8.—CONCLUSION

Since for many applications, an approximate flow net is perfectly adequate, the main use of the computer relaxation method is seen in checking the accuracy of important flow nets, however they have been constructed.

This paper is oriented specifically towards the solution of flow nets. However, there are many other two-dimensional applications in which the Laplace distribution of function-values occurs. These include the flow of electricity and heat, the surface movements of water in open channels, and stress distributions in structural elements, including thin shells (Poisson distribution). All such applications are potentially capable of solution by using these programs.

9.—ACKNOWLEDGMENTS

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Shock Diffraction on Rounded Corners

BY B. W. SKEWS, PH.D.*

Summary.—An experimental study of the behaviour of a plane shock wave diffracting around a convex corner of cylindrical profile, over a Mach number range from 1.0 to 4.0, is described. The shape of the shock is compared with predictions from Whitham's (Refs. 4 and 5) diffraction theory. The results show that the wave does not decay as rapidly as the theory indicates. The perturbed region behind the shock exhibits all the features of diffraction around plane-walled convex corners although the detailed behaviour is somewhat different.

1.—INTRODUCTION

This paper describes an extension of the work on the behaviour of a shock wave diffracting around plane-walled convex corners (Refs. 2 and 3). Here the corner consists of an arc of a circle as shown in Fig. 1. The significant features of the flow are identified in this diagram.

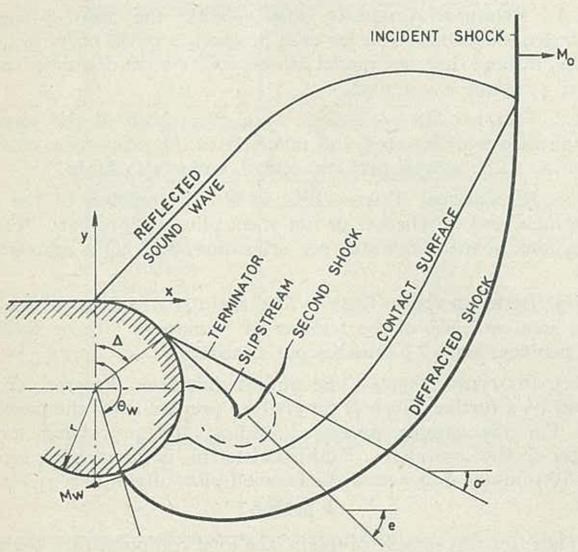

Fig. 1.—Features of the Diffraction Pattern.

The experiments were conducted in a double-driver air/air shock tube having a contraction a short distance downsteam of the second diaphragm. The 3-in. wide by 2-in. high channel opens out into a 3-in. wide by 10-in. dia. working section in which the models were mounted. A 10-in. field Schlieren system with a short-duration spark light source was used to record the phenomena.

2.—THE EFFECT OF ARC RADIUS

In the case of diffraction around a circular arc there is, unlike for the case of plane walls, a fundamental length, namely the radius of the arc; the process is thus not pseudo-stationary. A non-dimensional plot applicable to all radii may be obtained by using x/r and y/r as co-ordinates. In order to compare the flow pattern it is necessary to photograph the shock at the same relative position on the arc; that is, tests having the same value of α/r should be compared. ($\alpha = a_0 t$ where a_0 is the sonic speed in the undisturbed gas ahead of the shock and t is the time elapsed from the shock reaching the corner.)

Unfortunately, due to experimental difficulties in setting the delay times to trigger the flash a certain amount of scatter in the values of α/r was obtained. Results for $M_0 = 1.2$, 1.5, 2.0 and 3.0 are given in Fig. 2. The three radii tested were 1.0, 1.5 and 2.5 inches and are denoted in the figure by R, S and T, respectively.

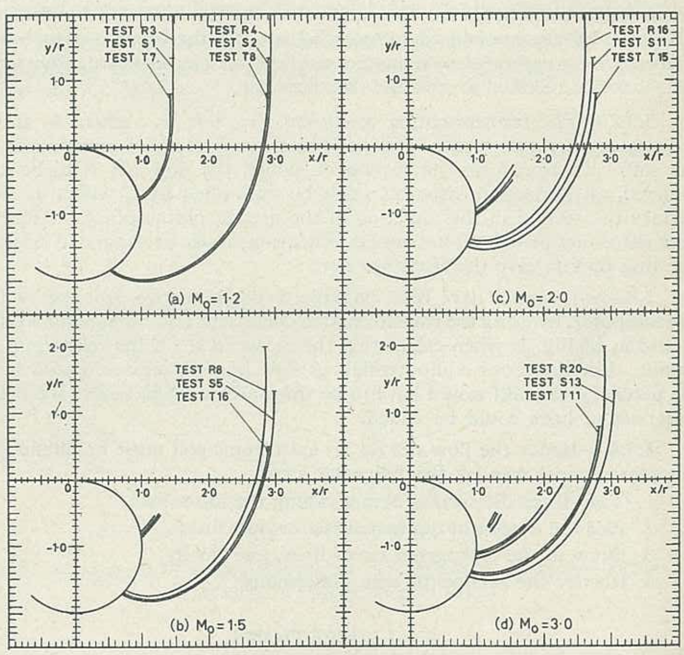

Fig. 2.—Effect of Arc Radius.

For any set of tests at a given incident shock Mach number, M_0, the characteristic gas properties (wherever they may be taken) are essentially constant and a change of arc radius may be interpreted as giving the effect of a Reynolds number change.

Taking into account the fact that the shock in contact with the wall moves more slowly than the undiffracted portion of the incident shock, it is clear from the figure that the radius of the arc has no apparent effect on the overall shock pattern for a given Mach number and value of α/r. Solutions to Whitham's theory for the circular arc wall (Ref. 1) are thus confirmed in this respect.

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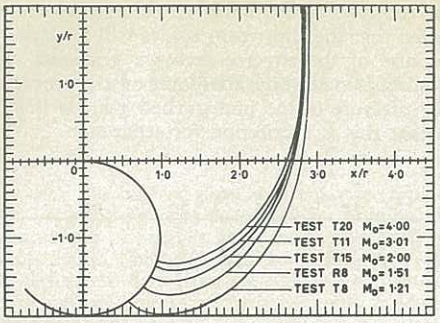

Fig. 3.—Shock Profiles at Various Mach Numbers.

3.—THE SHAPE OF THE SHOCK

In order to compare the shock profiles caused by different initial Mach numbers the shock positions may be plotted in terms of $\alpha M_0/r$. For each value of this parameter a family of curves exist, each member of which corresponds to a given initial Mach number. Fig. 3 gives a comparison for Mach numbers of 1.2, 1.5, 2.0, 3.0 and 4.0 from tests for which the value of $\alpha M_0/r$ is within 2% of 2.845. The tendency for the wall shocks to group together for the higher Mach numbers, as predicted by the theory, is noted.

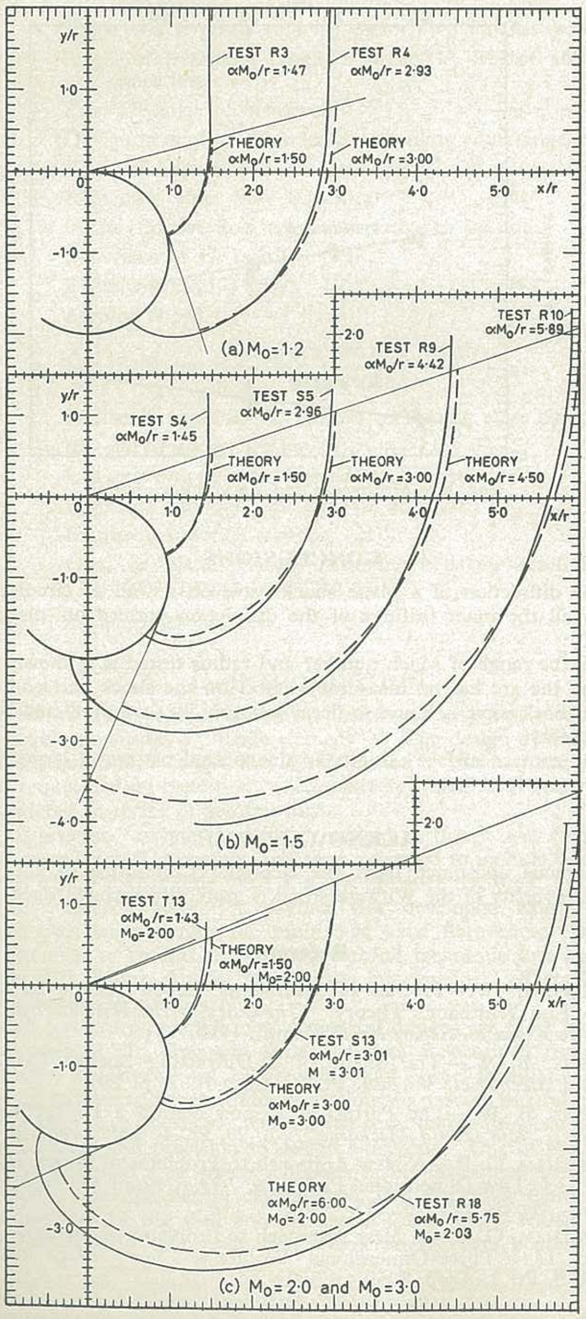

Fig. 4.—Comparison of Theory and Experiment.

The theoretical predictions for the shape of the shock are compared with those obtained experimentally in Fig. 4. The behaviour is very much as would be expected from the results obtained on plane-walled corners. In all cases the theoretical curve has the higher mean curvature. This results because the shock wave does not decay as rapidly as the theory indicates. It is found, as it was for the plane walls, that the best agreement is obtained at $M_0 = 3.0$ (centre curve of Fig. 4 (*c*)). This curve is a good example for comparing profiles as the theoretical and experimental values of $\alpha M_0/r$ are the same. In the other cases the slight differences in the undisturbed shock positions must be taken into account when comparing the results.

The instantaneous Mach number of the wall shock cannot be obtained from the photographic records. However, mean Mach numbers may be calculated and compared with the theoretical equivalent. This is done in Fig. 5. The theoretical curves cannot extend beyond the short vertical line as there $M_w = 1.0$ and the theory cannot describe the subsequent behaviour of the shock.

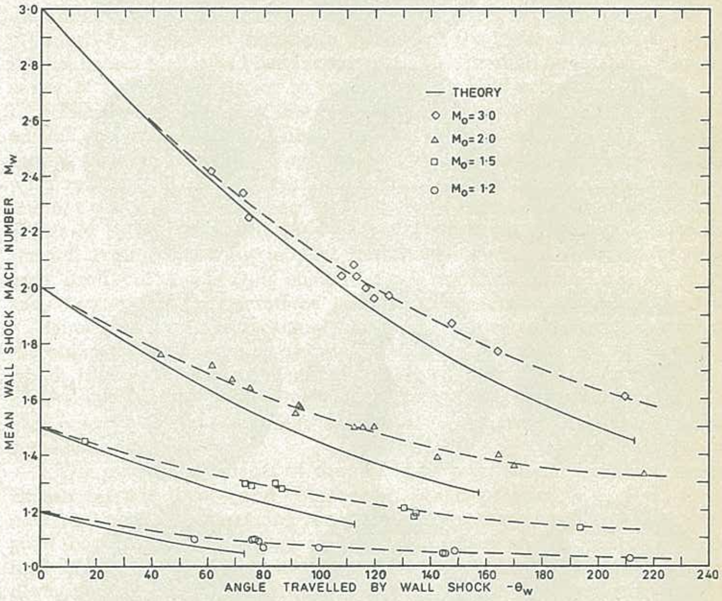

Fig. 5.—Decay of Mean Wall-Shock Mach Number.

The main points noted from Fig. 5 are first that the shock decays more rapidly at the higher Mach numbers and second that the decay is not as rapid as predicted theoretically. The result at $M_0 = 2.0$ was expected from that obtained for the plane-walled corners. At $M_0 = 3.0$, however, the plane walls show very good agreement between theory and experiment, whereas, for the circular arc the same behaviour as that for $M_0 = 2.0$ is apparent. In comparing the shock profiles at Mach 3.0 (Fig. 4) the tendency for the theoretical and experimental curves to cross over, as occured for the plane walls at this Mach number, is not noted, and the results obtained in Fig. 5 could be forseen.

4.—THE REGION BEHIND THE SHOCK

The various discontinuities in the perturbed region are indicated in Fig. 1. Their behaviour is conveniently dealt with by considering the results at specific Mach numbers. A selection of photographic results is given in Fig. 6.

At $M_0 = 1.2$ there is no sign of separation occurring on the wall even at the highest values of $\alpha M_0/r$ obtained. The same is true at $M_0 = 1.5$ except that at the higher values of $\alpha M_0/r$ the boundary layer thickens over the first quadrant of the arc and increases doing so as the shock progresses. This may be seen in test T 16 (Fig. 6). It is probable that the boundary layer would eventually separate from the wall at a higher value of $\alpha M_0/r$ than can be obtained on the present apparatus.

For an incident shock Mach number of 2.0 the thickening of the boundary layer is again noted (Test T 15). At later times the boundary layer separates and a slipstream, terminator and second shock are clearly discernible (Tests S 12 and R 17). The present series of tests does not enable the value of $\alpha M_0/r$, where these phenomena first occur, to be determined. Also, at present, too few results are available for determining the second shock velocity.

Tests S12 and R 17 are particularly interesting as regards the behaviour of the contact surface. The vortex is fairly well defined and the contact surface passes fairly close to it; but instead of being rolled up between the wall and the vortex, as happens on the plane walls, it continues past the vortex and meets the wall at an acute angle.

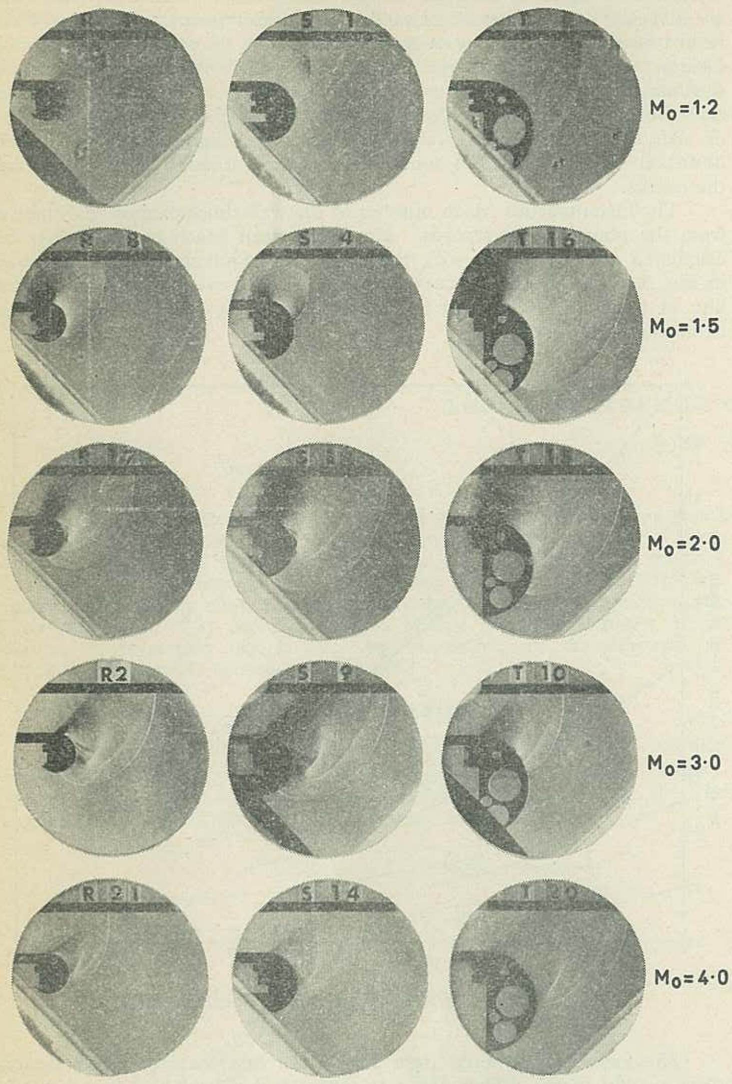

Fig. 6.—Schlieren Photographs.

The behaviour of the contact surface at Mach 3.0 and large values of $\alpha M_0/r$ (Tests R 2 and S 9) is again unexpected. It curves in towards the vortex as if it is to fold under against the wall but then undergoes a sharp change of curvature just before reaching it. A further discontinuity appears near the wall midway between the vortex and the point of separation. Reasons for the above behaviour are not known. These phenomena should preferably be studied with a horizontal knife-edge because of their orientation. The remainder of the diffraction pattern remains very much the same as for $M_0 = 2.0$.

The photographs for $M_0 = 4.0$ are similar to those at the previous Mach number except that the terminator and the discontinuities noted above are not visible. This may be a real effect or due to knife edge orientation and decreased Schlieren sensitivity at low channel pressures.

Once the slipstream and terminator are established it is noted that they both start at a definite point on the wall. This point has been defined in terms of the angle Δ (Fig. 1). Because of the very small angle that the slipstream makes with the tangent to the wall it is difficult to determine Δ very accurately. In cases where a terminator is present the point of separation is better defined. The variation of the point of separation with Mach number is shown in Fig. 7. It appears that the point of separation is delayed as the Mach number increases. This result is not conclusive because of the scatter occurring.

The variation of terminator and slipstream angles with Mach number is given in Fig. 8. It is noted that the slipstream angle increases with Mach number, whereas the reverse was found to be true for the plane wall case. It is to be expected that the slipstream angles will be larger than those for a plane wall because of the adverse pressure gradients being less steep. It may be interesting to determine the point on the arc where the pressure becomes p_0, (the pressure of the undisturbed gas), as it appears from the plane wall tests that this is a criterion for separation.

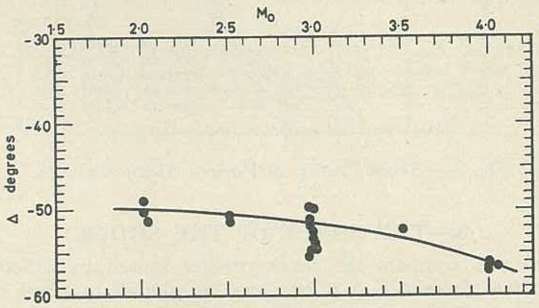

Fig. 7.—The Point of Separation on the Arc.

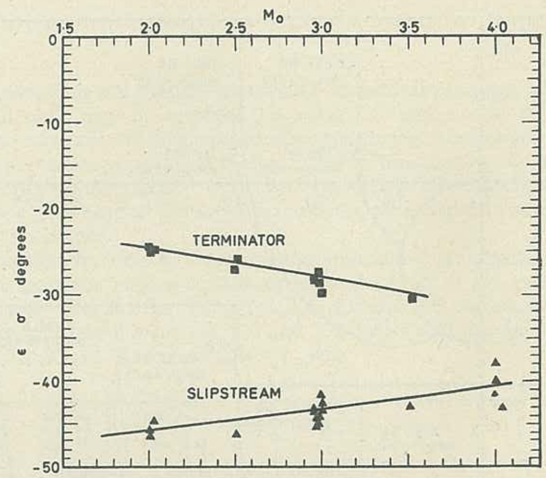

Fig. 8.—Slipstream and Terminator Angles.

CONCLUSIONS

The diffraction of a plane shock wave on a wall of circular profile exhibits all the main features of the diffraction pattern on plane-walled corners.

For the range of Mach number and radius tested it is shown that the radius of the arc has no measurable effect on the shock pattern.

The shock wave is found to decay less rapidly than predicted by Whitham's theory.

The contact surface exhibits an unexpected pattern at large values of $\alpha M_0/r$ and $M_0 = 3.0$.

ACKNOWLEDGMENT

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