On Water Waves Generated by Landslides

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INTRODUCTION

Landslides, on impact with water, produce a displacement of water by the transfer of momentum and the phenomenon is one in which a finite initial disturbance is to be applied to a finite region in a body of water which then translates or disperses through the infinite medium.

In order to deal with the propagation of the disturbance using the criteria based on the Cauchy-Poisson problem, it is important to realize that the waves which may have started out in relatively shallow water will, in effect, be starting to travel as deep water waves. After a dispersive process, they become water waves of finite depth in any depth of water. By the time they reach a shoaling coastline, they are, therefore, shallow water waves of great wave-length.

As shallow water is approached, that is, when the water is about one tenth as deep as the average wave-length of the individual wave in the group, each wave loses individual identity with the group and travels at a speed of $c = \sqrt{g(d + \eta)}$ where c = wave celerity, g = gravitational acceleration, d = depth of water and η = elevation of water surface above mean level. The number of waves thus remains constant, and each behaves as a " solitary wave " or " tidal bore ", being transformed into an asymmetrical surge with a steep front and a flat rear, as indicated by Van Dorn (Ref. 1).

In all cases, the initial disturbance propagates away from the point of impact. It is anticipated that the wave characteristics and the depth of water can be related to the magnitude and geometrical factors of the landslides in appropriate dimensionless parameters.

With an understanding of the general nature of impulsively generated waves, the study of waves generated by landslides can be analysed in the following steps:

(1) The initial stage of the wave generation due to the impact of the landslide on the surface of water: after the distribution of energy and dispersion, a relatively stable form (wave-profile) is established at some distance downstream from the point of impact. This observation follows from the fact that when waves are generated by a landslide near a slope, the leading wave will cease accelerating from the point of impact when a stable wave height is formed and when the limiting speed of $\sqrt{g(d + \eta)}$ is approached. Consequently, the periodicity of the wave remains a constant also.

(2) The mechanism of the transfer of energy from the impact to the waves: energy losses in different forms have to be accounted for.

(3) The propagation of the wave system so generated over a body of water: once a stable wave system is formed, it can be assumed that such a system can travel long distances with negligible change in form and little loss of energy. This assumption is in accordance with the analogy with the propagation of tsunamis.

(4) The decay of the wave system: an accurate prediction of the wave characteristics at a distance from the disturbance is important in dealing with design criteria for waterfront structures or other hydraulic structures such as dams.

In general, a designer is primarily concerned with the last step, namely the investigation of the wave characteristics as it approaches the limit of travel. To assist in evaluating the wave system observable near a shore or a structure, it is necessary to determine the input characteristics of the wave generating mechanism due to a landslide, as well as may be practicable.

DIMENSIONAL CONSIDERATION

In considering the wave making effect of sub-aerial slides, the assumed known factors include:

(1) the geometrical characteristics of the slide, such as slide slope and the dimensions of the slide,
(2) the falling height of the slide, h,
(3) the velocity of impact of the slide on the surface of water, V_i,
(4) the depth of water, d,
(5) distance and time domain, x and t.

From these given data, it is desired to know:

(6) the wave height, crest to trough distance, H, of the leading wave or wave of maximum height,
(7) the wave period, crest to crest, T, of the leading wave,
(8) the wave length, crest to crest, L, of the leading wave,
(9) the number of waves, n, in the train,
(10) the surface amplitude from the mean, η,
(11) the wave velocity, c.

To simplify the experimental study, it was decided that one particular slide slope would be studied in detail and that the geometrical front of the sliding object would be held perpendicular to the still water surface. The influence of the falling height of the slide was incorporated in the determination of the velocity of impact. Measurements of wave records were made at different locations along the channel—one of these locations remaining relatively constant as a reference for comparison.

Further, it was assumed that higher waves could be obtained by increasing the initial kinetic energy of impact until a certain limit was reached, a limit governed by the breaking of the wave and the process of energy transfer. It was also observed that different wave forms could be produced in water of differing depths for the same input initial kinetic energy. The parameters involved in this study were thus reduced to

$$f(L, d, H, T, \rho_w, c, h, V_i, W, g) = 0$$

where W is the sliding mass weight which is the product of ρ_s and $\forall$, with ρ_s the equivalent density of the slide, $\forall$ the volume per unit width of the slide. ρ_w is the density of water and g is the acceleration of gravity.

The existence of a relationship between T and L for progressive waves further simplifies the parameters involved to

$$f(L, d, H, \rho_s, \rho_w, c, \forall, V_i, g) = 0$$

The following dimensionless parameters were formed:

(1) H/d—a general wave parameter which relates the wave height with the depth of water.

(2) $c/\sqrt{g(d + \eta)}$ which is a wave velocity parameter.

(3) K—a kinetic energy parameter relating to the water depth which has the form $(\rho_s/\rho_w) . (V_i^2/gd) . (\forall/d^3)$. This parameter is important in the sense that it contains the elements of volume of the slide, the velocity of impact and the equivalent density of the sliding mass which in certain cases take care of the property of the slide.

(4) H/L which is the steepness of the leading wave in the case of a wave train.

Subscripted wave parameters include $H_{\max}/d$ and H_{st}/d. $H_{\max}$ is the maximum water elevation above the still water level near the point of impact. The value of $H_{\max}$ is taken at the point where the wave breaks or where the effect of splash is important and the process of dispersion is imminent. These criteria are evident as shown in the water level indicator to be described later. H_{st} is the wave height measured at about 30 ft. from the point of impact. It was assumed that at this location the process of energy redistribution and dispersion of the waves had been completed and a stable wave form had been established. A low H_{st} value would be recorded at this location if a stable wave had been formed further upstream owing to wave dissipation in the channel. However, an upper envelope of H_{st}/d for these cases should indicate the condition that a stable wave profile was formed not far from the point of measurement.

Additional parameters were also formed in this study. They are WE/KE and s/d. In these expressions, WE is the wave energy in the wave system evaluated according to the wave characteristics as either a bore or the summation of energies in the first few waves in a wave train, as the case may be. KE is the kinetic energy of impact of the landslide. The thickness of the slide is denoted by s. A small value of s/d indicates a relatively thin slide and its efficiency in wave generation is correspondingly low.

EXPERIMENTAL FACILITY

The study undertaken at Queen's University at the present time was restricted to sub-aerial slides. A two-dimensional channel 2 ft. wide was used. A 20-ft. roller with ball-bearings was used to convey boxes of different lengths, heights and weights into water of various depths. The major part of the results concern a slope angle of 17.7°. The structure of the conveying system was easily adjusted to change the slope angle.

Wave measurements were recorded as the surface profile history at several distances along the channel. The wave recorder used was a Helcoscripter He-4 with pre-amplifying system. The total weight of the sliding mass was composed of the weight of the well-soaked wooden box and the

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*Paper No. 2561, presented at the Third Australasian Conference on Hydraulics and Fluid Mechanics held in Sydney from 25th to 29th November, 1968.

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total weight of the steel plates placed inside the box. Enough time elapse between successive tests was allowed to drain away excessive water adhering to or contained in the box (Fig. 1).

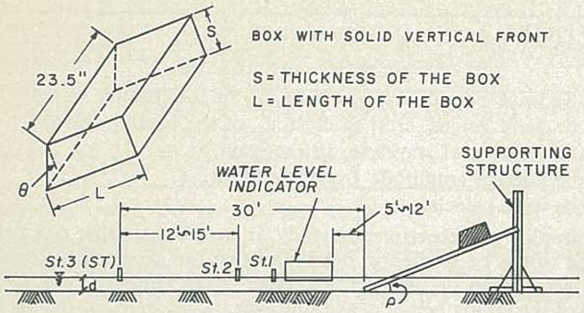

Fig. 1.—General Laboratory Set-up.

For the study of wave breaking phenomenon and the measurement of maximum water elevation near the point of impact, a water level indicator was used. It consisted of a piece of 6-ft. long aluminium sheet reinforced at the top edge along the whole length. It was painted black and could be immersed in water of any desired level. A well-defined trace of the maximum water level could be recognized as the wave passed. Evidence of splash and wave breaking were clearly shown on the indicator (Fig. 2).

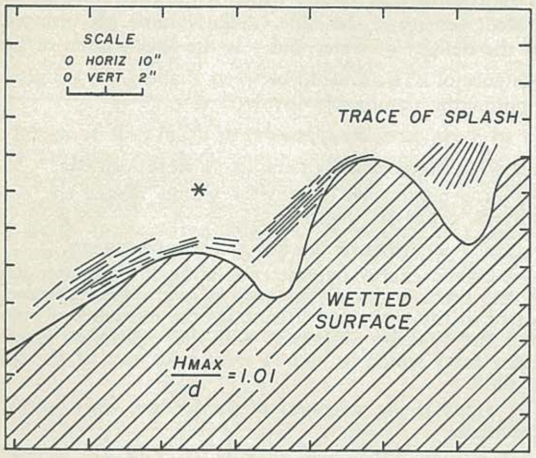

Fig. 2.—Wave Breaking as Shown on Level Indicator—Distorted Scale Location for H_{max} Measurement.

The measurement of impact velocity was facilitated with the use of strobo-photographic technique, velocity of impact for each test being found. On the cover of the box was marked a white line whereby streaks formed in the picture corresponding to the region of impact were used to calculate the velocity.

To facilitate the analysis of experimental results, a computer program on the IBM System 360 was prepared for the evaluation of data. A program for plotting the experimental data on the Calcomp plotter was also available to be used with the main program.

EXPERIMENTAL PROCEDURE

For each test, the well-soaked wooden box with the steel plates in it was placed at the desired height on the roller. The box was released by unhooking and a picture was taken when the box hit the water. The camera was mounted in such a way that it was centred with the white line on the box top when the box was stationed just touching the water surface. The slope of the platform where the camera was placed was made parallel to the slope of the conveyor. From time to time, the box was placed just touching the water and a steel scale was placed across the white line. A picture of the scale was taken which was then used to calibrate the velocity of impact.

The interval in which the box travelled corresponding to the distance traversed by the white line in the picture was measured by the lighting frequency of the stroboscope usually set around 10,000 r.p.m. The velocity of impact was then set equal to the distance travelled by the box divided by the time required. The stroboscope is a very accurate instrument with an error of less than 1%. In this respect, no calibration for the camera shutter speed was necessary.

It was observed that maximum impact velocity occurred when the sliding box was slightly below the still water level. In order to determine the maximum impact kinetic energy, it was decided that the maximum impact velocity would be used. The difference between the velocities was of the order of 5%.

After the impact, splash and great turbulence were formed. The water level indicator was placed in such a way that the splash would fall around the quarter length of the indicator. Thus, a general idea of the splash effect, wave breaking and adjustment of the disturbing water surface was recorded.

Wave profiles were then recorded further downstream by the system of wave probes and recorder. The wave probes were calibrated after each series of tests by successive small increments of immersion. A discrepancy of less than 1% could be expected in the wave height measurement.

The recording speed was 5 mm./sec. with traces of timing lines of 1/5 sec. on the recording paper. The refinement of the recording paper allowed accurate wave velocity to be measured.

A total of 47 sets of experiments were performed, three different slopes being investigated. For each slope, various weights were used to slide into water of differing depths. In general, the depths used were of 9 in. and 17 in. Solid vertical front boxes (with respect to the still water level) were used in most cases. Some cases with rectangular box front (inclined to the still water level) and porous vertical front boxes were also investigated.

PRESENTATION OF RESULTS

Experimental results for a slide slope of 17.7° are presented in terms of dimensionless parameters as defined earlier. Results for other slopes will be given later in this section for comparison.

In terms of K and H_{st}/d, Fig. 3 shows a considerable scatter of results. The maximum value of H_{st}/d is 0.53 at high input kinetic energy or at low water depths or a combination of both. Two general trends are observed. With slides characterized by high s/d values, bores are usually formed with the maximum values of H_{st}/d at 0.5. The value of K in this range is greater than 15. However, when the equivalent density of the slide, ρ_s, or the velocity of impact decreases, a sharp decrease in the wave generating efficiency is observed.

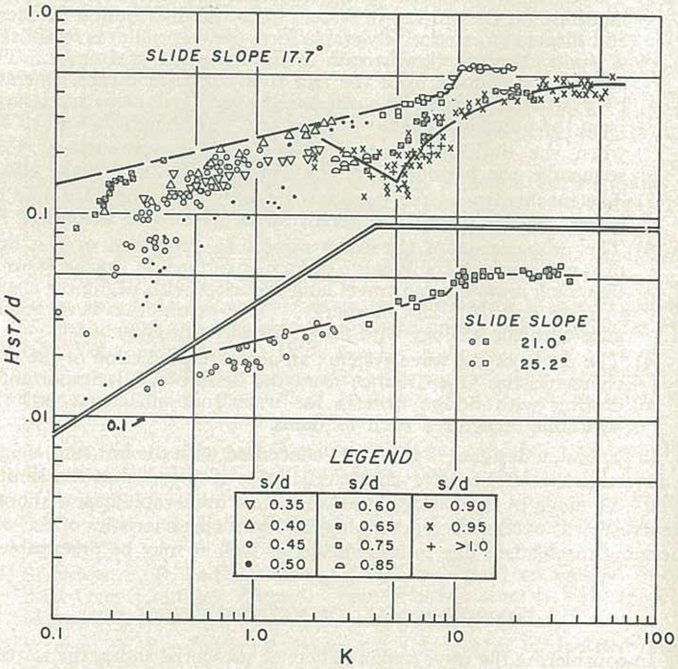

Fig. 3.—H_{st}/d vs. K.

When $s/d < 0.5$, the wave generating efficiency drops even at a faster rate with decreasing values of ρ_s or V_i. In general, the maximum values of H_{st}/d produced in this range of s/d form an upper envelope which has a maximum value of 0.5 and decreases steadily as the value of K decreases, caused by an increase in water depth or slower impact velocity or both.

For a value of K between 3 and 10, the wave system is complicated and unpredictable. It can be classified as a transition region where the wave transforms from a bore in relatively shallow water to the development of a wave train in relatively deep water.

When H_{max}/d is plotted against K as shown in Fig. 4, it is found that there is less scatter of data. The maximum difference in the value of H_{max}/d for a constant K can be as high as 80% in the transition region and

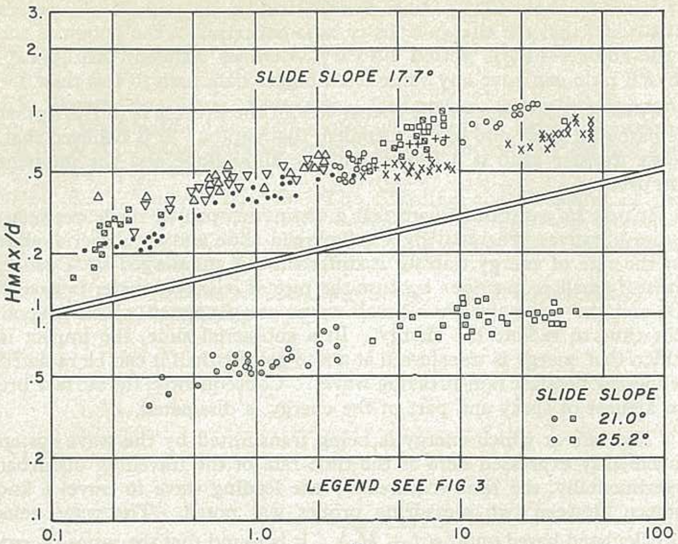

Fig. 4.—*H_{max}/d (from Level Indicator) vs. K.*

at low values of *K*. This plot is useful in the sense that it reveals the possible maximum water surface elevation near the point of impact. Values of $H_{\max}/d$ closed to 1.1 have been observed.

The important wave characteristic of wave length in terms of wave steepness, H_{st}/L, is related to H_{st}/d in Fig. 5. A very unique relationship is obtained. The dropping off of the curve and the scatter of points around H_{st}/d values greater than 0.15 signifies the transition region where the wave length cannot be determined precisely. Indeed, it is even difficult to classify whether the observed wave system is an impulsive train or merely a bore followed by minor disturbances.

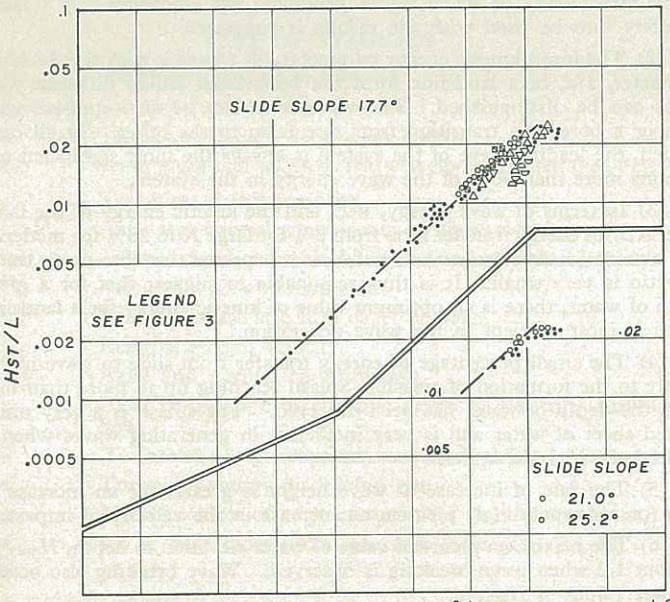

Fig. 5.—*Relationship between H_{st}/L and H_{st}/d.*

With a combination of Figs. 3 and 5, it is possible to obtain the values of *L* and *H* for a given landslide which plunges into a body of water of known depth.

After a wave type has been classified as either an impulsive wave train or a bore, the wave energy, *WE*, is calculated. Fig. 6 shows the variation of *WE*/*KE* with respect to *K*. When *K* approaches zero in the case of a small scale slide or at an infinite depth of water, the wave generating efficiency is low and *WE*/*KE* is low. It is expected that *WE*/*KE* increases as *K* increases. At $K = 0.4$, *WE*/*KE* approaches a maximum value of 28%. Then it decreases rapidly with increasing value of *K*, and approaches 2% at $K = 60$. It is believed that the rate of decrease of *WE*/*KE* will slow down substantially for any further increase in *K*. Indeed, it is expected that *WE*/*KE* approaches zero asymptotically as *K* approaches infinity.

Results for slide slopes of 21.0° and 25.2° are compared with those of a slide slope of 17.7°. In the plot of H_{st}/d against *K*, the points for the higher slide slopes follow closely to the envelope curve as shown in Fig. 3. There is hardly any distinction between the observations for the two slopes.

Fig. 4 for the plot of $H_{\max}/d$ against *K* shows a maximum value of $H_{\max}/d = 1.2$ for slide slope of 25.2° and the values of $H_{\max}/d$ are in general higher than those obtained for slide slope of 17.7°.

When H_{st}/L is related to H_{st}/d, it is found that the results obtained for the two larger slopes follow very closely to those for a slide slope of 17.7°. Since the values of H_{st}/d for these two larger slopes lie on the upper envelope curve of that for the smaller slope, it has only a narrow range of H_{st}/d values from 0.13 to 0.29.

The *WE*/*KE* values for these two larger slopes are found to have a maximum value of 28% at $K = 0.4$ as is shown in the case for a slope of 17.7°. The other values of *WE*/*KE* for these two slopes fall right on the envelope curve of slide slope of 17.7° as shown in Fig. 6.

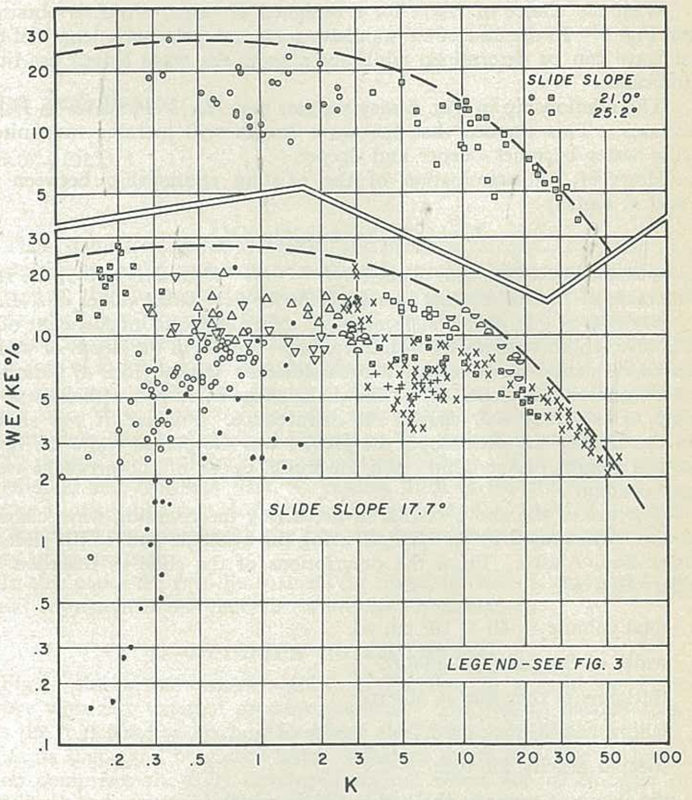

Fig. 6.—*Energy Transfer.*

With the similarities as shown above for the different slide slopes, the following discussion intended for the slope of 17.7° can be assumed to be valid for other larger slopes with slight modifications.

DISCUSSION OF RESULTS

In relating the wave height to the impact kinetic energy, roughly three regions can be identified. The value of *K* serves as a convenient factor to classify the wave system while *s*/*d* supplies further information about the slide which helps to indicate its wave generating efficiency (Fig. 3).

For a particular slide striking a body of water of depth, *d*, a value of *K* can be estimated. When $K > 15$, the wave generated has the form of a bore—an almost vertical front of great height which decreases gradually to the still water level only at a great distance. When $K < 3$, a wave train is formed. Usually the leading wave is the highest with a distinct crest and trough. The following waves can be of different magnitudes and, in general, the first two or three waves account for over 98% of the energy in the whole train. For design purpose, the upper envelope values of H_{st}/d should be used. Smaller values of H_{st}/d are caused by factors such as deep water, small impact velocity, a " thin " slide (*s*/*d* small) or a combination of these. One other important factor is that a stable wave may have been formed further upstream. The values of H_{st} recorded are therefore very much reduced due to wave dissipation. The distance required for the establishment of a stable wave form depends on many factors and no definite relationship to relate them is possible at the present time.

In the case of very deep water, it is an experimental observation (in an explosion and a landslide study) that little or no wave system can be started (Ref. 2). It is also evident from the fact that a landslide of very small thickness will simply pierce through the water surface without generating a disturbance of any significant magnitude even at high velocity. The small s/d values correspond to small values of K.

The values of $H_{\max}$ were obtained from the water level indicator. Very often, the profile was masked by the splash or the breaking of waves. The choice of the location for the measurement of $H_{\max}$ is quite arbitrary other than the fact that the localized effect of the splash appears to be important and that the process of dispersion is imminent. Any discrepancy in the measurement of $H_{\max}$ within 30% can be made. The main interest in this representation is to show the possible maximum water elevation near the slide site and is not intended for design purposes. A maximum $H_{\max}/d$ value of 1.2 has been recorded in the tests with slide slope of 25.2°. A comparison of test results of $H_{\max}/d$ for different slopes shows that higher values are obtained for steeper slopes. Ippen and Kulin (Ref. 3) have found in a study of solitary wave systems that H/d can be about 1.2 for a slight slope. For steeper slopes, the ratio is even higher.

With the choice of H_{st}/d for a designed K value, H_{st}/L is obtained from Fig. 5. In the case of an impulsive wave train, the wave length of the first wave can be determined while for a bore, the wave length has little significance.

The relationship in Fig. 5 may suggest that H_{st}/L decreases as H_{st}/d decreases. This implies that the wave length will increase indefinitely as the water becomes deeper and deeper.

However, an examination of the existing relationship between L, T and d, namely

$$L = (gT^2/2\pi) \tanh (2\pi\, d/L)$$

shows that L increases with d until $d/L = 0.5$. With $d/L < 0.5$, the rate of increase of L with d is not linear except for very small values of d/L.

For $d/L > 0.5$, deep water condition exists and L is independent of d. The wave length remains a constant for any increase in the depth of water. The experimental study on submerged slides by Wiegel (Ref. 4) indicated that T (or its equivalents, L) of the disturbance increased with increasing length of the body that caused the disturbance. Further, it was shown that the smaller the slide slope, the greater was the period while the body size and weight, water depth, and the initial depth of submergence were kept constant.

The use of the above results in predicting the resulting wave characteristics is illustrated by an example using the available data of the slide in Lituya Bay, Alaska. From the descriptions of the slide as recorded by Miller (Ref. 5),

total volume = 40×10^6 cu. yd.

width of the slide = 2,000 ft.

thickness of the slide = 300 ft.

falling height, measured from centre of gravity = 2,000 ft.

specific gravity for rock = 2.7

the depth of water at the bay is 400 ft. which is assumed to represent the average depth at the site of the slide.

Thus, $s/d = 0.75$, and $K = 0.23$.

For a slide of this size and water depth, Fig. 3 indicates that a wave similar to a bore (or solitary wave) will form with a complex system of smaller waves to follow. From Fig. 4, $H_{\max}/d = 0.3$ and from Fig. 3, $H_{st}/d = 0.16$. Thus, a maximum water elevation of 120 ft. in the slide site and a wave of about 64 ft. travelling down the bay should be expected, provided that the bay can be considered homologous to the model used in the present study. Owing to the geometrical complications in the slide site and in the bay, there were interference patterns caused by reflection, refraction and diffraction. The effects of these factors on the wave pattern can only be analyzed with difficulty even for a rough estimation. The wave height in the bay as observed by eyewitness was a straight wall of water possibly 100 ft. high extending from shore to shore. A model test of 1 : 1000 scale of this case was conducted by Wiegel (Ref. 6). It was observed that a large wave several hundred feet high with a complicated tail could be formed due to reflection and the effect of bottom hydrography.

The discrepancy of H_{st} between the observed and predicted values may be taken care of by a " factor of safety " of 1.5. This " factor of safety " accounts for the different assumptions made in assigning input dimensions of the landslide and the water depth and the fact that the simple model used to obtain generalized results are quite different from its counterpart in nature. Similarly, a " factor of safety " of 2 in the use of the predicted value of $H_{\max}$ is recommended.

When the wave energy, WE, is compared with the impact kinetic energy, KE, in this study, it is found that WE/KE can be as high as 28%, and the low value is about 2%. Other related studies on impulsively generated waves show that the energy transfer ratio (in terms of the potential energy of the source, PE) is within 1 to 2%. Under different conditions, the KE/PE ratio can have any value from bigger than zero to less than 1.

A large percentage of the energy loss in the process is in the formation of splash and spray and in the turbulent dissipation. It is believed that the energy transfer ratio is a function of the slide slope and the momentum of impact.

It may be mentioned here that a distinction can be made between the studies of waves generated by a submerged slide and a sub-aerial slide in that the rate of energy transfer is different. A submerged slide generates waves of small proportions because the rate of energy transfer between the slide and the water is slow. Small waves are generated which have sufficient time to radiate the energy. In a sub-aerial slide, the impact is so sudden that energy is transferred at a rate greater than it can be radiated by the highest possible non-breaking waves. Consequently, the surface breaks into a sheet of spray and part of the energy is dissipated.

The rate at which energy is being transmitted by the wave system is conveniently expressed here as the time rate of the travelling disturbance. Experimentally, the time required by the leading wave to travel a known distance between two measuring probes was noted. The wave velocity was calculated based on $\sqrt{g(d + H_{st})}$. It is found that the ratios of experimental to theoretical wave velocities lie between 0.9 and 1.2. This ratio is affected to a certain extent by the shape of the leading wave since measurements were taken from crest to crest of the waves. However, the experimental results show that at the points where measurements are taken, the wave system can be considered in the range of shallow water waves and its subsequent motion will be governed by the depth of water and the wave height.

CONCLUSIONS

(1) An experimental study of the generation of water waves by the impact of a sliding mass into a body of water was studied in a two-dimensional channel. Characteristics such as H, L, d and c are related to the kinetic energy of the slide. Dimensionless parameters are formed to enable predictions of water waves generated by landslides. A " factor of safety " to be used with the results is suggested.

(2) The input kinetic energy parameter, K, together with the thickness parameter, s/d, of a landslide form the basis from which different wave forms can be distinguished. The wave form can be an impulsive wave train or a bore or a transition from one form to the other. In all cases studied, the leading wave of the system is always the most significant and contains more than 90% of the wave energy in the system.

(3) In terms of wave energy, WE, and the kinetic energy of the landslide, KE, an energy transfer ratio from 2% for large K to 28% for moderate K is obtained. At very low value of K, it is expected that the energy transfer ratio is very small. It is thus reasonable to suggest that for a given depth of water, there is an optimum value of kinetic energy for a landslide which is most efficient in the wave generation.

(4) The small percentage of energy transfer from slide to wave is due mainly to the formation of splash. Splash reaching up to more than four times the depth of water has been observed. The splash is a very much aerated sheet of water and is very inefficient in generating waves when it strikes back into the water.

(5) The rate of increase of wave height is greater for an increase in mass (or correspondingly ρ_s) than an increase in the velocity of impact.

(6) The maximum recorded value of water elevation to depth, $H_{\max}/d$, is about 1.2 when wave breaking is observed. Wave breaking also occurs at lower values of $H_{\max}/d$.

(7) The effects of different slide slopes in the wave generation seems to be of secondary importance when compared with the sliding mass and the impact velocity. The geometrical factors of the slide front such as front angle and porosity are found to produce insignificant difference in the measurements of different wave characteristics.

(8) For the cases in which the values of H_{st}/d fall on the upper envelope, it is found that the early attenuation of the maximum wave is inversely proportional to the square root of the distance of travel.

(9) The foregoing results concerns mainly with the generating stage of the induced wave system. It is believed that the propagation and the shoaling effect of these waves can be studied with the theory as outlined by Unoki and Nakano (Ref. 7) and Kranzer and Keller (Ref. 8). In general, the Stokian wave theory can be used as well as the conoidal wave theory in studying the shoaling effect. In very shallow water, solutions developed for unsteady wave motion by the method of characteristics, might be helpful.

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The Turbulent Plane Jet-Wake

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Summary.—A jet-wake is defined to be the flow downstream of a jet which is surrounded by a parallel flow of fluid. The various regions of the flow—potential core or cavity, self-preserving jet, self-preserving wake, quasi-equilibrium flow, etc.—are mapped as a function of the jet velocity ratio, and the distance downstream. The effect of the initial conditions on the flow are discussed. Existing theories of jet and wake spreading are discussed as they apply to the fully developed jet-wake in a zero pressure gradient. The recent theory of Townsend (Ref. 1) has been extended in this way and the predictions are compared with other theories and the available experimental data.

LIST OF SYMBOLS

D	Width of initial jet or obstacle.
I_1, I_2, I_3	Velocity profile integrals defined by Eq. (17).
L	Turbulent energy dissipation length.
L_0	Jet or wake half width to half velocity increment (see Fig. 2).
$\overline{q^2}$	Turbulent energy intensity.
U, V	Axial and normal components of mean velocity.
u, v	Axial and normal components of velocity fluctuation.
U_j	Velocity of jet at nozzle exit.
U_0	Velocity excess at centre of jet or wake.
U_1	External stream velocity.
x, y	Axial and normal co-ordinates.
x_0	Axial co-ordinate of effective origin of the fully turbulent flow.
β	Townsend's "entrainment constant" defined by Eq. (13).
ε	Turbulent energy dissipation rate.
η_0	Mean position of edge of turbulent fluid from the axis.
θ	Jet or wake momentum thickness defined by Eq. (2).
λ	U_0/U_1.
ρ	Density.

1.—INTRODUCTION

A jet-wake is here defined to be the flow downstream of a jet issuing into a parallel moving stream of fluid; that is, it is the wake of the jet in the external flow. The initial velocity of the jet may be higher or lower than that of the external flow. At one extreme, that of negligible external flow velocity, the flow is that of the classical 'pure' jet in still surroundings. At the other extreme, negligible jet velocity is the classical wake flow. In the middle, with effective jet velocity equal to that of the external flow (giving zero momentum defect), is the interesting case corresponding to the wake of a self-propelled body.

These flows are of great theoretical and practical interest. Applications include the wakes of rocket plumes and re-entering bodies, of ships and submarines, the spread of turbulent flames in afterburners, ramjets, and "scramjets", and internal flows such as in ejectors and compressors. Of course in many of these situations the turbulent transport process is only a part of a much more complex phenomenon; however it is basic and usually the most difficult to predict. The flows are of great theoretical interest since they throw light on the mechanism of entrainment in free turbulent flows and the effect of history on the turbulent processes.

No general survey of the literature is intended here. Two recent papers, Hill (Ref. 2) and Newman (Ref. 3), discuss most of the recent work and bring up to date such standard texts as Hinze (Ref. 4). Most of the work done has been concerned with the flow far downstream from the jet for a variety of geometries.

In this paper the two-dimensional (or plane) jet-wake is discussed. A rational approach to the generalized problem is attempted.

2.—REGIMES OF THE FLOW

Fig. 1 shows the various regimes of the flow in a turbulent plane jet-wake with zero external pressure gradient. The shaded portion comprises the initial region of the flow where the initial conditions of the jet, such as its shape and boundary layers, influence the flow. The unshaded portion comprises the fully developed region, where the *development* of the flow is independent of the initial conditions of the jet. The only effect of the initial conditions in this region is in the determination of the

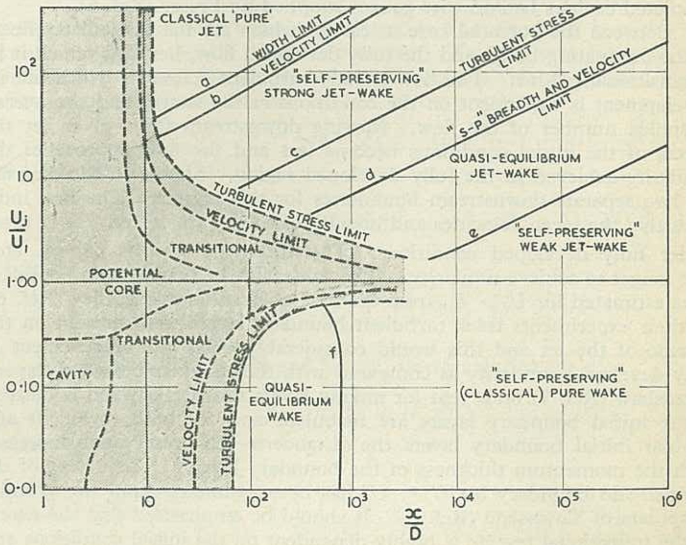

Fig. 1.—*Flow Regimes of the Turbulent Plane Jet-Wake.*

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[*] Paper No. 2629, presented at the Third Australasian Conference on Hydraulics and Fluid Mechanics held in Sydney from 25th to 29th November, 1968.
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