The Motion of Solitary Particles on the Fixed, Granular Bed of a Stream

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Summary.—This progress report upon a research now going on refers to the speeds of solitary particles travelling along the beds of water streams. Experiments have been made on smooth surfaces and on immovable rough surfaces, the latter being composed of particles of the same size as the moving particles. Experimental results have been successfully correlated for particles of each shape, but differences between shapes of particles seem to be very important in governing their speed. The force system on single moving particles has been investigated, and a theoretical model is proposed.

LIST OF SYMBOLS

V_p	The mean velocity of the particles.
$\overline{V}$	The mean velocity of the stream.
V_x	The shear velocity.
V_s	The settling velocity of particles in still water.
C	Coefficient of drag.
d	Diameter of particles.
F	Driving force on particle.
g^1	Effective gravity under water.
M	Mass of particle.
u	Stream velocity at height y above bed.
k	Roughness size in Prandtl rough boundary velocity distribution.
n	Ratio of $u : V_x$.
ρ_w	Density of water.
ω	Angular velocity of particle.

INTRODUCTION

In the study of the movement of sediments by rivers, there has been much work on the stream conditions necessary just to initiate motion of sand particles (the threshold conditions). There is far less known about the motion of the particles after they have started, largely because of the difficulty of observing individual particles amid a host of others. The properties of their motions must be inferred by indirect means, or the product of the number of grains moving and their speed must be observed as the transport rate. In the course of an investigation into the erosion of sewer pipes by grit, some measurements were taken of the speed of solitary particles moving along a smooth surface, and these have now been extended to rough surfaces. When the solid, immovable surface consists of roughness elements of the same size as the moving grains, there is an obvious analogy with the initial stages of movement of a bed of sand not made so immovable. Of course, the moving grains do not collide with other grains that are moving, or can be moved, so that this aspect is not reproduced. The reduction of the number of parameters concerned with sand transport, however, allows some simplification. The research is in progress at Imperial College, London, and Portsmouth College of Technology.

Little work appears to have been done previously on this aspect. Ippen and Verma (1953, Ref. 1) observed the motion of spheres rolling over a sand-roughened bed of a water stream, but they did not carry their experiments to the stage where the bed roughness and moving grains were the same. Binnie and Kuo (1966, Ref. 4) have observed the speed of large spheres in smooth bored pipes, but their densities were near that of water so they were largely in turbulent suspension in the stream. Spheres rolling in contact with solid surfaces in a stationary non-sheared fluid have been observed by Carty (1957, Ref. 3). There is little doubt that high concentrations of moving particles affect the distribution of fluid velocities; for example, Newitt et al (1962, Ref. 5) studied the movement of high concentrations of particles in a horizontal water pipe. Sinclair (1962, Ref. 6) showed that the limit deposit velocity was also affected by concentration. Thus an investigation of particle movement under conditions where concentration effects are not present is a simplification of natural conditions in rivers. Care must be taken in extrapolating results obtained from these solitary particle experiments to cases where many are in movement.

PARTICLE MOVEMENT ON SMOOTH SURFACES

Experiments were carried out in a 7-cm. dia. Perspex pipe, flowing full of water. Particles of a variety of sizes, shapes and densities were inserted singly into the stream, and their velocities observed. At low speeds visual observation with stopwatch was used; at higher speeds a photographic method was necessary. The shear stress was found by observing the longitudinal pressure gradient. The coefficients of solid friction of the particles with the wall were measured by tilting the pipe with static water in it until the particles just moved.

Some typical results are shown in Fig. 1, where it will be seen that the relationship between the shear velocity V_x and the particle speed V_p approaches a straight line. Indeed, V_p may be a useful method of determining the shear in a stream, and of plotting its variations from place to place in complicated situations. The method is preferable to Preston tubes in low-speed streams, where kinetic energy heads are low and difficult to measure. Initially, particles were rolling or sliding in continuous contact with the wall. Ultimately, as V_x increased, the sliding grains began to saltate. There was no significant difference in the $V_p - V_x$ relationship for all four cases.

A very simple theoretical model for the motion has been made which assumes two-dimensional flow around the particle at the relative velocity appropriate to the distance from the wall (as given by the usual smooth wall pipe-friction equation). Thus, near the wall, the water travels slower than the sliding particle, but near the top of the particle, travels faster. The particle is therefore driven near its top, retarded near the bottom, and is subjected to a friction force. On sloping pipes, a gravity force component is included. With these somewhat gross simplifications, a constant drag coefficient of $C = 1.2$ for angular, sliding grains gave values of V_p in good agreement with observed values. This model ignores any lift force by the curved flow over the particles. For rolling, spherical, particles a similar analysis using $C = 0.5$ gave equally good results. Since a purely force balance analysis appears to give adequate agreement with observations in this smooth wall case, there is some justification for using another force balance analysis in the case of particles moving over a rough surface. The selected values of C above are not far different from those which are appropriate to isolated particles falling in still fluid.

PARTICLE MOVEMENT ON ROUGH SURFACES

Sets of experiments have been carried out on two rough surfaces, being the plane beds of 10 cm. wide, glass sided channels. Water flowing over the beds was measured by volume or weight, and the depth of flow by pointer gauges. The longitudinal slope was adjusted until normal flow (i.e., constant depth along the channel) was found, and the shear stress thereby inferred. The grain speed V_p was again measured visually, the average speed of several grains being taken. Depths varied from 1.6 cm. to 5 cm.

The surfaces concerned were made by gluing particles to a flat base by a suitable cement, care being taken to ensure that the mean surface of the bed did not have undulations greater in length than about 2 particle diameters. The bed particles were.—

(*a*) Well-rounded quartz gravel, mean size 0.75 cm. (described as pea-gravel).

(*b*) Uniform pieces of polystyrene, rhombic in shape, cut from a nearly square section rod 0.22 × 0.28 cm., with the ends about 45° to the axis. The size across the diagonal was 0.42 cm. These particles were obtained in black and white colours at the same density and size, so that moving particles can easily be seen, as white against a black bed.

Experiments are being done with a variety of moving particles, but from the aspect of sediment movement the significant tests were those where the moving particles were the same size and shape as the fixed ones. The particles (*b*) could be drilled with a watchmaker's drill and pieces of lead wire inserted, so that a range of settling velocities V_s could be obtained with the same size and shape of particle.

Some early results are shown in Fig. 2, with the observed velocity V_p on bed (*a*). It is clear that V_x is a primary parameter, but at the time of writing there appears a small unexplained additional effect of the bed slope. Observations on bed (*b*) were made with particles weighted to three different settling velocities, and were correlated on Fig. 3, using axes of $V_p/\overline{V}$ and V_s/V_x where $\overline{V}$ is the mean velocity of flow in the channel. The experiments on quartz particles are also shown on the same axes.

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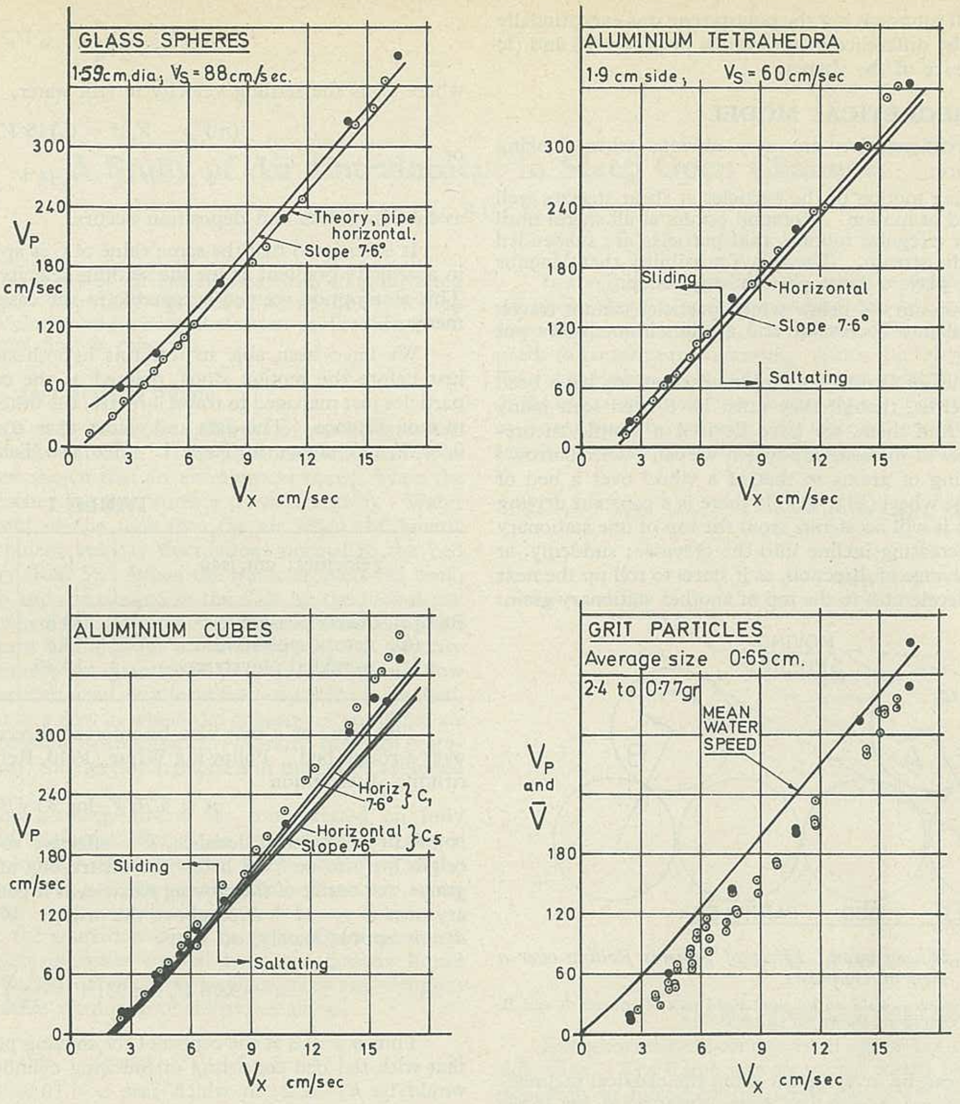

Fig. 1.—The Mean Speed V_p of Individual Particles of Different Shapes in a Smooth Bore Water Pipe.

The theoretical two-dimensional flow system, with coefficients of drag as described in text gave the lines shown. Aluminium cubes had properties—*C*1, 1.75 cm. side, weight 14.8 g.; *C*5, 0.79 cm. side, weight 1.32 g. Mean water speed in pipe $\bar{V}$. Open circles pipe horizontal; solid circles pipe at 7.6°.

For the particles of bed (*b*), the variables were changed in the ranges

$$3.4 \text{ cm./sec.} < V_s < 10.35 \text{ cm./sec.}$$

$$12.6 \text{ cm./sec.} < \bar{V} < 44.5 \text{ cm./sec.}$$

$$1.19 \text{ cm./sec.} < V_x < 4.86 \text{ cm./sec.}$$

As a consequence

$$6.6 \text{ cm./sec.} < V_p < 39.8 \text{ cm./sec.}$$

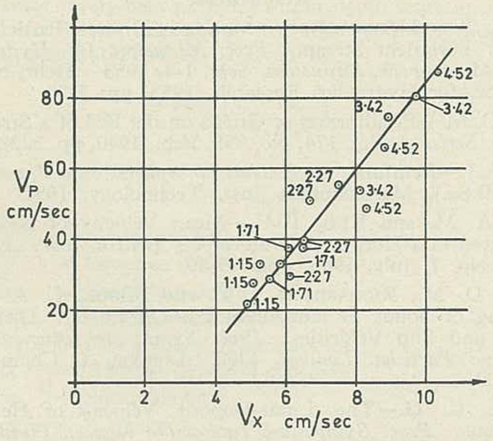

Fig. 2.—The Mean Speed of Pea-Gravel 0.75 cm. dia. in a Water Stream over a Fixed Rough Bed of the Same Gravel.

Figures against points indicate the bed slope, parts per 100. Particle weight 1.69 g. average.

In these wide ranges, it appears that the above non-dimensional quantities give a fairly satisfactory correlation of results for particles of the same shape.

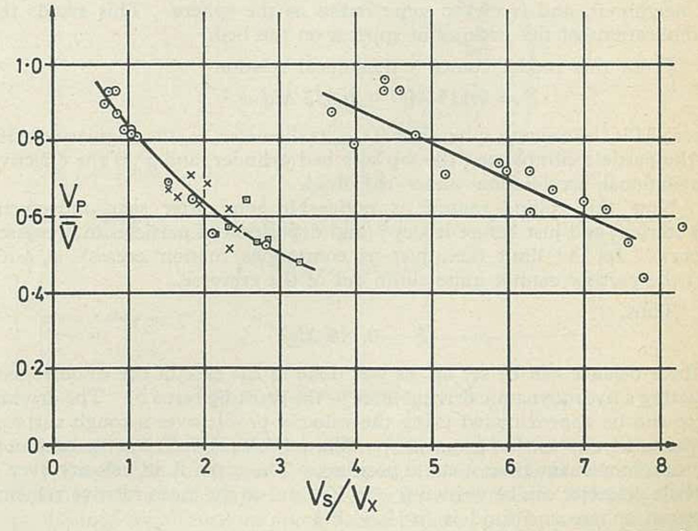

Fig. 3.—Non-Dimensional Plot of Particle Velocity V_p over Rough Beds.

Settling velocity in still water, V_s, mean stream velocity $\bar{V}$. Particles of bed (*a*) on right, data from Fig. 2, $V_s = 39$ cm./sec. Particles of bed (*b*) on left. The latter particles were weighted to give $V_s = 3.4$ cm./sec. (circles); $V_s = 7.4$ cm./sec. (crosses); $V_s = 10.5$ cm./sec. (squares).

Since the pea-gravel was well rounded, but the polystyrene was exceptionally angular, it is thought that the difference of the results of group (*a*) and (*b*) is mainly due to the difference of the shapes.

A THEORETICAL MODEL

Two qualitative observations that are very obvious when looking at single moving particles are:

(i) the essentially rolling motion of the particles at shear stresses well above the threshold of motion. Rotation occurs at all stages until it is clear, by their irregular motion, that particles are suspended by turbulence in the stream. There is a possibility that Magnus effect on particles plays a large part in suspension.

(ii) that there is a minimum V_p below which particles cannot travel, since they then fall into a crevasse and are not immediately put into motion again.

It is thought that neither of these two simple observations have been described before in these terms, though they must have been seen many times by experimenters. From them, we have devised a simple theoretical model for the first stages of motion, and which we call ' wheelbarrow ' motion. We liken the rolling of grains to that of a wheel over a bed of stones of the same size as the wheel (Fig. 4). If there is a constant driving force F on the moving grain it will accelerate from the top of one stationary grain, and roll down the increasing incline into the crevasse; suddenly, at the bottom, it undergoes a change of direction, as it starts to roll up the next gradient; the particle then decelerates to the top of another stationary grain.

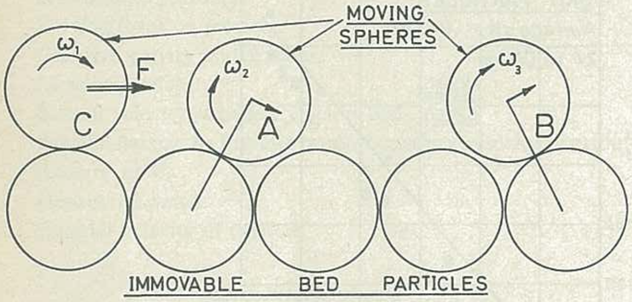

Fig. 4.—Diagram to Show ' Wheelbarrow ' Effect of Spheres Rolling over a Bed of Cylinders.

Change of direction at bottom of crevasses shown by right hand pair of spheres A and B. Initial conditions shown on left at C.

The first and third motions can be investigated using the classical inclined-plane analysis; the intermediate motion has been investigated by Inglis (1951, Ref. 7) who, by assuming conservation of angular momentum, showed how there must be a loss of kinetic energy as an inelastic wheel goes over a pot-hole in a road. A driving force is therefore necessary which depends on the pot-hole : wheel radius ratio. This model ignores hydrodynamic lift forces (which White (1940, Ref. 2) thought small) and viscous damping of the motion as fluid is displaced from the bottom of the crevasse by a falling grain. As a further simplification, we consider a sphere as rolling over a two-dimensional bed of cylinders. Each cylinder touches its neighbour, and is of the same radius as the sphere. This avoids the complications of the packing of spheres on the bed.

From this model comes a dynamical relation

$$F = 0.115\, Mg^1 + 0.575\, Md\, \omega_1{}^2$$

where M is the moving sphere's mass, d its diameter, ω_1 the angular velocity as the particle climbs over the top of a bed cylinder, and g^1 is the effective gravitational acceleration under the fluid.

Now this rolling motion is noticeable soon after sand movement has started, and just before it stops (and deposition of particles in crevasses occurs). In the limit (i.e., just as continuous motion ceases) $\omega_1 = 0$, and the particle cannot quite climb out of the crevasse.

Thus,

$$F = 0.115\, Mg^1$$

A force balance can be set up, as was done in the case of the smooth bed, equating a hydrodynamic driving force to the resisting force F. The driving force can be approximated using the velocity profile over a rough surface, in the same way as that proposed by White (1940, Ref. 2) for the initiation of grain movement from a static position. The mean fluid velocity over a particle diameter can be written $u = n\, V_x$, and so the mean relative velocity between sphere and fluid is $(n\, V_x - V_p)$.

Thus driving force $= \frac{C}{2}\frac{\pi}{4} d^2\, \rho_w (n\, V_x - V_p)^2$ where C is the coefficient of drag. Equating this force to F, and substituting,

$$M g^1 = \frac{C}{2}\frac{\pi}{4} d^2\, \rho_w\, V_s{}^2$$

where V_s is the settling velocity in still water,

$$(n\, V_x - V_p)^2 = 0.115\, V_s{}^2$$

or

$$V_p = n\, V_x - 0.34\, V_s$$

as motion ceases and deposition occurs.

It is assumed that the same value of C is appropriate for a driving force in a velocity gradient as for the settling velocity under gravity in still fluid. This assumption seemed acceptable in the case of the smooth bed experiments.

We have been able to test this hypothesis by observing V_p and V_x just before the motion stops, defined as the condition when 50% of the particles just managed to travel 1 metre, the other 50% stopping in crevasses in that distance. The data and values of n, computed from $V_p = n\, V_x - 0.34\, V_s$, are shown in Table I. (See also Table IA on p. 241—*Ed.*)

TABLE I

Velocities: cm. /sec.	V_s	V_x	V_p min.	n
Bed (*a*), quartz pea-gravel	38.6	4.96	21.6	6.6
(*b*), natural polystyrene	3.4	1.17	5.99	6.1
(*c*), weighted polystyrene	10.35	2.98	15.0	6.2

The value of n may also be inferred directly from the velocity profile over a rough bed. Following White (1940, Ref. 2), we assume that a logarithmic distribution

$$u = 5.76\, V_x \log 33\, y/k$$

is valid (y = vertical distance, k = effective roughness size). Taking the origin for y to be $k/33$ below the centre-line of the top layer of stationary grains, the centre of the moving particle, as it goes over a crest of the stationary ones is $y = d + k/33$ above the origin. With typical sand roughness, $d = k$ approximately, so that

$$u = 5.76\, V_x \log 33\left(1 + \frac{1}{33}\right) = 8.8\, V_x$$

Thus $n = 8.8$ at the centre of the moving particle. It might be argued that with the bed consisting of touching cylinders, a better approximation would be $k = d/2$; in which case $n = 10.5$.

It is no doubt somewhat untrue to believe that the logarithmic velocity profile is appropriate very close to the bed particles; this could account for the discrepancies between the observed and theoretical values of n.

The variable geometry of the crevasses in the case of the polythene grains, and their notably non-spherical shape doubtless makes our simple model less realistic than in the case of the well-rounded pea-gravel of bed (*a*). Further experiments, on a bed of glass cylinders across the stream over which glass spheres will be observed, are intended; these will be reported at the Conference. This bed, though further from the practical case of sand grains, will be a better approximation to our simplified conditions of the above theory.

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