14. Valensi, J.—Oscillations d'un Liquide Pesant et Visqueux dans un Tube en U de Fiable Diamètre. I. Etude Théorique. *Comptes-Rendus de l'Académie des Sciences,* Vol. 224, Feb. 17, 1947, pp. 446-48.

15. Valensi, J. and Clarion, C.—Oscillations d'un Liquide Pesant et Visqueux dans un Tube en U de Fiable Diamètre. II. Vérification Expérimentale. *Comptes-Rendus de l'Académie des Sciences,* Vol. 224, Feb. 24, 1947, pp. 532-35.

16. Valensi, J.—Oscillations d'un Liquide Pesant et Visqueux dans un Tube en U de Fiable Diamètre. Expressions de λ/ω_p et ω/ω_p dans les Régimes Laminaire et Turbulent. *Comptes-Rendus de l'Académie des Sciences,* Vol. 224, Feb. 24, 1947, pp. 893-94.

17. Valensi, J. and Vogel, T.—Sur l'Oscillation d'un Liquide Pesant et Visqueux dans un Tube en U. *Comptes-Rendus de l'Académie des Sciences,* Vol. 224, June 16, 1947, pp. 1695-97.

18. Valensi, J., Clarion, C. and Zerner, F.—Sur les Deux Régimes d'Oscillation d'un Liquide Pesant et Visqueux dans un Tube en U. *Comptes-Rendus de l'Académie des Sciences,* Vol. 230, June 5, 1950, pp. 2002-4.

19. Williams, T. J.—Pulsation Errors in Manometer Gages. *Trans. A.S.M.E.,* Vol. 78, Oct., 1956, pp. 1461-69.

20. Smith, A. M. O.—Remarks on Transition in a Round Tube. *Jour. Fluid Mechanics,* Vol. 7, Pt. 4, April, 1960, pp. 565-76.

Simulation of Ocean Wave Spectra in Laboratories—Some Non-Linear Effects

By Alex J. Sutherland, Ph.D., M.E.*

Summary.—Spectral measurements made during a laboratory investigation of wind waves are presented. The spectra show two features which have not previously been reported in laboratory studies. Firstly, a second peak was found at a frequency close to twice the frequency of the main spectral peak. Some speculations as to the origin of this peak are given. Secondly, in the range of frequencies usually associated with the equilibrium range, the spectral density retained a strong dependence upon wind speed. For each wind speed studied there was no fetch dependence for frequencies greater than twice the peak frequency. Both features have been observed in studies of ocean waves. This, coupled with previous work on the simulation of ocean waves, and the knowledge that the spectrum develops in a similar manner in the two environments leads to the conclusion that ocean conditions can be successfully simulated in laboratory facilities.

1.—INTRODUCTION

Common to all who work in oceanography is the desire for more detailed information about the physical processes with which they are dealing. To obtain this information from experiments performed in the ocean is always difficult and sometimes impossible. Consequently many laboratories have developed facilities in which ocean waves may be simulated by blowing an airstream over a body of water. These facilities are used for a wide variety of studies the results from which are applied to oceanic problems. Fortunately for many studies the means of wave generation is unimportant and this extrapolation can be done successfully if the resultant laboratory waves sufficiently resemble those found in the ocean. The extent to which the statistical properties of wind-generated waves can be modelled in a laboratory has been studied in the Stanford facility by Colonell (Ref. 3). He concluded that the statistical description of wave heights proposed by Longuet-Higgins (Ref. 7) was reliable for both laboratory and full-scale sea conditions. This suggested that the waves produced by wind action in the laboratory can be used in model studies where statistical properties of the waves are important.

Two aspects of spectra measured in the Stanford facility, the equilibrium range and the presence of a second peak in the spectrum, are discussed herein. Comparisons with ocean wave spectra are made and the results can be used to help evaluate the usefulness of laboratory studies made using wind-generated waves.

2.—EXPERIMENTAL APPARATUS

The experimental measurements were made in the Stanford Hydraulic Laboratory's wind, water-wave facility, the main section of which is a channel 115 ft. long with a rectangular cross-section 3 ft. in width and 6 ft. 4 in. in height. Fetches of up to 70 ft. can be obtained in the glass-walled test section which ends in a beach used to absorb wave energy and to reduce wave reflections. A slightly sloping (approximately 1:100) smooth plate, 5 ft. in length, formed an upstream beach and provided a smooth transition for the airstream on to the water surface. The air is drawn through the facility by a centrifugal fan the speed of which controls the air velocity. With a 3-ft. water depth winds up to 80 ft./sec. could be attained with good reproducibility. Under these conditions wave heights of approximately 0.75 ft. were obtained at a fetch of 60 ft. A schematic representation of the facility is given in Fig. 1 and further details may be found in the report by Sutherland (Ref. 17).

Time-average velocity profiles in the airstream were measured using a Pace differential transducer, Model P90D, to monitor the pressure difference between a total head and a static pressure tube. The resultant voltage output was converted to velocity by means of a calibration curve.

Wave height measurements were made with surface penetrating, capacitance-type wave gauges. Data were first recorded on analog tape and then converted to digital form on data cards. An IBM 7090 computer at the Stanford Computation Center was used to compute spectra of water surface displacement from these data. The methods of Blackman and Tukey (Ref. 2) were used with time series of 5000 points and 200 degrees of freedom. A smoothing process (Hamming) using consecutive weights of 0.23, 0.54 and 0.23 was used on the computed values. Most calculations were done with a Nyquist frequency f_{ny} of 16 Hz. and some, at short fetches, with $f_{ny} = 20$ Hz. The 80% confidence band on each spectrum was estimated to be + 14% and − 12%. The effect of a sharply peaked spectrum, which is to increase the width of the confidence band, was offset by repeating runs and averaging. The final results have an 80% confidence band of approximately ± 12%. A frequency resolution of 0.333 Hz. was used throughout.

3.—RESULTS

Four series of experiments were performed, each with a different mean air velocity. A reference air velocity was defined as the free stream velocity at a fetch of 19.1 ft. and is shown in Table I. In each series velocity profiles were measured at six different stations and spectra of water surface

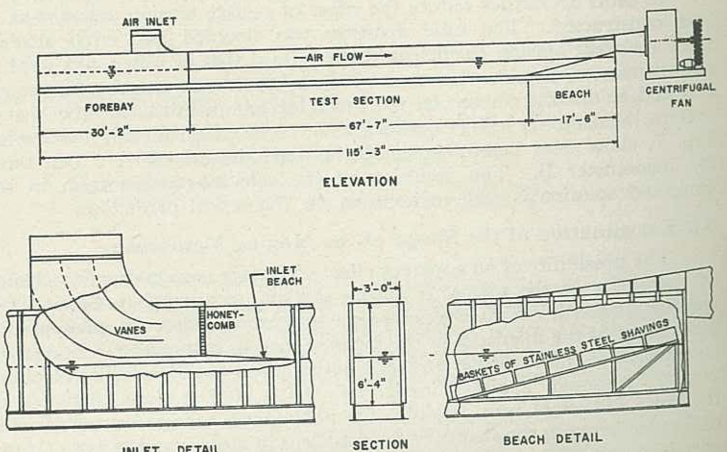

Fig. 1.—The Stanford Wind, Water-Wave Facility.

---

*Paper No. 2581, presented at the Third Australasian Conference on Hydraulics and Fluid Mechanics held in Sydney from 25th to 29th November, 1968.

The author is a Lecturer in Civil Engineering, University of Canterbury, New Zealand.

displacement were determined at twelve stations ranging in fetch from 3.9 to 59.0 ft.

3.1 Characteristics of the Air Flow :

The velocity profiles were not truly logarithmic, the lower portion tending to droop slightly towards the water surface, see Fig. 2.

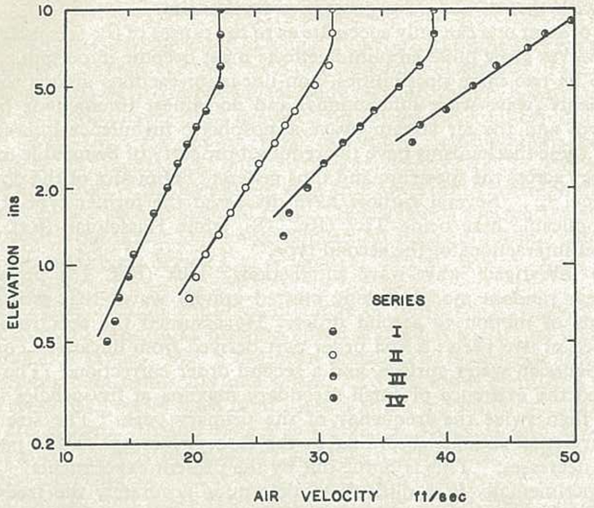

Fig. 2.—Air Velocity Profiles; Fetch = 19.1 ft.

All experimental measurements experience a degree of uncertainty near the water surface. Two effects can be important. A fixed velocity probe continuously changes streamlines and also the instantaneous velocities may differ over the crest and the trough (Miles, Ref. 8). The former causes values smaller than the actual to be recorded, while the latter causes recording of larger values. Shemdin (Ref. 16) has shown that these effects can be significant at elevations less than three times the wave amplitude. The lowest points on the profile were thus disregarded, the shear velocity being determined from the slope of the best fit straight line through the remaining data (Fig. 2). The resultant values of the shear velocity increased monotonically with fetch, the average over the length of the facility corresponding closely with that measured at a fetch of 19.1 ft. and shown in Table I.

TABLE I
Experimental Conditions

Series	Reference Air Velocity, ft. /sec.	Shear Velocity, ft. /sec. (Fetch = 19.1 ft.)
I	22.0	1.67
II	31.0	2.63
III	39.0	3.40
IV	50.1	5.45

3.2 The Spectrum of Water Surface Displacement :

Table II lists the main results of the spectral density measurements. Fig. 3, derived from Series III, is a typical contour map of spectral density ϕ, as a function of wave frequency f, and fetch. The contours are based on approximately 15 values of ϕ vs. f at each of the twelve different fetches (180 points). In drawing the contours a minimum of smoothing was done in an attempt to retain as much detail as possible. Fig. 3 illustrates the essential features in the development of the wave spectrum, for by taking sections at constant fetch the original wave spectra are obtained. The most obvious features are the reduction in the frequency f_m, as the fetch increases, of the spectral peak and the steepness of the low frequency side of the spectrum. These are in accord with previous work by Hidy and Plate (Ref. 5) in the laboratory and Barnett and Wilkerson (Ref. 1) in the ocean.

At the shorter fetches (0-8 ft.) the wave train contains very little energy which is spread over a broad band of frequencies. This suggests that resonance between pressure fluctuations in the air stream and the possible modes of surface waves is responsible for the initial wave generation. This resonance occurs over a broad frequency band. Subsequent growth would seem to be the result of an instability mechanism which amplifies the high frequency components more rapidly than those of lower frequency. The spectrum thus develops a peak which moves to lower frequencies as the fetch increases and the high frequency components become saturated. A second peak appears at a frequency of approximately $2f_m$ for fetches greater than about 10 ft. (Fig. 4). It is this second peak that causes the closed contours of Fig. 3*.

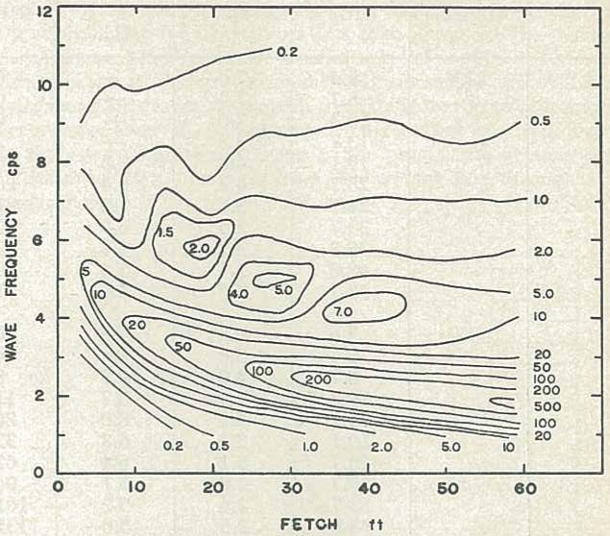

Fig. 3.—Contour Plot of Spectral Density as a Function of Wave Frequency and Fetch. Contour Values are given in ft.^{2} sec. × 10^6.*

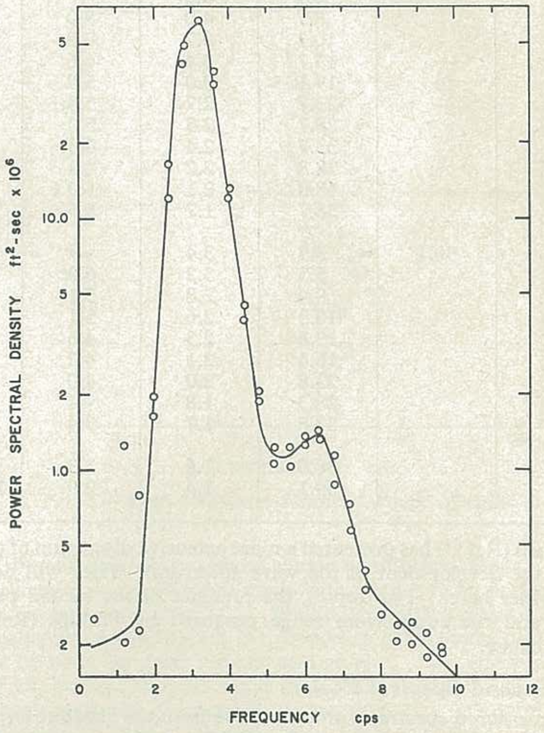

Fig. 4.—Power Spectrum from Series II at a Fetch of 23.7 ft. Note the Logarithmic Scale used for the Ordinate.

By cutting the contour diagram parallel to the fetch axis growth curves (spectral density as a function of fetch) for any frequency component can be derived. It is interesting that the power contained in the higher frequency components reaches a maximum and then decreases before attaining an eventual equilibrium value. This " overshoot effect ", which is contrary to the accepted shape of a growth curve, is discussed more fully by Sutherland (Ref. 17) and by Barnett and Wilkerson (Ref. 1) who observed the same effect in the open ocean. Those components of frequency less than the f_m at the maximum fetch were gaining energy throughout the length of the facility. Furthermore, even at the highest wind speed, Series IV, the power contained at the peak frequency did not reach a maximum in the available fetch. Thus all wave conditions studied were of the fetch-limited type.

*See p. 248—*Ed.*

TABLE II

Results of Spectral Density Measurements

Series	Reference Velocity, ft. /sec.	Fetch, ft.	Peak Frequency, Hz.	Frequency of 2nd Peak, Hz.	Power at Peak Frequency, ft.^{2} sec. $\times 10^6$.
I	22.0	6.4	7.5	—	0.012
		8.7	7.4	—	0.14
		10.3	7.0	—	0.60
		13.7	5.8	—	2.4
		19.1	4.8	9.6	6.9
		23.7	4.2	8.4	20.5
		28.7	3.8	7.4	22.0
		33.7	3.5	6.9	30.0
		38.8	3.2	6.3	33.5
		48.8	2.7	5.4	59.0
		58.7	2.5	4.9	71.0
II	31.0	3.6	6.8	—	0.85
		6.4	5.4	—	4.3
		8.7	4.9	9.8	8.2
		10.3	4.5	8.7	11.5
		13.7	4.1	8.0	26.0
		19.1	3.5	6.8	32.0
		23.7	3.2	6.4	62.0
		28.7	2.9	5.7	94.0
		33.7	2.6	5.2	101
		38.8	2.5	5.0	135
		48.8	2.3	4.5	170
		58.7	2.1	4.1	240
III	39.0	3.6	5.2	—	3.9
		6.4	4.4	—	14.0
		8.7	4.0	8.0	18.0
		10.3	3.7	7.2	27.0
		13.7	3.4	6.6	48.0
		19.1	3.0	5.9	79.0
		23.7	2.7	5.3	97.0
		28.7	2.5	5.0	146
		33.7	2.4	4.5	160
		38.8	2.2	4.4	210
		48.8	2.1	4.0	400
		58.7	1.7	3.5	590
IV	50.1	3.5	3.4	—	34.0
		5.3	3.2	6.0	60.0
		8.4	2.8	5.1	130
		10.3	2.6	5.1	180
		13.3	2.3	4.6	300
		18.4	2.1	4.2	360
		23.8	2.0	4.0	470
		28.3	1.8	—	520
		33.8	1.7	3.4	710
		38.9	1.6	3.3	940
		59.1	1.4	2.9	1400

Colonell (Ref. 3) has presented a more extensive discussion of the overall trends in the development of the wave spectrum. They will not be discussed further here. Two topics, the presence of the second peak in the spectrum and the equilibrium range proposed by Phillips (Ref. 10) are discussed below.

3.3 The Second Spectral Peak:

The measured spectra, with only those from the shortest fetches being exceptions, showed a local maximum at a frequency close to twice the peak frequency. The second maximum is a definite feature in that it spans four or five frequency bands and the points defining the peak do so in an ordered way. It is only a remote possibility that they might have resulted by chance from the inherent statistical variations. The possibility that they may be a characteristic of the Stanford facility is lessened by reference to spectra measured in the ocean. Kinsman (Ref. 6) and Moskowitz (Ref. 9) have both measured spectra that show a local maximum at twice the peak frequency. Neither Hidy and Plate (Ref. 5) nor Colonell (Ref. 3) reported secondary peaks. A possible reason is that the confidence bands used were so large as to preclude proper definition of the second peak. In Colonell (Ref. 3) the 80% confidence band was + 37% and − 23%.

The magnitude of the second peak can be defined as the difference in spectral density between the peak and the trough immediately preceding it. With this definition, the magnitude of the second peak increases as the fetch, and hence the energy at the peak frequency, increases. The increase is such that the magnitude of the second peak remains an essentially constant percentage of the peak energy. This percentage increases with wind velocity from approximately 0.4% in Series I to 1.0% in Series IV. The width of the second peak is always close to that of the main peak, decreasing with increasing fetch.

One concludes that the two peaks are closely related, indeed the frequency ratio suggests a basic wave and a first harmonic. Furthermore the time scale of the process by which the second peak develops must be comparable with that of the wave generative mechanism.

At present one can only speculate as to the causes of the secondary peak. As the waves grow finite amplitude effects must become important. There seem to be two main possibilities, non-linear interactions among the wave components (wave-wave interactions) and non-linear interactions between the waves and the air stream (wave-atmospheric turbulence interactions). Both of these mechanisms have the required property of being able to transfer energy across the spectrum and thus generate harmonics of the dominant frequency, f_m. Several authors have discussed the former type, the one most applicable here being Tick (Ref. 18), while Hasselman (Ref. 4) has proposed interactions of the second type.

To investigate wave-wave interactions, Tick (Ref. 18) proposed a non-linear random model of long crested gravity waves that satisfied the equations of motion to second order. He assumed the spectrum to be composed of two parts: a first order part derived from linearised equations and a Gaussian water surface, and a second order correction. The theory predicted the existence of small secondary maxima at frequencies slightly greater than twice the frequency of the primary peak. The size of the secondary peak should, according to the theory, increase as the peak frequency decreases. This is borne out by the present experiments.

Experimentally it is difficult to determine accurately the frequencies corresponding to the spectral peaks. With a resolution of only 1/3 Hz. the accuracy probably is limited to ± 0.1 Hz. The ratio of the two frequencies is, to within these limits, equal to 2.0 (see Table II). However, the average of all the values would be slightly less than 2.0. A more definitive study of this point would require a finer frequency resolution in the spectrum. With this reservation, experimental confirmation for Tick's theory is provided by these results. It is the consistency with which the second peak occurs that is significant. It is of less importance that Tick predicts the frequency of the second peak to be slightly greater than twice the peak frequency and that the measurements indicate values which might be slightly less.

In general wave-wave interactions are considered to be too weak to account for the observed energy transfer across the spectrum. See, for example, Pierson (Ref. 14). In addition the time scale of these interactions is much larger than those associated with wave generation processes (Phillips, Ref. 13, p. 108). It is known, however, that the wave-wave interactions become stronger as the forward face of the spectrum becomes steeper. Since the smoothing processes used in spectral computations tend to decrease the slope of this face it is possible that these interactions have been underestimated. Further calculations supplemented by appropriate experiments are needed before more definite conclusions can be drawn.

Non-linear aspects of the wave-atmospheric turbulence interactions have not yet been calculated but a possible mechanism may be envisaged as follows. At small wave amplitudes, interaction between the air stream and the water surface is linear. It has two main effects, the mean flow perturbations and perturbations on the turbulence patterns. For a sinusoidal water surface these perturbations would be sinusoidal. As the wave amplitude increases the interactions become non-linear. Firstly in the mean flow an asymmetry develops between the flow over the wave crests and that over the troughs. For example, the velocity profile over the crest has been shown to differ from that over the trough (Shemdin, Ref. 16), the former having a greater curvature near the water surface than the latter. Consequently the turbulence intensities become larger over the crests. Hence any distortion, produced by non-linear interaction with the water surface, of the linearly perturbed turbulence patterns is accentuated. The perturbation profile will most probably assume greater crests and lesser troughs or, alternatively, may become asymmetric with one face being steeper than the other. Both possibilities are equivalent mathematically to the addition of higher harmonics to a base frequency, the phase difference determining the final shape. These harmonics can in turn act on the water surface and excite wave components of the same frequency. In a wave spectrum all components will tend to develop higher harmonics with the first harmonic being dominant. When the spectrum is sharply peaked the dominant harmonic will be that corresponding to the peak frequency and will appear at frequency $2f_m$, viz., the second peak. Confirmation or rejection of such a mechanism must await theoretical calculations.

3.4 The Equilibrium Range:

Phillips (Ref. 10) has proposed the existence in the spectrum of an equilibrium range for the high frequency components. The range is bounded above by a frequency which is small compared with those for which surface tension is an important restoring force (approximately 13 Hz.). The form of the spectrum in this range is derived by dimensional analysis under the assumption that a balance has been attained between the energy gained from the wind and that lost by wave breaking. The only important parameter is the gravitational acceleration g, which governs the conditions for the attachment or breaking of wave crests. Hence the equilibrium portion of the spectrum is given by

$$\phi(f) \sim \beta g^2 f^{-5}$$

where β is a constant, the value of which must be determined by experiment.

Clearly this range cannot extend to frequencies below that of the spectral peak because the wave components of frequency less than f_m are still growing. A more restrictive lower limit has been placed on the equilibrium range by Pierson (Ref. 14). He shows, from non-linear considerations based on Tick's (Ref. 18) model, that only frequencies greater than $2f_m$ can form the equilibrium range. In the present case this lower limit corresponds to the frequency of the second spectral peak. For some of the spectra measured by the writer the upper and lower bounds on the equilibrium range practically coincide, e.g., Series I at a fetch of 19.1 ft. where $2f_m = 9.6$ Hz. (see Table II).

The concept of an equilibrium range of frequencies requires that all spectra from all conditions of wind speed and fetch should be identical in that frequency range. Fig. 5 has been prepared to investigate this. Each plot presents data from only one series. To minimise confusion only six spectra from each series have been plotted and a minimum number of data points are shown. In the ranges 2-4 Hz. for Series II and 1.7-3.5 Hz. for Series III there is no tendency towards overlapping. Instead there is a definite ordering of the spectra, those from the longer fetches being towards the left and those from shorter fetches at the right. The slopes of all the spectra in this region are approximately − 9. The definite ordering and the steep slope are both contradictory to the concept of an equilibrium range. This is as predicted by Pierson (Ref. 14).

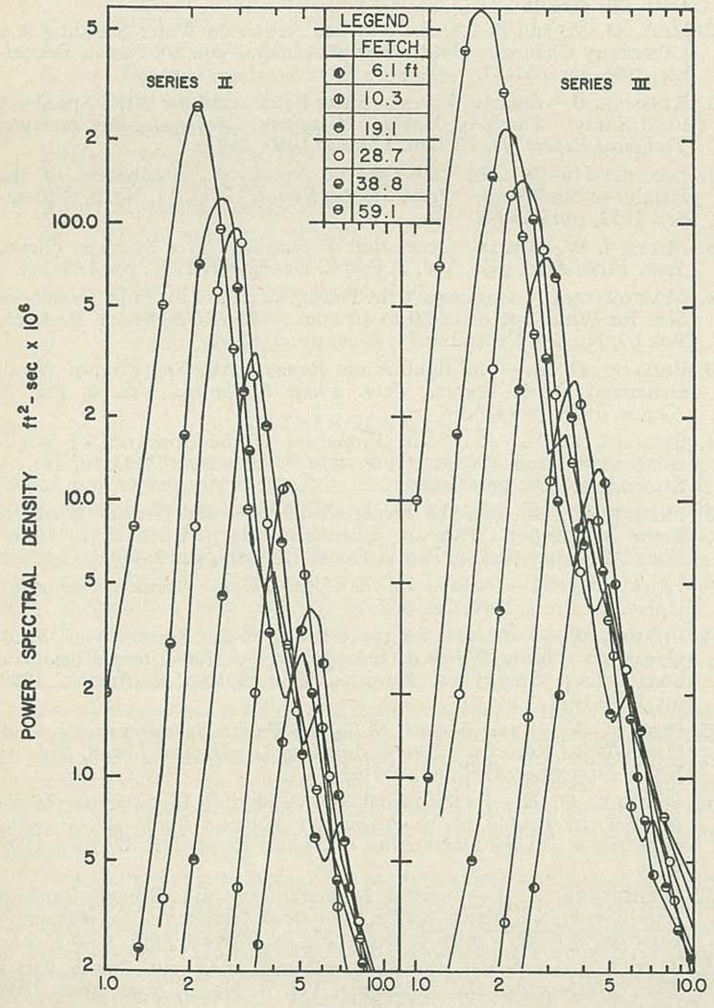

Fig. 5.—Power Spectra from Series II and III.

At approximately 4 Hz. the secondary peaks start occurring and there is some overlapping of the spectra. For frequencies greater than 7 Hz. in Series II and 6 Hz. in Series III there does appear to be a region where the curves genuinely collapse and an average spectrum can be defined. Fig. 6 is an expanded plot of this region with data from all series. The points are differentiated by their series number, i.e., according to their corresponding air velocity. Data from the spectra measured at the five longest fetches in each series are used, only points at frequencies greater than that of the second peak being plotted. Within each group the points define a straight line for frequencies less than about 10 Hz. These lines which are indicated in Fig. 6 have a mean slope of − 4.25. This compares well with values measured in the ocean by Kinsman (Ref. 6). A line of slope − 5 could easily be drawn through all the data from top to bottom and thus in effect average over all the spectra. This cannot be done legitimately because there is a definite separation of the points into groups, each with its own wind velocity. In such cases, an average line through all points has no significance.

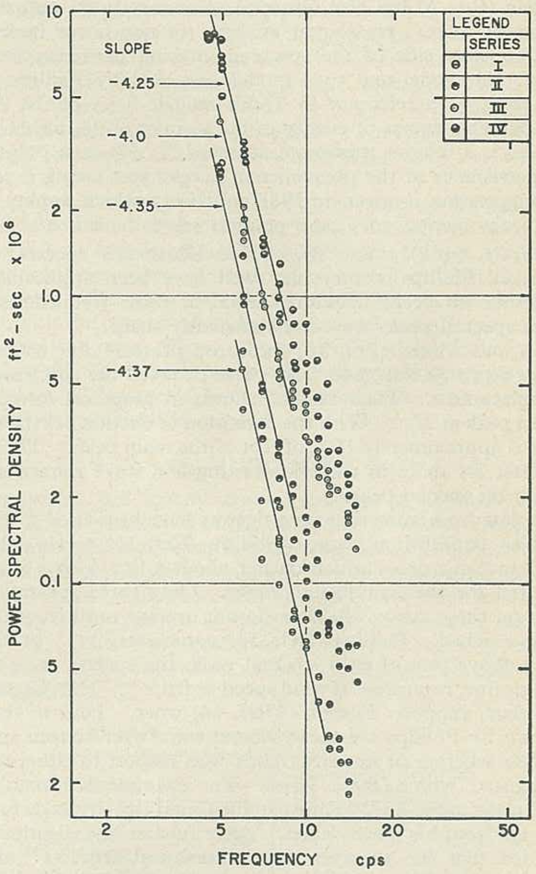

Fig. 6.—High Frequency Range of Power Spectra.

The present data do not show a true equilibrium range. However, within each series an equilibrium range with respect to fetch does exist for those frequencies greater than $2f_m$. Phillips (Ref. 10) predicted that before the establishment of equilibrium the properties of the wave field would be influenced by both the shear velocity and gravity. The effect of the shear velocity in the present experiments is seen in the magnitude of the spectral densities associated with frequencies greater than $2f_m$. Only in Series IV and occasionally in Series III were any whitecaps noticed. Therefore a mechanism other than whitecapping must have limited the growth of frequency components in this range. Again non-linear interactions between wave components and possibly between the waves and the air turbulence seem most likely to be responsible. The equilibrium spectral density for each frequency greater than $2f_m$ is an increasing function of wind speed (or shear velocity). Presumably this is true until white caps appear at which time a true equilibrium range may exist.

At frequencies greater than 10 Hz. the data on Fig. 6 deviate from the lines discussed above. This is where surface tension may be important and the − 7/3 slope which, according to Phillips (Ref. 11) was proposed by Hicks, might be expected. There are insufficient experimental points to define any line in this region.

4.—COMPARISON WITH OCEAN WAVE SPECTRA

Colonell (Ref. 3) has shown that the wave spectrum produced in a laboratory facility corresponds statistically with that found in the ocean.

That spectra in the two environments develop along similar lines has also been well documented (Barnett and Wilkerson, Ref. 1; Hidy and Plate, Ref. 5). An initially broad spectrum which develops a sharp peak with a very steep forward face is typical of both situations. The chief difference between spectra from the laboratory and spectra from the ocean is the magnitude of the peak frequency; approximately 2 Hz. in the former and usually much less than 1 Hz. in the latter. This is a reflection on the fetch-limited nature of laboratory work.

Evidence for the occurrence of the second spectral peak in ocean data is more difficult to obtain. Definition of such a peak with magnitude considerably less than that of the major peak requires greater resolution and higher accuracy than is usually attained in ocean work. However, Moskowitz (Ref. 9) did measure some spectra, in fully developed seas, that had a second peak near twice the peak frequency. The peaks occurred only in results obtained at wind speeds of 35 and 40 knots. They could not be discerned in results from lower wind speeds.

Kinsman (Ref. 6) has also observed secondary peaks and done a full investigation of them. He sought evidence of non-linear interactions in the high frequency side of the spectrum making particular reference to Tick's (Ref. 18) model and to a model proposed by Phillips (Ref. 12). Kinsman states, with reference to Tick's model, "... of the 24 spectra, 23 show a palpable excess of energy in the vicinity of $2f_m$ although in only two instances is a relative maximum achieved". He then points out that the "... persistance of the phenomenon necessitates taking it seriously". Greater confirmation is given to Phillips' theory which applies only to a single frequency component f, and predicts relative maxima at frequencies $\sqrt{2}f$, $\sqrt{3}f$, $2f$, $(\sqrt{2})^3 f$ and $3f$. Since Kinsman's spectra were very sharply peaked Phillips' theory may well have been applicable. In the writer's spectra no peaks could be found at these frequencies, possibly because the spectral peaks were not sufficiently sharp.

Pierson and Marks (Ref. 15) measured pressure fluctuations on the ocean floor (depth 32.5 ft.) and from these deduced the spectrum of water surface displacement. Their results, shown in graphical form, exhibit a very marked peak at $2f_m$. With the definition of Section 3.3 the magnitude of the peak is approximately 16% of that of the main peak. These authors, whose interest lay more in wave forecasting and wave refraction, did not comment on the second peak.

Ocean data from numerous investigators have been used to support the concept of an equilibrium range. Phillips (Ref. 13, p. 113) shows data, from seven investigations, which cluster about a line with a slope of -5 as is required for the equilibrium range. Only three spectral peaks are shown and on these curves the data do not overlap until frequencies close to $2f_m$ are reached. Phillips (Ref. 13) comments "... at frequencies appreciably above that of each spectral peak, the spectra are all clustered about a single line, regardless of wind speed or fetch". This, like the writer's laboratory data, supports Pierson's (Ref. 14) work. Four of the series of points shown by Phillips are mean values, some over sixteen spectra, and it is not clear whether or not any trends with respect to either wind speed or fetch existed within these series. For example Kinsman's data, as shown, fall close to the line. Kinsman too found this evidence for the equilibrium range from his mean values. After further investigation, however, he concluded that the averages were "statistical artifacts" showing an equilibrium range in the average spectrum when the individual spectra did not show one. The same can be said, with respect to wind speed, about the writer's results.

One must conclude that the approach to, and establishment of an equilibrium range is not yet understood. It is certainly more complex than at present envisaged.

5.—CONCLUDING REMARKS

Measured spectra of water surface displacement obtained from a laboratory investigation of the growth of wind waves have been discussed herein. The spectra showed the same trends as those found by previous investigators and helped to confirm that the spectrum develops in the same manner under both ocean and laboratory conditions.

Two topics, the appearance of a second peak in the spectrum, at twice the frequency of the main peak, and the presence of an equilibrium range with respect to fetch only, were discussed in detail. Previous work in laboratories has not revealed these features, although they have both been observed previously in ocean wave spectra. In general, any reference to them has been only of a cursory nature but their presence becomes more significant in view of the findings of this study. Certainly when one is formulating models to explain the physics of wind-wave generation they should not be overlooked. However the extra energy contained in those frequency components defining the second peak is probably insufficient to vitiate most engineering calculations based on spectra without such a peak.

In conjunction with Colonell's (Ref. 3) work this study has shown that wave spectra produced in laboratory facilities do have the same properties as those produced in the ocean. Results obtained from laboratory studies can thus be used with confidence for the solution of problems occurring in the ocean.

ACKNOWLEDGMENTS

The writer would like to acknowledge a series of valuable discussions with Dr. P. J. Bryant who was also kind enough to read the original manuscript and to suggest a number of improvements to it. Financial support for this investigation was provided by the National Science Foundation under Grant GK-736 and by the Office of Naval Research through Contract Nonr-225(71).

References

1. BARNETT, T. P. and WILKERSON, J. C.—On the Generation of Ocean Wind Waves as Inferred from Airborn Radar Measurements of Fetch-Limited Spectra. *Jour. Marine Research*, Vol. 25, No. 3, September, 1967, pp. 292-328.
2. BLACKMAN, R. B. and TUKEY, J. W.—*The Measurement of Power Spectra from the Point of View of Communications Engineering.* New York, Dover, 1959, 190 p.
3. COLONELL, J. M.—Laboratory Simulation of Sea Waves. *Stanford University, Department of Civil Engineering, Technical Report No.* 65, July, 1966, 136 p.
4. HASSELMAN, K.—Non-Linear Interactions Treated by the Methods of Theoretical Physics (with Application to the Generation of Waves by Wind). *Proc. Roy. Soc., Series A*, Vol. 299, No. 1456, June 13, 1967, pp. 77-110.
5. HIDY, G. M. and PLATE, E. J.—Wind Action on Water Standing in a Laboratory Channel. *Jour. Fluid Mechanics*, Vol. 26, Part 4, December, 1966, pp. 651-87.
6. KINSMAN, B.—Surface Waves at Short Fetch and Low Wind Speed—A Field Study. *The Johns Hopkins University, Chesapeake Bay Institute, Technical Report No.* 19, Vol. 1, May, 1960, 169 p.
7. LONGUET-HIGGINS, M. S.—On the Statistical Distribution of the Heights of Sea Waves. *Jour. Marine Research*, Vol. 11, No. 3, September, 1952, pp. 245-66.
8. MILES, J. W.—On the Generation of Surface Waves by Shear Flows. *Jour. Fluid Mechanics*, Vol. 3, Part 2, November, 1957, pp. 185-204.
9. MOSKOWITZ, L.—Estimates of the Power Spectrums for Fully Developed Seas for Wind Speeds of 20 to 40 knots. *Jour. Geophysical Research*, Vol. 69, No. 24, December 15, 1964, pp. 5161-79.
10. PHILLIPS, O. M.—The Equilibrium Range in the Spectrum of Wind-Generated Ocean Waves. *Jour. Fluid Mechanics*, Vol. 4, Part 4, August, 1958, pp. 426-34.
11. PHILLIPS, O. M.—On Some Properties of the Spectrum of Wind-Generated Ocean Waves. *Jour. Marine Research*, Vol. 16, No. 3, September, 1958, pp. 231-40.
12. PHILLIPS, O. M.—On the Dynamics of Unsteady Gravity Waves of Finite Amplitude. Part I, The Elementary Interactions. *Jour. Fluid Mechanics*, Vol. 9, Part 2, October, 1960, pp. 193-217.
13. PHILLIPS, O. M.—*Dynamics of the Upper Ocean.* London, Cambridge University Press, 1966, 262 p.
14. PIERSON, W. J.—A Note on the Growth of the Spectrum of Wind-Generated Gravity Waves as Determined by Non-Linear Considerations. *Jour. Geophysical Research*, Vol. 64, No. 8, August, 1959, pp. 1007-1011.
15. PIERSON, W. J. and MARKS, W.—The Power Spectrum Analysis of Ocean-Wave Records. *Trans. American Geophysical Union*, Vol. 33, No. 6, December, 1952, pp. 834-44.
16. SHEMDIN, O. H.—Experimental and Analytical Investigation of the Air Velocity Profile Above Progressive Waves. *Stanford University, Department of Civil Engineering, Technical Report No.* 82, July, 1967, 57 p.
17. SUTHERLAND, A. J.—Spectral Measurements and Growth Rates of Wind-Generated Water Waves. *Stanford University, Department of Civil Engineering, Technical Report No.* 84, August, 1967, 64 p.
18. TICK, L. J.—A Non-Linear Random Model of Gravity Waves, Part I, *Jour. Mathematics and Mechanics*, Vol. 8, No. 5, September, 1959, pp. 643-51.