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An Evaluation of the First-Order Approximation to the Direct Supersonic Blunt Body Problem

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Summary.—The first-order solution to the direct supersonic blunt body problem, using the method of Integral Relations, has been studied in order to evaluate the soundness of the approximation. Typical results are presented.

LIST OF SYMBOLS

Symbol:

c	Dimensionless sonic velocity.
C_p	Pressure coefficient, Fig. 11.
f	A function and its derivatives.
I	An integral.
j	Operator, $j = 0$ in plane flow, $j = 1$ in axisymmetric flow.
M	Mach number.
p	Dimensionless pressure.
r	Radial co-ordinate, Fig. 2.
R	Body radius of curvature, Fig. 2.
s, n	Body orientated curvilinear co-ordinates, Fig. 2.
v	Dimensionless velocity component.
V	Dimensionless velocity.
x, y	Cartesian co-ordinates, Fig. 10.
γ	Ratio of specific heats.
δ	Shock detachment distance, Fig. 2.
θ	Body angle, Fig. 2.
$\varkappa$	$(\gamma - 1)/2\gamma$.
μ	$1 + (\delta/R)$.
ξ	n/δ.
ρ	Dimensionless density.
τ	Streamline density function, Section 2.
χ	Shock wave angle, Fig. 2.

Subscript:

s	Component in s-direction.
n	Component in n-direction.
0	Surface value.
1 or δ	Shock value.
∞	Free-stream value.
χ, v_{s0}	Denotes type of coefficient, e.g., C_χ is a coefficient in the continuity equation of the term $d\chi/ds$, see Section 3.

Superscript:

*	Indicates sonic value.

Computer Variables of Tables Ia and Ib:

DELTO	δ_0, Stagnation point shock standoff distance, dimensionless.
GATE	Extrapolation gate, as a percentage of sonic velocity.
STEP	Integration step size, dimensionless.
SLAMDA	λ, Radians subtended at centre of sphere by arc length s.
DELTA	δ, Dimensionless shock standoff distance.
CHI	χ, Shock wave angle.
VSO	v_{s0}, Dimensionless surface velocity.
PO	p_0, Dimensionless surface static pressure.
PNE	p_{NE}, Dimensionless Newtonian surface pressure.
VS, VN	v_s, v_n, Dimensionless local velocity components.
P	p, Dimensionless local static pressure.
RHO	ρ, Dimensionless local density.
N/DELTA	$n/\delta = \xi$.

1.—INTRODUCTION

A blunt body moving at supersonic speed through a gas, supports an oblique, curved shock wave which is detached from the nose of the body. Given the body contour and free stream gas conditions, the direct problem is to find the shape and position of this shock, and gas conditions in the shock layer between body and shock and on the surface of the body. A detailed survey of the problem and of various methods of solution is given in Hayes and Probstein (Ref. 10).

The principal features of flow are shown in Fig. 1. Behind the curved shock the flow field is both rotational and compressible. All variables immediately behind the shock are functions of the wave angle (see Fig. 2).

The integral method for solving first-order partial differential equations of the mixed type was first proposed by Dorodnitsyn (Ref. 1) and termed " The Method of Integral Relations ". The first application of the method to solve the blunt body problem was by Belotserkovskii (Refs. 2, 3 and 4). He divided the shock layer into N strips to yield a set of N integral equations and demonstrated that as N was increased the numerical solution rapidly converged to that obtained by experiments. The numerical procedure calls for an iteration on guesses for starting values of the integration which must satisfy a singularity condition at the sonic position. Belotserkovskii has since extended the method to cover real gas flows, viscous effects and bodies at incidence and has published extensive tables (Refs. 2, 3, 4, 6 and 9). From time to time he has published important reviews (Refs. 7, 8 and 9).

No other worker outside the U.S.S.R. and China has used more than a first-order (one-strip) approximation, presumably because the increased accuracy yielded by the multi-strip solutions is insufficient to justify the increase in time needed to solve the equations. Some of Belotserkovskii's tabulated solutions are given to the second approximation only, and above Mach number 10 he considers that a first-order result is sufficiently accurate (Ref. 8, p. 845).

All of the following results are first-order (or one-strip) solutions. In the United States the first application of the method was to symmetric sharp-cornered bodies. The flat plate, flat faced cylinder and spherical caps were studied by Chubb (Ref. 12), Holt (Refs. 13 and 15), Gold and

*Paper No. 2620, presented at the Third Australasian Conference on Hydraulics and Fluid Mechanics held in Sydney from 25th to 29th November, 1968.

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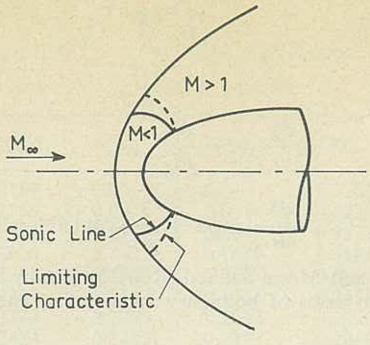

Fig. 1.—Flow Field.

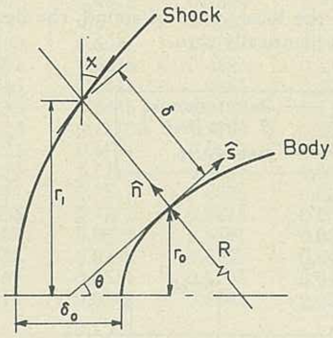

Fig. 2.—s-n Co-ordinate System.

Holt (Ref. 14). Holt and Hoffmann (Ref. 16) also treated ellipsoids. Except for flat-nosed bodies for which Cartesian co-ordinates were used, all these workers had followed Belotserkovskii and used polar co-ordinates even for body shapes far removed from circular. Chenoweth (Ref. 31) points out that this can lead to additional singularities and that this is more likely to occur in two-dimensional plane flow than axisymmetric. Traugott (Ref. 17) first proposed the " *s-n* ", " body oriented " or " boundary layer " type of co-ordinate system now used by most workers for bodies of general shape. Using these co-ordinates, Traugott (Ref. 18) has examined the validity of the first-order theory for a variety of blunted-cone nose shapes throughout the subsonic and supersonic flow field. He has defined a region on a Mach number-cone angle diagram in which the theory may be used. Archer has examined elliptic noses (Refs. 19, 20 and 21).

Xerikos and Anderson (Ref. 27) have studied the computational problems of the one-strip solution. Their results demonstrate the nature of the instability of the solution near the sonic singularity. Kentzer (Refs. 28 and 29) has commented on this problem and interprets the singularity as the left characteristic from the sonic point (Fig. 1). Kao (Ref. 30) has found that by retaining one of the dependent variables as the mass flux along the surface instead of transforming to the physical variable for surface velocity the sonic singularity is replaced by an algebraic maximum. This simplifies but does not eliminate the numerical difficulties at the sonic point (Ref. 26).

Kao's transformation is reminiscent of that used by Melnik (Ref. 35) on an elliptic cone at incidence. He retained all of the dependent variables as mass and momentum fluxes. This holds promise for easier manipulation of the algebra since the computer itself can be relied on to perform the matrix inversion required to obtain the solution in terms of physical variables. Van Dyke (Ref. 32) has recently suggested that other methods be used to overcome the " villain of convergence ".

Vaglio-Laurin (Ref. 24) was the first to consider a blunt asymmetric nose shape. He eliminated the need for numerical iteration by perturbing both dependent and independent variables. However, the equations are extremely tedious. Also, Brong and Leigh (Ref. 25) have criticised Vaglio-Laurin's arbitrary assumption about the location of the maximum entropy streamline and have offered a rational approach to overcome this difficulty. This has been used by Bailey (Ref. 26) who also adopted Kao's technique at the sonic point.

In the course of studies of the application of the integral method to the flow about pointed cones, South and Newman (Refs. 36 and 37) have evolved a criterion for maximum allowable integration step size when performing numerical integration in the supersonic regime. This is particularly interesting because they have shown that results there can be expected to be as good as a characteristic solution. Traugott (Ref. 18) has also commented on this. Archer (Refs. 20 and 21) has compared experiment and the one-strip Belotserkovskii solution in the supersonic regime on an elliptic nose and found favourable agreement at Mach number 7.

Belotserkovskii's method has been used for solutions to three-dimensional flows (Refs. 8 and 9). Circular cones at incidence and elliptic cones were solved first. Chushkin and Shchennikov (Ref. 34) obtained results for angles of incidence less than 10°. Melnik (Ref. 35), by contrast, did not assume constant entropy on the cone surface. Both results are one-strip solutions and are close at zero incidence. Belotserkovskii reports a solution for an ellipsoid of revolution at 5° incidence (Ref. 9).

Several authors have extended the integral method to include real gas effects for the flow about circular, elliptic, spherical and ellipsoidal noses (Refs. 9, 38, 39, 40, 41, 42, 43 and 44). These differ in whether or not equilibrium (Refs. 9, 40, 41, 42 and 43) or non-equilibrium (Refs. 9, 38, 39 and 44) effects are treated and in the way dependent variables are defined. Of importance is the finding of Xerikos and Anderson in Ref. 42 and used in Ref. 43 that entropy changes must be included in a formulation of the continuity equation for real gas flow.

All workers other than in the U.S.S.R. group have worked in one-strip. The work of Yalamanchili (Ref. 38), Hermann and Thoenes (Ref. 39) is a noteworthy and successful approach to describe both equilibrium and non-equilibrium effects by a simplified but rational air model. They consider only two components—nitrogen and oxygen—and allow only for oxygen dissociation. The only empirical data required is the oxygen dissociation rate constant. Belotserkovskii (Ref. 9) reports equilibrium solutions using an empirical but accurate fit to the data for air dissociation. He uses an oxygen dissociation model for non-equilibrium solutions. He also gives a solution for an air model of oxygen and nitrogen although the first to do this were evidently Shih et al (Ref. 44). Springfield (Ref. 45) has extended their work to include vibrational as well as chemical relaxation on arbitrary body shapes.

In this paper a report of a study to examine the validity of the first-order perfect gas solution is presented. A computer program of the solution is now available for axisymmetric and cylindrical bodies of arbitrary shape at zero incidence and at any free-stream Mach number. Program printout presents shock wave, body surface and shock layer data.

2.—EQUATIONS

The steady flow equations expressing conservation of mass, momentum and energy for a perfect, inviscid gas respectively are

$$\nabla \cdot \rho \vec{V} = 0 \quad \ldots\ldots(2.1)$$

$$\nabla \frac{V^2}{2} + \frac{1}{\rho} \nabla\, \varkappa p = \vec{V} \times \text{curl}\, \vec{V} \quad \ldots\ldots(2.2)$$

$$\frac{p}{\rho} + V^2 = 1 \quad \ldots\ldots(2.3)$$

where $\varkappa = \dfrac{\gamma - 1}{2\gamma}$.

Velocity, pressure and density have been non-dimensionalised through use of the maximum free-stream velocity and the free-stream stagnation pressure and density.

Re-arranging the equations gives

$$\nabla \left(\ln \frac{p}{\rho^\gamma} \right) = \frac{-\gamma(\gamma - 1)}{c^2} \vec{V} \times \text{curl}\, \vec{V} \quad \ldots\ldots(2.4)$$

and therefore

$$\vec{V} \cdot \nabla \left(\frac{p}{\rho^\gamma} \right) = 0 \quad \ldots\ldots(2.5)$$

i.e., $\dfrac{p}{\rho^\gamma}$ is a function of vorticity and is constant along a streamline.

In the (s, n) co-ordinate system (Fig. 2) for symmetric plane ($j = 0$) or axisymmetric flow ($j = 1$), Eqs. (2.1) and (2.2) become non-linear, first-order partial differential equations. These may be put into " divergence " form and transformed into ordinary differential-integral equations by integrating with respect to n.

Using Leibnitz's rule, there results the integral flow equations:

Continuity:

$$\frac{d}{ds} I_1 + (r^j h)_\delta \left(1 + \frac{\delta}{R} \right) - (r^j g)_\delta \frac{d\delta}{ds} = 0 \quad \ldots\ldots(2.6)$$

n-momentum:

$$\frac{d}{ds} I_2 + (r^j H)_\delta \left(1 + \frac{\delta}{R} \right) - (r^j H)_0 - (r^j P)_\delta \frac{d\delta}{ds} = I_3 \quad \ldots\ldots(2.7)$$

From geometry of the shock:

$$\frac{d\delta}{ds} = \left(1 + \frac{\delta}{R}\right) \cot(\theta + \chi) \qquad \text{.................(2.8)}$$

where—

$$I_1 = \int_0^\delta (r^j g)_s\, dn$$

$$I_2 = \int_0^\delta (r^j P)_s\, dn$$

$$I_3 = \int_0^\delta (G)_s\, dn \qquad \text{..........................(2.9)}$$

the new dependent variables g, h, G, H and P are mass and momentum fluxes and are defined as:

$$g = \tau v_s; \qquad h = \tau v_n; \qquad P = \rho v_s v_n$$

$$G = \frac{r^j}{R}(\varkappa p + \rho v_s^2) + j\left(1 + \frac{n}{R}\right) \varkappa p \cos\theta; \qquad H = \varkappa p + \rho v_n^2$$

where $\tau = (1 - V^2)^{1/(\gamma-1)}$ and represents density along a streamline. This is interchangeable with ρ in the continuity Eq. (2.1) through use of Eq. (2.5).

The integral I_1 represents the mass flow across any section and I_2, I_3 the n, s direction momentum flow rates respectively. The s momentum equation vanishes on the stagnation streamline and hence is redundant.

To solve the integral equations it is necessary to assume some form for the three integrands, consistent with physical reasoning and the known boundary values. A linear approximation leads to the first-order equations.

3.—FIRST-ORDER APPROXIMATION

The first-order equations are obtained by approximating the integrands in the integral flow equations by linear functions, i.e.:

$$r^j g = (r^j g)_0 + \frac{n}{\delta}[(r^j g)_\delta - (r^j g)_0] \qquad \text{...............(3.1)}$$

$$r^j P = \frac{n}{\delta}(r^j P)_\delta; \qquad P_0 = 0 \qquad \text{.....................(3.2)}$$

$$G = G_0 + \frac{n}{\delta}(G_\delta - G_0) \qquad \text{.....................(3.3)}$$

Any integral $I = \int_0^\delta f\, dn$, Eq. (2.9), then becomes $I = \frac{\delta}{2}(f_0 + f_1)$ and the integral equations reduce to the following three coupled, first-order, linear differential equations in δ, χ, v_{s0} and which can be integrated by standard numerical methods:

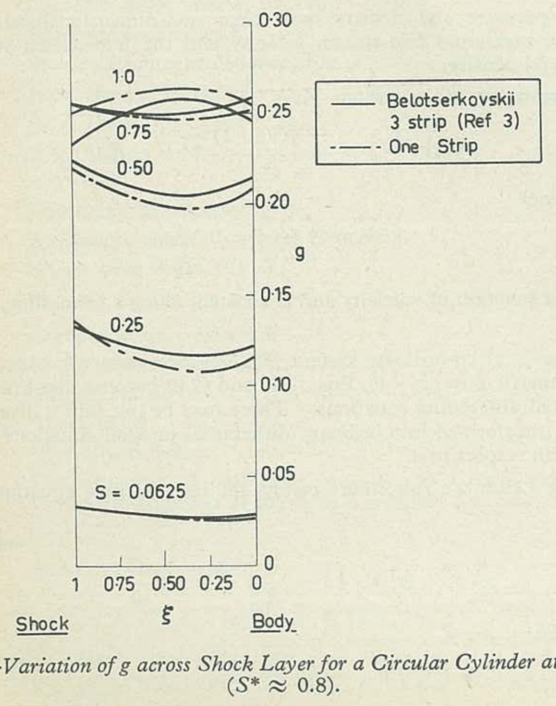

Fig. 3.—Variation of g across Shock Layer for a Circular Cylinder at $M_\infty = 5$ ($S^* \approx 0.8$).

$$\frac{d v_{s0}}{ds} = -\frac{C_\chi \frac{d\chi}{ds} + C_0}{C_{v_{s0}}} \qquad \text{........................(3.4)}$$

$$\frac{d\chi}{ds} = -\frac{M_0}{M_\chi} \qquad \text{........................(3.5)}$$

$$\frac{d\delta}{ds} = \left(1 + \frac{\delta}{R}\right)\cot(\theta + \chi) \qquad \text{...............(3.6)}$$

The coefficients C and M are defined (Ref. 23) first in terms of g, h, G, H and P and then in terms of boundary values of v_s and v_n, δ, χ, θ, M_∞ and γ.

3.1 Integration Procedure:

At the start of integration δ_0, the stagnation point shock standoff distance, is unknown.

When $v_{s0} = c_0{}^*$ the local speed of sound, the denominator of dv_{s0}/ds in Eq. (3.4) becomes identically zero.

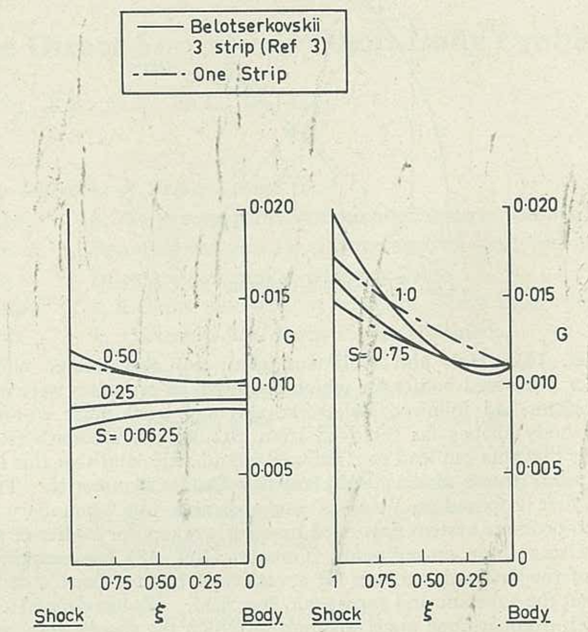

Fig. 4.—Variation of G ($= kp + \rho v_s^2$) across Shock Layer for a Circular Cylinder at $M_\infty = 5$ ($S^* \approx 0.8$).

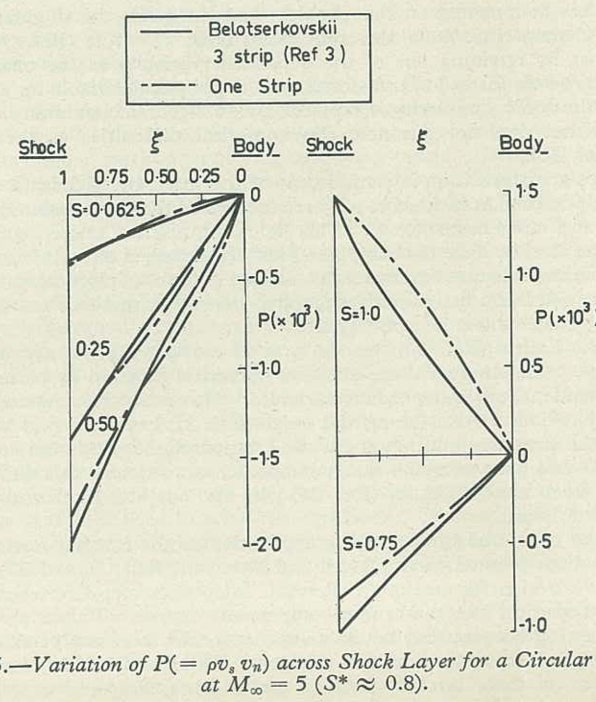

Fig. 5.—Variation of $P(= \rho v_s v_n)$ across Shock Layer for a Circular Cylinder at $M_\infty = 5$ ($S^* \approx 0.8$).

TABLE Ia

SPHERE MACH NO = 3.000

DELTO = 0.2159293664 GATE = 0.95 STEP = 0.00625

SLAMDA	DELTA	CHI	VSO	PO	PNE
0.0	0.2159	0.0	0.0	0.3283	0.3283
0.0625	0.2165	0.0477	0.0346	0.3270	0.3271
0.1250	0.2182	0.0952	0.0692	0.3229	0.3232
0.1875	0.2210	0.1422	0.1038	0.3161	0.3169
0.2500	0.2251	0.1887	0.1382	0.3069	0.3082
0.3125	0.2305	0.2344	0.1723	0.2955	0.2973
0.3750	0.2372	0.2790	0.2063	0.2820	0.2843
0.4375	0.2454	0.3225	0.2399	0.2668	0.2694
0.5000	0.2552	0.3646	0.2731	0.2503	0.2529
0.5625	0.2668	0.4053	0.3058	0.2328	0.2350
0.6250	0.2804	0.4443	0.3380	0.2148	0.2159
0.6875	0.2961	0.4816	0.3694	0.1965	0.1961
0.7500	0.3143	0.5172	0.4000	0.1784	0.1758
0.8125	0.3352	0.5511	0.4297	0.1607	0.1553
0.8750	0.3591	0.5834	0.4583	0.1439	0.1349
0.9375	0.3863	0.6141	0.4858	0.1280	0.1150
1.0000	0.4173	0.6435	0.5120	0.1133	0.0959
1.0625	0.4524	0.6718	0.5368	0.0999	0.0778
1.1250	0.4922	0.6991	0.5602	0.0879	0.0610
1.1875	0.5373	0.7257	0.5821	0.0771	0.0459
1.2500	0.5881	0.7517	0.6026	0.0677	0.0326
1.3125	0.6455	0.7772	0.6215	0.0595	0.0214
1.3750	0.7104	0.8023	0.6391	0.0523	0.0124
1.4375	0.7838	0.8269	0.6553	0.0461	0.0058
1.5000	0.8669	0.8511	0.6703	0.0407	0.0016
1.5625	0.9613	0.8748	0.6842	0.0360	0.0000

Hence we have a two-point boundary value problem in which the correct initial guess of δ_0 is determined by the simultaneous vanishing of the numerator and denominator of dv_{s0}/ds at the sonic point. The sonic singularity is floating in that its position is unknown at the start of integration. This leads to a computational procedure of a non-routine nature. The illuminating study by Xerikos and Anderson (Ref. 27) presents a complete treatment of this problem. Integration will then yield shock and surface variables such as shock shape and surface pressure distribution.

One virtue of the method of integral relations is that the integration may be continued over as much of the body as desired regardless of the changing nature of the differential equations between the subsonic and supersonic flow regimes. However, if desired, a switch to a characteristics solution may be made at the sonic line for the integration into the supersonic region.

For the axisymmetric case ($j = 1$), the equations are singular at the start of integration on the stagnation streamline. Application of L'Hospital's Rule gives appropriate starting values (Ref. 23).

3.2 Inverse Procedure:

The solution of the integral equations yields boundary values only. To obtain information about the flow field an inverse procedure is necessary. For this purpose Xerikos and Anderson (Refs. 27 and 42) developed the " gradient " method. An alternative method is proposed here.

Expressions may be derived for the normal derivatives of the variables (p, ρ, v_s, v_n) at the shock and body through use of the equations of vorticity and continuity. This allows a cubic equation in n/δ to be fitted to the values on the boundaries and the subsequent evaluation of the flow field. The details are given in Ref. 23.

4.—RESULTS AND DISCUSSIONS

4.1 The Linear Assumption:

Because the first-order equations are obtained by representing the functions g, G and P as linear across the shock layer, three integral truncation errors are introduced into the flow equations. Consequently, the difference between the first-order and the exact solutions will depend on the extent to which this approximation is justified, particularly in the subsonic zone.

Any integral becomes:

$$I = \frac{\delta}{2}(f_0 + f_1) - \frac{\delta^3}{12}f''(\xi) \qquad 0 < \xi < 1 \qquad \text{...........(4.1)}$$

The validity of this linear approximation may be examined " a posteriori " from the results of the first-order solution which, if the integral truncation terms are of second-order, should not differ greatly from the exact solution.

TABLE Ib

FLOW FIELD MACH NO = 3.000

SLAMDA		N/DELTA 0.0	.25	.50	.75	1.0
0.0	VS	0.0	0.0000	0.0000	0.0000	0.0000
	VN	0.0	−0.0564	−0.1079	−0.1574	−0.2079
	P	0.3283	0.3252	0.3160	0.3012	0.2813
	RHO	0.3283	0.3261	0.3195	0.3087	0.2940
0.1250		0.0692	0.0711	0.0739	0.0777	0.0824
		0.0	−0.0560	−0.1068	−0.1556	−0.2052
		0.3229	0.3203	0.3118	0.2978	0.2787
		0.3244	0.3229	0.3170	0.3070	0.2931
0.2500		0.1382	0.1415	0.1467	0.1538	0.1628
		0.0	−0.0545	−0.1035	−0.1501	−0.1972
		0.3069	0.3063	0.2999	0.2881	0.2713
		0.3129	0.3134	0.3097	0.3019	0.2902
0.3750		0.2063	0.2101	0.2170	0.2268	0.2395
		0.0	−0.0519	−0.0976	−0.1405	−0.1836
		0.2820	0.2844	0.2813	0.2730	0.2596
		0.2945	0.2983	0.2981	0.2939	0.2856
0.5000		0.2731	0.2762	0.2837	0.2954	0.3109
		0.0	−0.0476	−0.0885	−0.1261	−0.1639
		0.2503	0.2568	0.2580	0.2541	0.2450
		0.2705	0.2786	0.2829	0.2832	0.2795
0.6250		0.3380	0.3387	0.3457	0.3582	0.3755
		0.0	−0.0410	−0.0753	−0.1063	−0.1376
		0.2148	0.2259	0.2321	0.2330	0.2285
		0.2425	0.2556	0.2652	0.2708	0.2720
0.7500		0.4000	0.3964	0.4017	0.4143	0.4323
		0.0	−0.0315	−0.0571	−0.0804	−0.1047
		0.1784	0.1946	0.2058	0.2115	0.2114
		0.2123	0.2312	0.2464	0.2574	0.2636
0.8750		0.4583	0.4482	0.4508	0.4627	0.4806
		0.0	−0.0184	−0.0333	−0.0479	−0.0650
		0.1439	0.1652	0.1810	0.1909	0.1946
		0.1821	0.2070	0.2277	0.2437	0.2544
1.0000		0.5120	0.4931	0.4921	0.5032	0.5205
		0.0	−0.0012	−0.0036	−0.0090	−0.0193
		0.1133	0.1393	0.1586	0.1716	0.1784
		0.1536	0.1844	0.2100	0.2301	0.2448
1.1250		0.5602	0.5302	0.5253	0.5358	0.5522
		0.0	0.0201	0.0316	0.0353	0.0317
		0.0879	0.1178	0.1392	0.1538	0.1629
		0.1280	0.1647	0.1939	0.2168	0.2348
1.2500		0.6026	0.5589	0.5501	0.5609	0.5762
		0.0	0.0453	0.0718	0.0841	0.0867
		0.0677	0.1007	0.1227	0.1373	0.1480
		0.1063	0.1485	0.1797	0.2036	0.2241
1.3750		0.6391	0.5789	0.5665	0.5788	0.5927
		0.0	0.0750	0.1170	0.1367	0.1446
		0.0523	0.0878	0.1089	0.1220	0.1336
		0.0884	0.1361	0.1677	0.1907	0.2127
1.5000		0.6703	0.5897	0.5739	0.5892	0.6017
		0.0	0.1109	0.1688	0.1933	0.2044
		0.0407	0.0785	0.0977	0.1081	0.1197
		0.0739	0.1275	0.1583	0.1786	0.2007

Using the results of Section 3.2 on Inverse Procedure, expressions may be developed for the normal derivatives of g, G, P and the cubic fit will then give the n-direction variation as before. The details are given in Ref. 23. The results are examined in Figs. 3, 4 and 5 for the particular case of the circular cylinder at $M_\infty = 5$. These show a comparison between the exact values of Belotserkovskii (Ref. 4) and those obtained from the first-order solution of the variation of g, G and P across the shock layer. At any value of s, a straight line joining the ordinates of the one-strip boundary values would represent the initial linear approximation.

The agreement between exact and first-order values of g, G and P is seen to be quite close over a wide range of s, especially for the function P which represents the normal momentum flux.

Although agreement progressively deteriorates as s increases, the first-order approximation is well justified. Furthermore, it can be expected

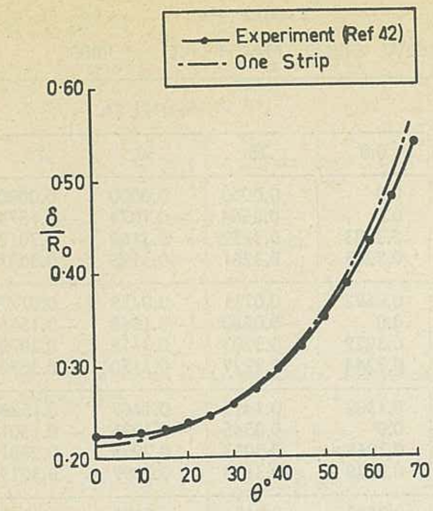

Fig. 6.—Comparison of Shock Shapes. Sphere $M_\infty = 2.996$.

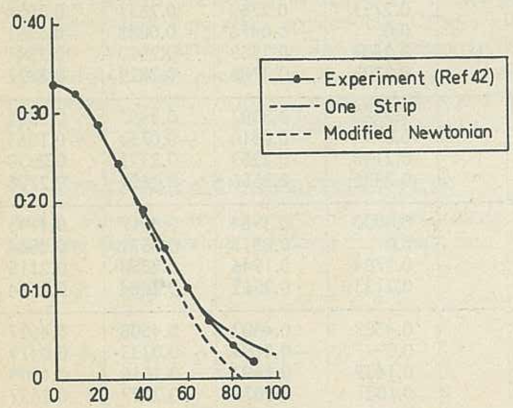

Fig. 7.—Surface Pressure Distribution. Sphere $M_\infty = 2.996$.

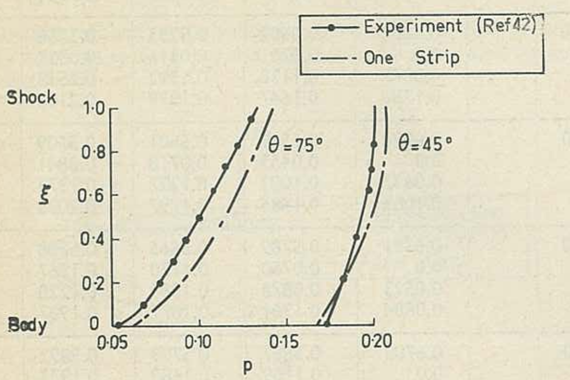

Fig. 8.—Shock Layer Static Pressure Profiles. Sphere $M_\infty = 2.996$.

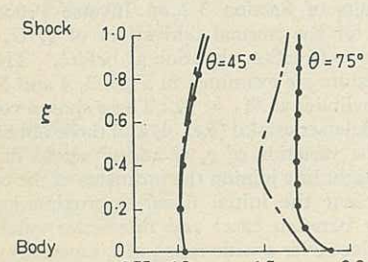

Fig. 9.—Shock Layer Mach Number Profiles. Sphere $M_\infty = 2.996$.

that the approximation will improve away from the stagnation point with increasing Mach number.

4.2 Results for a Sphere:

The convergence test calls for 10 decimal places on the value of stagnation shock standoff distance. See DELTO in Table Ia. Once this test is passed, integration is allowed to proceed into the supersonic region over the whole quadrant of the sphere. As an example, the flow solution over the whole face of a hemisphere at Mach number 3 is presented in Tables Ia and Ib. The first-order solution for shock shape (δ, χ), surface velocity (v_{s0}) and surface pressure (p_0) are presented in Table Ia, together with a column of the pressure predicted by the Newtonian theory (p_{NE}). The independent variable λ represents the circular angle measure appropriate to the arc length s on the surface of the sphere. The shock layer flow field data are presented next in Table Ib. The calculation used the new inverse procedure described earlier (Section 3.2 on Inverse Procedure). The chosen intervals on λ and n/δ make possible direct comparison with the tabulated results of Belotserkovskii (Ref. 4), if desired.

Taking the measurements of Xerikos and Anderson (Ref. 42) of the flow around a sphere as a basis for comparison, Figs. 6, 7, 8 and 9 illustrate the accuracy of the first-order solution and its corresponding shock layer flow field at Mach number 3. The numerical results of Xerikos and Anderson (Ref. 42) are the same as those obtained here.

4.3 Results for a Power-Law Body:

To demonstrate the application of the method to arbitrary body shape and also to evaluate the effect of increasing M, Horning's study (Ref. 11) of a power-law cylinder at $M_\infty = 8.3$ using blast wave theory and tested in a gun-tunnel is chosen for comparison. The one-strip solution of the method of integral relations is run (Ref. 22) far into the supersonic region. Shock shape and surface pressure are shown in Figs. 10 and 11. It should be noted that the sonic line occurs at a body slope of about 50° (Fig. 10), and close to surface pressure coefficient of 1 (Fig. 11).

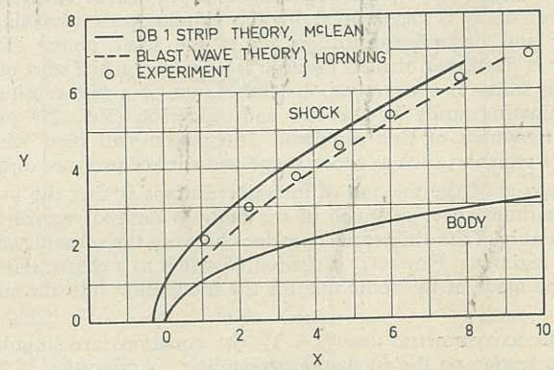

Fig. 10.—Shock Wave Shape at $M_\infty = 8.3$ *for Two-Dimensional Power Law Body,* $X = Y^2$.

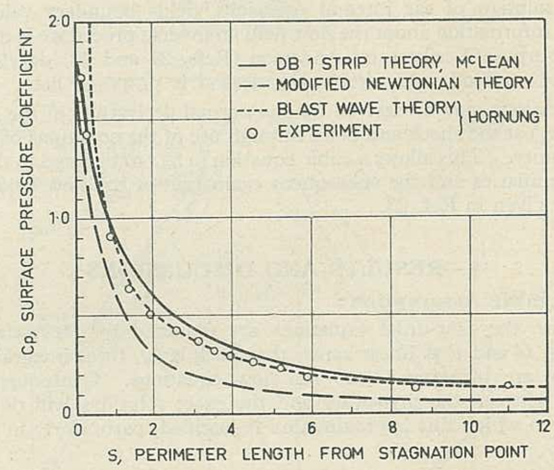

Fig. 11.—Surface Pressure at $M_\infty = 8.3$ *for Two-Dimensional Power Lau Body,* $X = Y^2$.

The first-order method of integral relations is seen to give good agreement with experiment, especially near the nose, although the blast wave theory is clearly superior far from the nose. Both theories are significantly better than Newtonian far downstream.

CONCLUSIONS

Examination of the flow field by a new method has shown that the first-order solution using the method of integral relations for the flow of a perfect inviscid gas around a supersonic blunt body of arbitrary shape represents a very good approximation to the physical processes involved especially in the nose region. The accuracy of the method improves throughout the whole flow field with increasing Mach number.

As examples of typical first approximation solutions, tables for the flow over the whole nose of a hemisphere at Mach number 3 and results for the shock shape and surface pressure distribution for a power-law body at Mach number 8.3 are presented.

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