using gamma-ray attenuation methods.

Two new approaches to the measurement of the K-θ relation have recently been published and in both of these the water content is measured by gamma-ray attenuation techniques. The first method, Watson (Ref. 5), is termed the method of instantaneous profiles. In it the K-θ relation is determined under quite severe transient conditions. By contrast, the second method, Watson (Ref. 1), is based on steady state conditions, but differs from other steady state methods in that a water content variation in the column is achieved by entrapping a zone of air in the profile and allowing it to be maintained at a pressure above atmospheric.

Such methods make it apparent that the research worker who has the facilities for the non-destructive measurement of the water content of a rigid porous material can now conveniently and accurately determine the K-θ relation. However, the establishment of such a measurement facility will probably remain a specialized research tool for some time. There is therefore merit in the proposal for establishing yet another method of measuring K-θ which utilizes, if possible, the simplest of experimental techniques combined with a computer-executed numerical analysis. Whisler and Watson (Ref. 6) have suggested such a method. In the method a saturated column of the porous material draining to a water table is used. With such a system there are three pieces of experimental information which can be obtained easily without recourse to complex measuring equipment. These are the saturated hydraulic conductivity (K_{sat}), the discharge (Q)-time (t) curve and the pressure head (h)-water content (θ) relationship measured by sectioning the column at equilibrium. With this information it is a simple matter to draw possible K-θ curves using as a fixed point the measured K_{sat} and θ_{sat} values, and then to obtain the respective Q-t curves from a suitable computer solution of the differential flow equation. These are then matched against the measured Q-t relationship to obtain the K-θ curve giving the best fit. This method is in contrast to methods such as those of Childs and Collis-George (Ref. 2), Millington and Quirk (Ref. 7) or Brutsaert (Ref. 8) where empirical formulae are used to calculate the K-θ relation from the h-θ relation. Even when these latter methods are modified by fitting constants as suggested by Jackson et al (Ref. 9) they do not have the inherent advantage of comparing the predicted outflow with the experimentally determined values. This paper studies the feasibility of the proposed method in detail using two sands and also discusses questions arising in the analysis, such as the effect of not using a dynamically determined moisture characteristic and the sensitivity of the method to small changes in the geometry of the K-θ curve.

Basic to the whole approach in this paper is the confidence that the numerical analysis used predicts accurately the pressure head and water content changes occurring in a one-dimensional gravity drainage system. This confidence is based on extensive testing of the analysis against several sets of available data as given by Whisler and Watson (Ref. 6).

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*Paper No. 2611, presented at the Third Australasian Conference on Hydraulics and Fluid Mechanics held in Sydney from 25th to 29th November, 1968.

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2.—NUMERICAL ANALYSIS

The numerical analysis has been previously detailed by Whisler and Watson (Ref. 6) and in this paper an outline only will be given.

The equation for isothermal vertical flow in a homogeneous, non-compressible porous material is

$$C(h)\,\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(K(h)\,\frac{\partial h}{\partial z}\right) + \frac{\partial K(h)}{\partial z} \qquad \text{.................(1)}$$

where h is the pressure head of water in the porous material measured in centimetres of water (negative for unsaturated conditions), t is the time from the start of drainage, z is the vertical dimension defined as positive in the upward direction and $z = 0$ at the soil surface. The volumetric water capacity is $C(h) = \frac{d\theta}{dh}$ where θ is the volumetric water content. The hydraulic conductivity $K(h)$ is expressed as a function of h in the analysis although as input data it is presented in terms of $K - \theta$.

The upper and lower boundary conditions for the draining system are respectively

$$-\,K(h)\left(\frac{\partial h}{\partial z} + 1\right)_{z=0} = 0 \qquad t > 0 \qquad \text{..............(2)}$$

$$h(z, t)_{z=-L} = 0 \qquad t > 0 \qquad \text{..............(3)}$$

where L is the length of the column.

To solve Eq. (1) numerically a grid of points was superimposed on the region $t \geqslant 0$, $-L \leqslant z \leqslant 0$. The z axis was divided into N intervals ($N = 100$). The mesh points are defined by

$$t_m = m\,\Delta t \qquad m = 0, 1, 2, \ldots \qquad \text{.................(4)}$$

$$z_n = -(N - n + 1)\,\Delta z \qquad n = 1, 2, \ldots, N + 1 \qquad \text{.......(5)}$$

$$\Delta z = \frac{L}{N} \qquad \text{..................................(6)}$$

The partial derivatives in Eq. (1) were approximated by the finite differences

$$\frac{\partial}{\partial z}\left[K(h)\,\frac{\partial h}{\partial z}\right] \approx \frac{1}{2(\Delta z)^2}\,[K_{n+\frac{1}{2},\,m-\frac{1}{2}}\,(h_{n+1,\,m} + h_{n+1,\,m-1} - h_{n,\,m} - {} \\ - h_{n,\,m-1}) - K_{n-\frac{1}{2},\,m-\frac{1}{2}}\,(h_{n,\,m} + h_{n,\,m-1} - h_{n-1,\,m} - h_{n-1,\,m-1})] \quad (7)$$

$$\frac{\partial h}{\partial t} \approx \frac{h_{n,\,m} - h_{n,\,m-1}}{\Delta t} \qquad \text{...........................(8)}$$

$$\frac{\partial K(h)}{\partial z} \approx \frac{K_{n+\frac{1}{2},\,m-\frac{1}{2}} - K_{n-\frac{1}{2},\,m-\frac{1}{2}}}{\Delta z} \qquad \text{.....................(9)}$$

where arbitrarily

$$K_{n+\frac{1}{2},\,m-\frac{1}{2}} = \frac{K_{n,\,m-1} + K_{n+1,\,m-1} + K_{n+1,\,m} + K_{n,\,m}}{4} \qquad \text{.........(10)}$$

and a similar definition is true for $K_{n-\frac{1}{2},\,m-\frac{1}{2}}$.

The finite difference approximations Eqs. (7) through (10) were substituted in Eq. (1) to obtain a set of $N - 3$ algebraic equations of the form

$$E_n\,h_{n-1,\,m} - F_n\,h_{n,\,m} + G_n h_{n+1,\,m} = -\,H_n \qquad \text{..............(11)}$$

When Eq. (11) is applied at $n = 2$ and the boundary condition Eq. (3) is invoked the result is

$$-\,F_2 h_{2,\,m} + G_2\,h_{3,\,m} = H_2 \qquad \text{....................(12)}$$

At $n = N$ with boundary condition Eq. (2), the result is

$$E_n\,h_{N-1,\,m} - (F_N - G_N)h_{N,\,m} = -\,(H_N - G_N\,\Delta z) \qquad \text{......(13)}$$

Eqs. (11), (12) and (13) constitute a set of $N - 1$ algebraic equations in $N - 1$ unknowns. Additional details on the method of solution are given in Whisler and Watson (Ref. 6).

3.—EXPERIMENTAL METHOD AND RESULTS

The acrylic column used in the experimental work was 5.75 cm. internal diameter and was built up of sections 1 cm. and 2 cm. in thickness. The sections were made waterpoof at their junction faces by using O-ring seals. When the full column length of 85 cm. was built up, the sections were lightly cramped together using longitudinally positioned screwed rods. The screen unit at the base of the column was constructed in a similar manner to that described by Watson (Ref. 10). Two clean, reasonably uniform sands were used in the experiment, these being designated *G*1 and *R*8 by the supplier (Processed Sand Pty. Ltd.)*. Sand *G*1 lay within the sieve range 500 microns to 150 microns, whilst the finer sand *R*8 passed the 250 micron sieve and was retained on the 150 micron sieve.

A completely saturated column of the sand was formed and the saturated hydraulic gradient (K_{sat}) determined. At 23° centigrade K_{sat} was measured for sands *G*1 and *R*8 as 2.214 cm./min. and 1.220 cm./min. respectively. The drainage of the saturated column was then commenced with the outflow being measured by piping it into a container placed on the pan of an automatic balance. Finally, when it was judged that the column had reached an equilibrium condition it was rapidly sliced and the gravimetric water content determined. From these readings the volumetric water content was then calculated and plotted against the height above the water table to give the h-θ relationship.

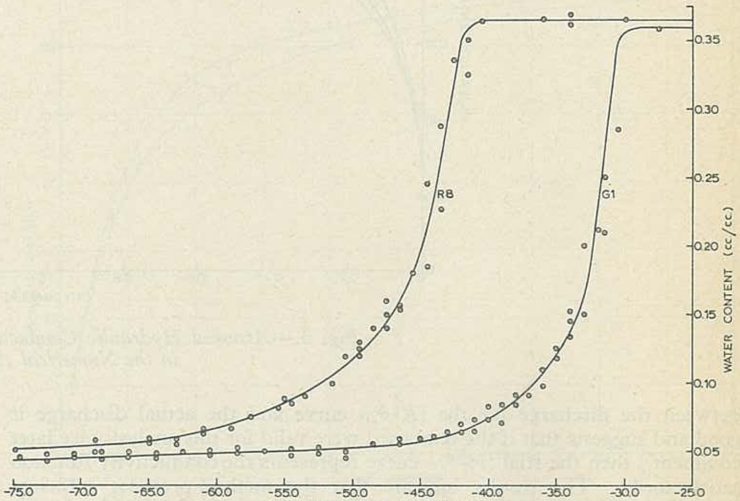

Fig. 1.—Pressure Head-Water Content Relationships for Sands G1 and R8 Determined by Sectioning the Column at Equilibrium.
The circled dots represent the experimental readings.

The h-θ curves for both sands are given in Fig. 1. It may be noted that the air entry values have been taken as − 29.0 cm. and − 40.0 cm. The line of demarcation between the saturated and partly saturated zones in the drained condition was somewhat diffuse, as represented by the rounded portion of the h-θ curve in the vicinity of the air entry value.

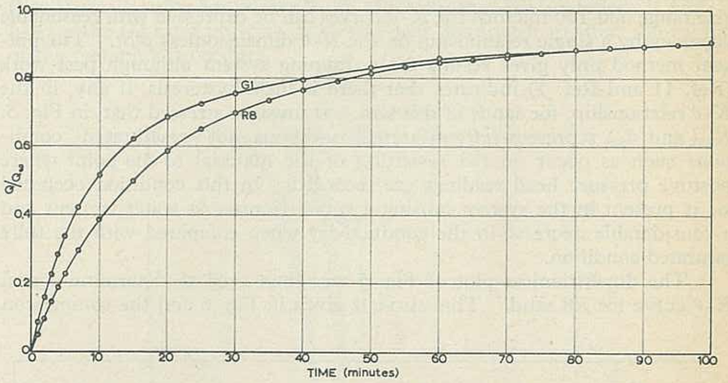

Fig. 2.—Discharge-Time Curves for Sands G1 and R8 in terms of Q/Q_∞.
The circled dots represent the experimental readings.

In Fig. 2 the discharge-time curves for both sands are plotted in the form $Q/Q_\infty - t$. Here Q_∞ represents the total outflow from the column at equilibrium.

4.—NUMERICAL RESULTS AND DISCUSSION

Fig. 3 gives three trial K-θ curves (designated A, B, C) used in the numerical analysis of sand *G*1. Curve A is linear from the θ_{sat} value of 0.36 cc./cc. to 0.23 cc./cc. From past experimental records this would seem a rather unlikely condition and, accordingly, represents a reasonable lower extreme. The computed discharge-time relationships are given in Fig. 4 in the form $Q/Q_\infty - t$ together with the actual discharge curve. The correspondence

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*Trade names and company names are included for the benefit of the reader and do not infer endorsement or preferential treatment of the product listed by the U.S. Department of Agriculture.

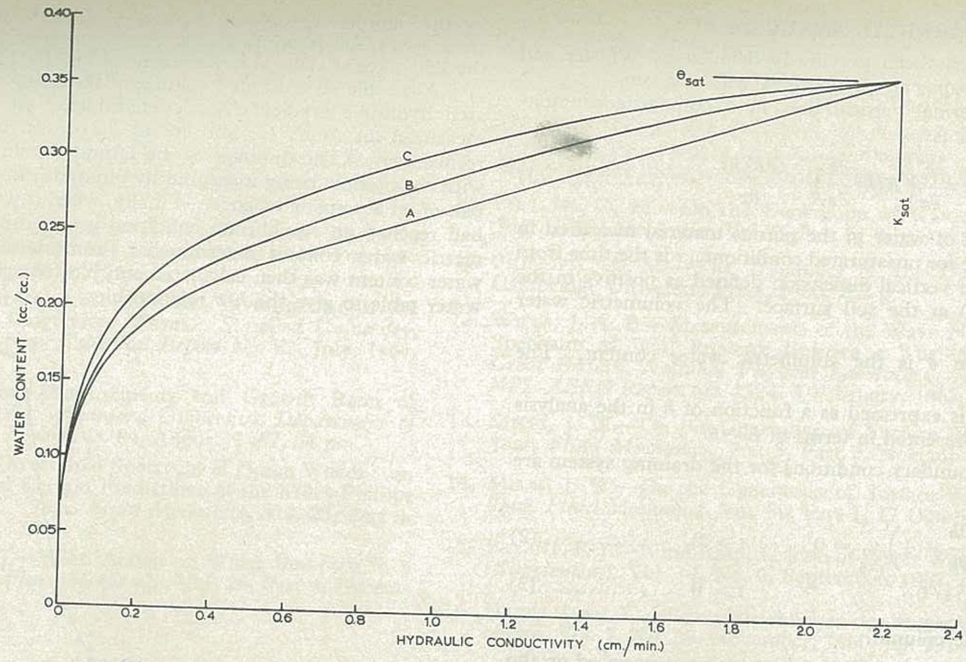

Fig. 3.—Assumed Hydraulic Conductivity-Water Content Relationships used in the Numerical Analysis of Sand G1.

between the discharge for the $(K\text{-}\theta)_C$ curve and the actual discharge is good and suggests that if the data used were valid for this analysis (see later comment), then the trial $(K\text{-}\theta)_C$ curve represents the conductivity function satisfactorily. The results indicate that the method provides sufficient sensitivity for reasonably small changes in the $K\text{-}\theta$ curve to be reflected in the computed $Q\text{-}t$ curve.

The established $(K\text{-}\theta)_C$ curve was then plotted in dimensionless form in terms of K_{sat} and θ_{sat} and compared with a similar dimensionless plot (Ref. 1) for two sands (no. 17 sand and Banding Sand) of the Ottawa Silica Co., U.S.A. For these sands the entrapped air method had been used to find $K\text{-}\theta$ and the saturated conductivities had been measured as 1.630 cm./min. and 0.475 cm./min. respectively. The nature of the entrapped air method only allowed the dimensionless curve to be drawn down to a K/K_{sat} value of 0.10. These dimensionless plots are given in Fig. 5. As Fig. 5 reveals they are almost identical and this suggests the tentative conclusion that for reasonably uniform and generally rounded sands in the size range 500-100 microns the $K\text{-}\theta$ curves can be expressed with reasonable accuracy by a single relationship on the $K\text{-}\theta$ dimensionless plot. The present method only gives results for a draining system although past work (Ref. 11 and Ref. 1) indicates that there is little hysteresis, if any, in the $K\text{-}\theta$ relationship, for sands of this size. It must be stressed that, in Fig. 5, K_{sat} and θ_{sat} represent *fully* saturated conditions, not 'resaturated' conditions such as occur on the rewetting of the material to the point where positive pressure head readings are recorded. In this condition occluded air is present in the system causing a small decrease in water content and a considerable decrease in the conductivity when compared with the fully saturated condition.

The dimensionless plot of Fig. 5 was then used to determine a trial $K\text{-}\theta$ curve for $R8$ sand. This curve is given in Fig. 6 and the comparison between the computed and actual discharges in Fig. 7. The comparison is very good particularly at the shorter times and lends further support to the general usefulness of the dimensionless plot. There is a variation between the computed and actual curves in Fig. 7 beyond a time of approximately 40 min. A study of pressure head water and content profiles at this time showed that it corresponded approximately with the arrival of the drainage front at the air entry value elevation. Further drainage beyond this time would then occur at much reduced water contents over the entire

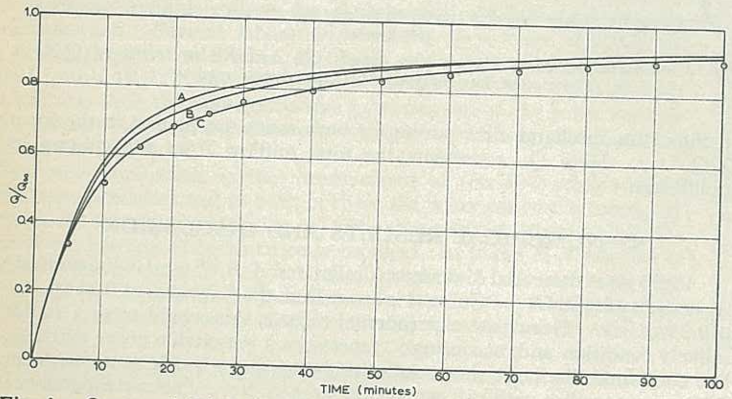

Fig. 4.—Computed Discharge-Time Curves in Terms of Q/Q_∞ for Sand G1 using the $K\text{-}\theta$ Curves Given in Fig. 3.
The circled dots represent values taken from the actual discharge curve given in Fig. 2.

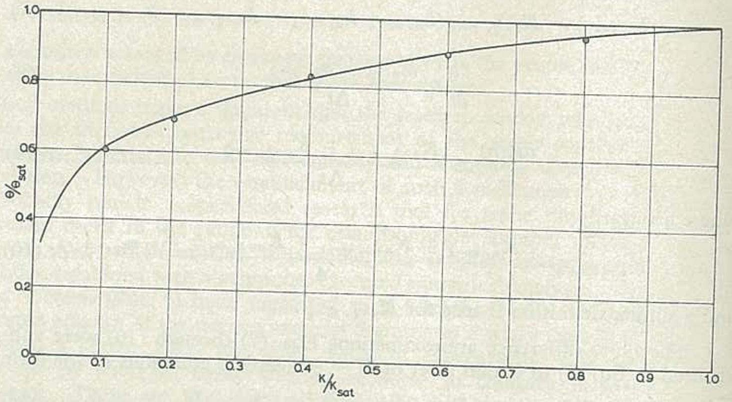

Fig. 5.—Dimensionless Plot of Conductivity Function in Terms of K/K_{sat} and θ/θ_{sat}.
The circled dots represent values from a similar dimensionless plot as given in Ref. 1.

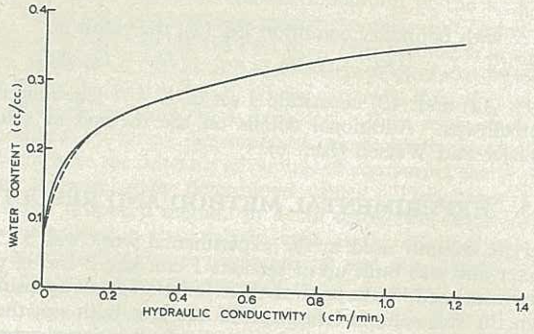

Fig. 6.—Hydraulic Conductivity-Water Content Relation for Sand R8 Determined from the Dimensionless Plot of Fig. 5.
The dashed curve represents the small change introduced into the relationship to test the sensitivity of the matching procedure at longer times.

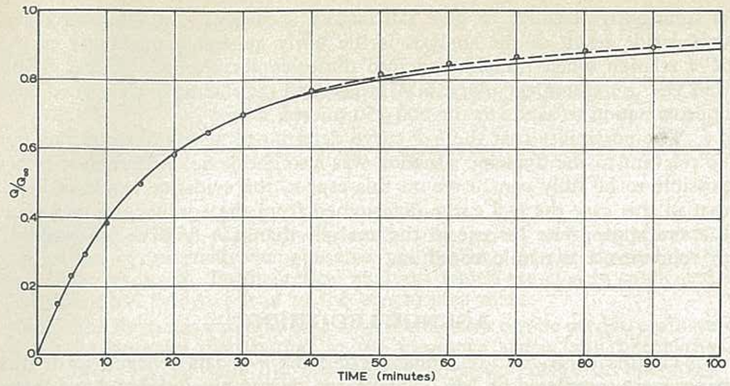

Fig. 7.—*Computed Discharge-Time Curves in Terms of Q/Q_∞ for Sand R8 using the K-θ Curves Given in Fig.* 6.

The dashed section of the curve corresponds to the small change introduced into the K-θ curve. The circled dots represent values taken from the actual discharge curve given in Fig. 2.

drainable section. To test the sensitivity of the method to changes in the K-θ curve at the lower water contents and to attempt to improve the correlation between the computed and actual curves, a small change in K-θ was made. The change is shown in Fig. 6 but can be appreciated more readily when plotted on semi-log graph paper. The induced change in K-θ produced the type of change desired in the Q-t curve and brought the computed curve closer to the actual curve at these times (see Fig. 7). The effect of this change on Q-t for the first 40 min. of drainage was negligible.

Following this it could be expected that small changes in the $(K\text{-}\theta)_C$ curve for $G1$ sand at the lower water contents would likewise improve the correlation at the longer times; however, this test was not carried out. The dimensionless plot covers the K/K_{sat} range from 1.0 to 0.01. The $\theta/\theta_{\text{sat}}$ value corresponding to the lower limit of the range is 0.30.

Although the above has shown that it is possible to determine a K-θ curve from matched discharge curves and indirect evidence to its validity has been provided by the consistency of the dimensionless plot, there is still need for discussion on the relevance of the h-θ data used. Topp et al (Ref. 12) and Sedgley (Ref. 13) have reported that different h-θ relationships are obtained depending on whether measurements are made during equilibrium, steady or unsteady state conditions. In general the results of these workers indicate that the h-θ curve determined during unsteady flow has a smaller pressure (i.e., a more negative value of h) for any particular water content than the relation obtained from a static equilibrium condition. The experiments were carried out with a small cell of the porous material rather than in a full scale column system. Insufficient research has been conducted at this stage either to explain the phenomenon completely satisfactorily or to define the extent of expected variations for different soils. However, sufficient evidence of differences in h-θ for the equilibrium and unsteady state conditions is available to warrant some discussion of it in regard to the present method.

It is clear that as equilibrium is approached the h-θ curve used in the analysis is valid. However, it is not clear how valid the curve is during the early stages of drainage, and what effect any unknown variations would have on the matching procedure. To check this point an assumed h-θ curve differing from the equilibrium curve by an amount approximating that reported by Sedgley (Ref. 13) was drawn for sand $G1$. This is given in Fig. 8 with the equilibrium curve included for comparison. The numerical analysis was repeated using this curve for the pressure head data and using the $(K\text{-}\theta)_A$ and $(K\text{-}\theta)_C$ curves. Under these conditions it was not possible to use the Q/Q_∞ plot since the Q_∞ values were different for each h-θ curve due principally to the different air entry values. This problem was overcome by using, for comparative purposes, the actual discharge in terms of cm.^{3} of water/cm.^{2} of column area. The Q-t curves are given in Fig. 9 for the assumed h-θ curve. It is apparent that reasonable matching can only be obtained if the $(K\text{-}\theta)_A$ curve is used.

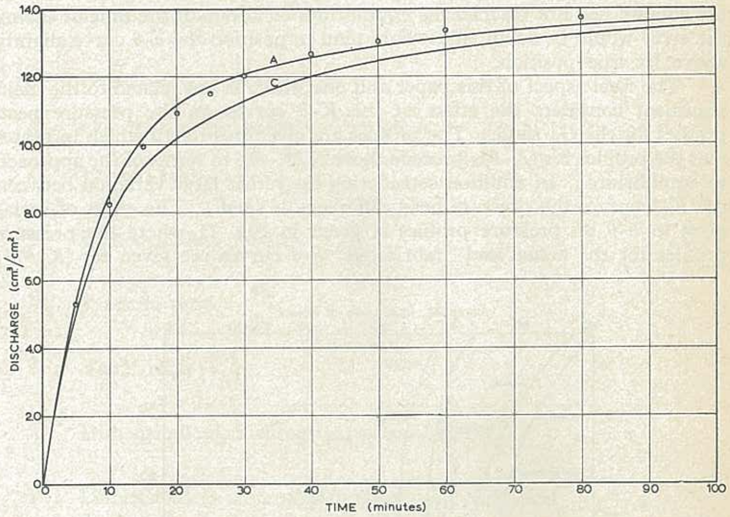

Fig. 9.—*Computed Discharge-Time Curves for Sand G*1 *using the 'Fabricated' h-θ Curve and Conductivity Relationships A and C of Fig.* 3.

The circled dots represent values of the actual discharge.

Since the $(K\text{-}\theta)_A$ curve has a geometry unlike published K-θ relationships due to its linear region, and since this shape is required if correspondence is to be attained, it is reasonable to assume that the 'fabricated' h-θ curve does not represent the pressure head-water content relationship at the commencement of drainage. This analysis suggests that for $G1$ sand the equilibrium h-θ curve must represent the unsteady h-θ curve very closely if unacceptable geometries of K-θ are to be avoided. It is not possible to be more conclusive on this point at this stage; however, it can be stated that there is sufficient evidence to conclude that the variation, if any, in h-θ for the $G1$ sand must be small and does not appear to affect the validity of the matching procedure.

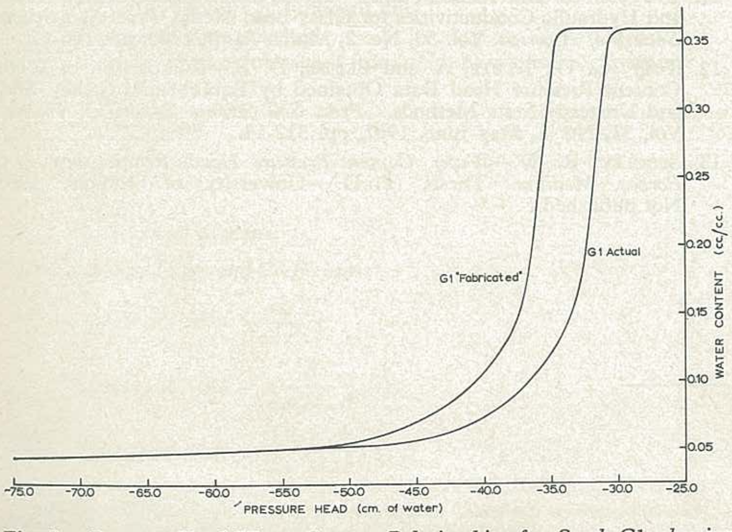

Fig. 8.—*Pressure Head-Water Content Relationships for Sand G*1 *showing the Actual and 'Fabricated' Curves.*

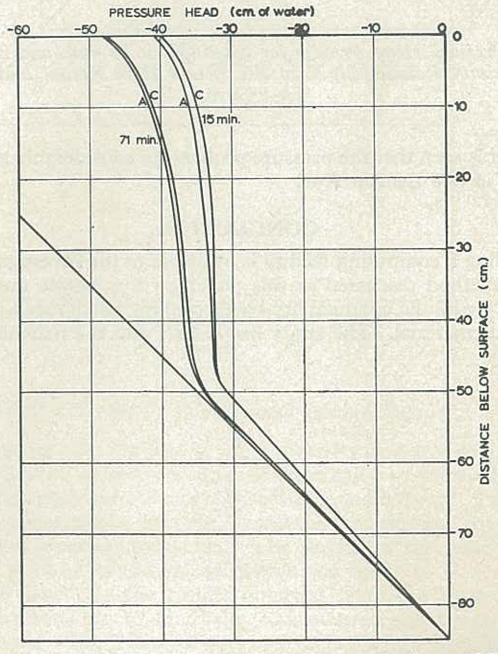

Fig. 10.—*Pressure Head Profiles for Sand G*1 *at* 15 *min. and* 71 *min. using the Actual h-θ Curve and Conductivity Relationships A and C of Fig.* 3.

Since pressure measurements are not made in this experiment it could be objected that the h-θ curves might be in error if equilibrium had not been reached at the time of slicing. This point was satisfactorily covered in this work by waiting for a period well in excess of the equilibrium time as measured for similar porous materials. However, in general, for reasonably short columns the position is not as critical as it may first appear. A column approaches the equilibrium condition of a linear pressure profile from the 'bottom up', that is the zone near the static drainage front, which is the most critical from the h-θ curve viewpoint, moves to the equilibrium position first and often quite rapidly. Above this narrow zone of small pressure change but large water content change, the h-θ curve gradually adopts the characteristic 'flat' shape in which there is a small water content change for a large pressure change. Accordingly, if a zone at the top of the column had not reached the equilibrium condition at the time of slicing the error would be small, and would tend to position the h-θ curve slightly above its true position.

The final aspect of this paper and one which is not related to the main argument considers the effect of the K-θ curves on the pressure head profiles for the $G1$ sand. The profiles are given in Fig. 10 which indicates that the profiles for $(K\text{-}\theta)_A$ precede those of $(K\text{-}\theta)_C$ in terms of the approach to equilibrium. In addition considering the rather large variation between the K-θ curves the pressure head difference is small. The effect of variations in h-θ on pressure profiles is given in Fig. 11 where the pressure profiles for the actual and 'fabricated' h-θ curves are given for $(K\text{-}\theta)_C$.

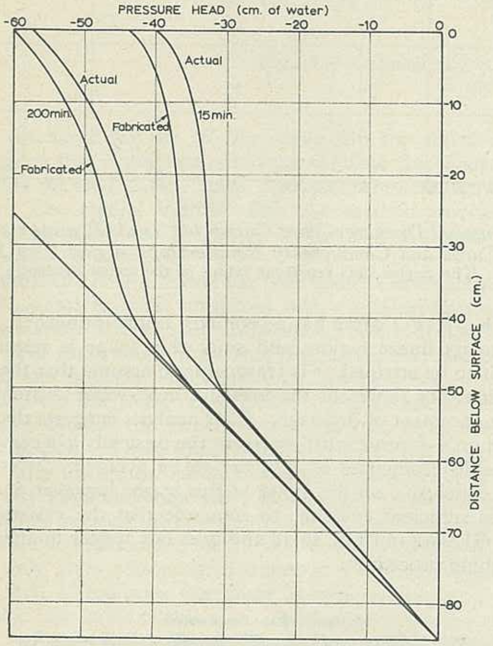

Fig. 11.—*Pressure Head Profiles for Sand G*1 *at* 15 *min. and* 200 *min. using the Conductivity Relationship C of Fig.* 3 *and Both Actual and 'Fabricated' h-θ Curves.*

From this it is seen that the pressure profiles are considerably more sensitive to changes in h-θ than in K-θ.

CONCLUSION

Providing a computing facility is available to the investigator the semi-numerical method discussed in this paper gives a simple means of determining the hydraulic conductivity-water content relationship of an unsaturated porous material. The study has shown that the matching procedure is sufficiently sensitive to give satisfactory accuracy. An interesting and useful side result of the analysis is the fairly general applicability of the K-θ relation when transformed into dimensionless terms. It was found that the dimensionless plot could be applied satisfactorily as a good first approximation to sands in the 500-150 micron range.

The possibility that the h-θ curve determined at equilibrium may not be relevant to the dynamic situation was also checked. Although it is not possible to be fully conclusive on this aspect, the evidence would indicate that in this case the h-θ curve determined from the equilibrium condition is more appropriate for use in the analysis than a h-θ curve 'fabricated' to represent a dynamic condition.

ACKNOWLEDGMENT

The discharge-time relationships and the h-θ data referred to in this paper were completed by Mr. J. Stevenson during his undergraduate thesis work at The University of New South Wales. The authors thank him for his contribution to this study.

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