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# Wind Generation of Water Waves, Part II

BY P. J. BRYANT, PH.D.*

*Summary.*—A connected theory for the initiation and amplification of water waves by wind is presented, and is compared with recent experimental work in the subject. A dimensional analysis, based on the idea that the interaction between the wave induced air motion and the turbulent air flow decreases uniformly with distance from the water surface, is used to develop an argument that the rate of wave amplification is a simple function of $(u_*/c)$, where $c$ is the phase velocity of the water wave motion and $u_*$ is the friction velocity of the wind at the water surface.

## 1.—INTRODUCTION

The three stages of water wave generation by wind are initiation, amplification, and saturation. In each waveband, the water wave motion is initiated by one or more of several processes, amplified by the turbulent pressure field by further processes, and reaches saturation when the energy transfer into and out of the waveband is in balance.

This paper is a sequel to one on the same subject presented at the Second Conference (Ref. 1). Attention was drawn in that paper to the importance of the air turbulence in water wave generation, and an estimate of the rate of wave amplification due to the air turbulence was made. The analysis of the initiation and amplification stages is extended and improved in this paper, mainly as a result of the stimulation provided by recent experimental work in the subject (Refs. 2, 3, 4, 5, 6, 7 and 8).

## 2.—INITIATION

The turbulent pressure field amplifies to a greater or lesser extent all waves that are initiated in any particular situation, so the initiation stage determines the range of frequencies in the resulting generated spectrum. The wave motion may be initiated by sources external to the area of observation, by motion of the water, or by the turbulent pressure field. The first cause is self-explanatory and is the main reason for the difficulty in interpreting observations at sea. The second cause is not as common as the first and third, but does apparently occur when waves are generated on running water (Ref. 9). Atmospheric motions are invariably turbulent, and the air flow in experimental situations is usually turbulent, so the third cause is almost always present when interaction occurs between wind and water.

The mechanism of wave initiation by the turbulent pressure field is described by Phillips (Refs. 10 and 11). It is based on the property that waves of a given wavelength on a free water surface have a given frequency related to this wavelength. This property influences the energy input from the turbulent pressure field to the water surface, since a form of resonance results from the forcing of the natural wave modes on the surface.

Recent wind tunnel experiments, e.g., by Wills (Ref. 12), have indicated that the spectral energy density of the turbulent pressure field is spread over a broad band of frequencies and wave numbers. Wind-water tunnel experiments, e.g., by Sutherland (Ref. 5), have shown that the initial response of an undisturbed water surface to the turbulent pressure field consists of waves propagating in the mean wind direction with a spectral energy density spread over a broad band of frequencies. It is suggested therefore that this observed initial broad waveband response of the water surface is the result of initiation and resonance by the turbulent pressure field.

Phillips assumed that the spectral energy density of the turbulent pressure field is peaked along a narrow band of wave numbers and frequencies, which implies that the turbulent pressure field initiates and resonates with only a narrow band of frequencies along the mean wind direction. It also implies that the turbulent pressure field can initiate and resonate with higher water wave frequencies at angles inclined to the mean wind, without such initiation being submerged in the initiation and resonance along the mean wind direction. If the wave initiation in the mean wind direction by the turbulent pressure field is spread over a broad waveband, as is suggested above, the directional nature of the water wave spectrum predicted by Phillips would be obscured by the broad waveband response in the mean wind direction.

Phillips' directional spectrum has not yet been observed in a wind-water tunnel, but has been observed at sea, e.g., by Gilchrist (Ref. 6). One possible explanation is that the spectral energy density of the turbulent pressure field is of different form in a wind-water tunnel than at sea.

## 3.—AMPLIFICATION

A non-zero covariance is set up between the water wave motion and the air pressure on the water surface, or in other words, the air pressure in any given waveband is partly free and partly forced by the water wave motion. The covariance exists whenever water wave motion is present, and in any given waveband it is a linear coupling when the wave motion is sufficiently small, becoming a non-linear coupling as saturation in the waveband is approached. The amplification stage, which is discussed in this section, refers to the stage when the covariance exists as a linear coupling. During this stage, the wave motion in the given waveband is amplified exponentially.

Water wave amplification has been analysed extensively by Miles, beginning in 1957 (Ref. 13) with a quasi-laminar model of the turbulent air flow. A more recent model (Ref. 14) includes some of the effect of the air turbulence on wave amplification, and neglects aspects of the non-linear interactions within the turbulent air flow. Phillips (Ref. 11) made a detailed study of the turbulent air flow over a water surface, but his analysis depends on several assumptions which have not yet been proven experimentally. An important assumption he made several times was that turbulent air flow over water waves is very similar to laminar air flow over water waves. This assumption appears in the mean streamlines sketched in his figure (4.3), which are almost identical with those found in laminar air flow, and in his approximation (page 95) that the explicit contribution of the air turbulence to the mean rate of momentum transfer into the water motion may be neglected.

Consider now the amplification of the Fourier wave component of the water surface propagating in the mean wind direction ($x$-direction) in the waveband centred on the wave number $k$ and the frequency $\omega$ ($= kc$). The water surface is assumed to be locally statistically stationary

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and homogeneous, for the purpose of Fourier analysis, though under the action of the wind it may amplify in the given waveband over distances large compared with the wavelength or during times large compared with the period. The wave component in this waveband is therefore written

$$\zeta_k = a_k \cos(kx - \omega t) \quad \ldots\ldots\ldots\ldots(3.1)$$

as a local approximation. Over larger distances or times, it should be written

$$\zeta_k = a_k(x, t) \cos[kx - \omega t + \varepsilon_k(x, t)] \quad \ldots\ldots\ldots\ldots(3.2)$$

where $a_k(x, t)$, $\varepsilon_k(x, t)$, are slowly varying functions of $x, t$ in the sense above (Ref. 15).

The air velocity field is written $(U + u_k + u', v', w_k + w')$ where $(U, 0, 0)$ is the mean velocity, $(u_k, 0, w_k)$ is the wave induced air velocity in the given waveband, i.e., the waveband centred on the wave number $k$ and the frequency $\omega(k)$, and $(u', v', w')$ is the turbulent velocity.

The profile of the mean velocity, $U(z)$, is dependent on the water wave motion (Ref. 4). The wave induced velocity $(u_k, 0, w_k)$ is the result of the interaction in the given waveband between the air velocity field and the water motion, and may be measured from the covariances $\overline{u\,\zeta_k}$, $\overline{w\,\zeta_k}$. The turbulent velocity $(u', v', w')$ is the random fluctuation remaining when the mean velocity and wave induced velocity in the given waveband are subtracted from the air velocity field.

The turbulent velocity gives rise to various mean velocity products such as $\overline{u'^2}$, $\overline{u'w'}$, etc. It contributes explicitly to the coupling between the air and water motions as a wave induced perturbation of these mean velocity products, denoted by $(u'^2)_k$, $(u'w')_k$, etc., and which may be measured from triple covariances of the form $\overline{u'^2\,\zeta_k}$, $\overline{u'w'\,\zeta_k}$, etc.

The equations for the wave induced air motion are then (Ref. 1)

$$(U - c)\frac{\partial u_k}{\partial x} + w_k \frac{dU}{dz} + \frac{\partial}{\partial x}(u'^2)_k + \frac{\partial}{\partial z}(u'w')_k = -\frac{1}{\rho}\frac{\partial p_k}{\partial x} \qquad (3.3)$$

$$(U - c)\frac{\partial w_k}{\partial x} + \frac{\partial}{\partial x}(u'w')_k + \frac{\partial}{\partial z}(w'^2)_k = -\frac{1}{\rho}\frac{\partial p_k}{\partial z} \quad \ldots\ldots(3.4)$$

where $p_k$ is the wave induced pressure in the given waveband. If it were possible to solve these equations, subject to the equation of continuity and the boundary conditions, the pressure $p_k$ on the water surface could be determined, and an expression for the rate of amplification in the given waveband derived.

The spectral energy density of the water wave motion in the given waveband is $\frac{1}{2}\rho_w g\, a_k^2$, which may be written alternatively as $\rho_w g\, \Phi(\omega)\, \Delta\omega$, where $\Phi(\omega)$ is the frequency spectrum of the mean square surface displacement, and $\Delta\omega$ is the width of the given waveband. The rate of change of $\Phi(\omega)$ during the initiation and amplification stages of wave generation satisfies

$$\left(\frac{\partial}{\partial t} + c_g \frac{\partial}{\partial x}\right)\Phi = \alpha + \beta\Phi \quad \ldots\ldots\ldots\ldots(3.5)$$

an equation first formulated by Hasselmann (Ref. 16), where $c_g$ is the group velocity. The processes of wave initiation determine $\alpha$, and the mechanisms of wave amplification determine $\beta$.

## 4.—DIMENSIONAL SIMILARITY

If it were possible to solve the equations of motion (3.3), (3.4), for the wave induced pressure, $p_k$, on the water surface, the solution would express $p_k$ as a function of the parameters describing the water wave motion and the turbulent air flow. An expression for the rate of wave amplification could then be obtained, and would be a function of the same set of parameters. In the analysis following, a subset of these parameters is found on which the rate of wave amplification depends dominantly, and the principle of dimensional similarity is applied to this subset of parameters to find an expression for the rate of wave amplification.

In Miles' quasi-laminar model (Ref. 13), the coefficient of wave amplification, $\beta$, (Eq. (3.5) above), is determined by the structure and position of the critical layer, the layer in the air flow where the mean wind velocity is equal to the wave velocity. The strong dependence of the rate of amplification on a narrow region of the turbulent air flow, requiring several parameters to describe it, rules out the possibility of a dimensional similarity analysis of a simple form for a turbulent version of this model.

The analysis below is based on the assumption that the effect of the water wave motion on the turbulent air flow decreases uniformly with distance from the water surface, and that the water wave amplification is caused by interaction between this wave induced air motion and the turbulent air flow over the whole extent of penetration of the wave induced air motion.

The structure of the wave induced air motion, in each waveband, is suggested to be as follows. In the immediate neighbourhood of the water surface, where both the mean velocity profile curvature and the turbulence level are greatest, the transfer of momentum from the air flow to the wave induced air motion is also greatest. This region is therefore the major contributor to the momentum transfer into the water wave motion and hence to the rate of wave amplification. Further from the surface, where the mean velocity profile curvature is smaller and the turbulence levels are less, the wave induced air motion approximates to potential flow. The extent of penetration of the wave induced air motion into the air flow, and the rate of wave amplification, are dependent on the set of parameters describing the air-water interaction.

Miles showed recently (Ref. 14) that the rate of wave amplification is partially dependent on the interaction between the wave induced air motion and the turbulent air flow throughout the whole extent of the air flow. This deduction is taken further here, in that it is assumed that the air turbulence diffuses the critical layer to such an extent that its contribution to the rate of wave amplification is not significantly different from the contribution made by neighbouring regions of the turbulent air flow.

There appears to be no experimental evidence yet of the existence of a special flow structure at the critical height in a fully turbulent air flow. The best measurements to date, by Shemdin (Refs. 2 and 4), are consistent with the description of the air flow structure given here.

The subset of the parameters describing the air-water interaction, which affects dominantly the rate of water wave amplification, is now determined. Attention is restricted to deep water waves greater in length than capillary ripples, in which case the water wave motion in the given waveband may be specified by two parameters, the frequency, $\omega$, and the gravitational acceleration, $g$. The densities of air and water appear in an expression for the rate of wave amplification as their ratio $\rho_a/\rho_w$ only, regarded as being constant.

The turbulent air flow is specified by only one parameter in this subset of parameters, namely, by $u_*$, the friction velocity of the turbulent air flow on the water surface. The main reason for this assumption is that $u_*$ determines both the curvature of the mean velocity profile and the level of turbulence in the neighbourhood of the water surface, and these are the two factors, it was suggested above, that determine dominantly the rate of wave amplification. Large variations in the external conditions, such as in the thermal stability of the turbulent air flow, could be expected to affect this assumption.

It appears reasonable therefore to describe the rate of wave amplification by the three parameters $g$, $\omega$, and $u_*$. Only one independent non-dimensional ratio can be formed from these three parameters, and it is taken to be $u_*/c$, where $c\ (= g/\omega)$ is the phase velocity in the given waveband.

The initial value of the spectral energy density, $\rho_w g\, \Phi\, \Delta\omega$, is determined by the processes of wave initiation, not by the mechanisms of wave amplification. Further, the rate of wave amplification is linearly dependent on $\Phi$ during the wave amplification stage (Eq. (3.5)). For dimensional reasons, the rate of wave amplification by the linear coupling between the air and water motions is therefore written*

$$\beta\Phi = g^2\, \omega^{-4} f_0\left(\frac{u_*}{c}\right) \Phi \Big/ \Phi_0 \quad \ldots\ldots\ldots\ldots(4.1)$$

where $f_0$ is a non-dimensional function proportional to the reference spectral density $\Phi_0$. The latter must appear explicitly as a factor of proportionality because the initial value of $\Phi$ is not determined by the linear coupling between the air and water motion, and because the rate of wave amplification is linearly proportional to $\Phi$ during the amplification stage.

The coefficient of wave amplification, $\beta$, from Eq. (4.1), is*

$$\beta = g^2\, \omega^{-4} f_0\left(\frac{u_*}{c}\right) \Big/ \Phi_0 \quad \ldots\ldots\ldots\ldots(4.2)$$

## 5.—MEASUREMENTS OF WAVE AMPLIFICATION

The only experimental results analysed so far (March 1968) are those of Shemdin and Hsu (Ref. 2), and these have only been partially analysed to determine the $\omega$-dependence of $\beta$ at constant $u_*/c$. Unfortunately, these data are not sufficiently smooth to make a more positive statement than that they are consistent with a $\omega^{-4}$ dependence at constant $u_*/c$, but that one could also fit equally well other similar $\omega$ dependences such as $\omega^{-5}$ dependence.

It is hoped that further experimental results can be analysed and compared (Refs. 2, 3, 5, 6, 7 and 8 in particular), and the form of the non-dimensional function $f_0$ determined also.

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*See p. 242—*Ed.*

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# A Semi-Numerical Approach for Determining the Hydraulic Conductivity of Unsaturated Porous Materials

BY K. K. WATSON, M.E., PH.D., M.I.E.AUST. and F. D. WHISLER, M.S., PH.D.*

*Summary.*—A method is presented for determining the hydraulic conductivity ($K$)-water content ($\theta$) relationship of an unsaturated porous material using a simple experimental arrangement and a computer solution of the differential flow equation. The experimental measurements involve the saturated hydraulic conductivity of the material, the discharge-time relationship of a draining column of the material and the pressure head-water content relationship as determined by sectioning the column at equilibrium. Trial $K$-$\theta$ curves are then used in the computer solution to obtain a discharge-time curve matching the measured curve. The method was used with two sands and found to be sufficiently sensitive and quite satisfactory.

## 1.—INTRODUCTION

The relationship between the hydraulic conductivity ($K$) of an unsaturated porous material and its volumetric moisture content ($\theta$) is a basic input requirement in analytical and numerical studies concerned with the movement of water in such materials. As recently noted by Watson (Ref. 1) there is a paucity of reliable experimental data on the relationship in the literature and this reflects the experimental difficulties inherent in its determination. Several laboratory methods have been available for some years; these include those of Childs and Collis-George (Ref. 2), Day and Luthin (Ref. 3) and Richards and Weeks (Ref. 4). One difficulty in the past has been the non-destructive measurement of the water content but this is now possible in experiments with rigid materials by using gamma-ray attenuation methods.

Two new approaches to the measurement of the $K$-$\theta$ relation have recently been published and in both of these the water content is measured by gamma-ray attenuation techniques. The first method, Watson (Ref. 5), is termed the method of instantaneous profiles. In it the $K$-$\theta$ relation is determined under quite severe transient conditions. By contrast, the second method, Watson (Ref. 1), is based on steady state conditions, but differs from other steady state methods in that a water content variation in the column is achieved by entrapping a zone of air in the profile and allowing it to be maintained at a pressure above atmospheric.

Such methods make it apparent that the research worker who has the facilities for the non-destructive measurement of the water content of a rigid porous material can now conveniently and accurately determine the $K$-$\theta$ relation. However, the establishment of such a measurement facility will probably remain a specialized research tool for some time. There is therefore merit in the proposal for establishing yet another method of measuring $K$-$\theta$ which utilizes, if possible, the simplest of experimental techniques combined with a computer-executed numerical analysis. Whisler and Watson (Ref. 6) have suggested such a method. In the method a saturated column of the porous material draining to a water table is used. With such a system there are three pieces of experimental information which can be obtained easily without recourse to complex measuring equipment. These are the saturated hydraulic conductivity ($K_{\text{sat}}$), the discharge ($Q$)-time ($t$) curve and the pressure head ($h$)-water content ($\theta$) relationship measured by sectioning the column at equilibrium. With this information it is a simple matter to draw possible $K$-$\theta$ curves using as a fixed point the measured $K_{\text{sat}}$ and $\theta_{\text{sat}}$ values, and then to obtain the respective $Q$-$t$ curves from a suitable computer solution of the differential flow equation. These are then matched against the measured $Q$-$t$ relationship to obtain the $K$-$\theta$ curve giving the best fit. This method is in contrast to methods such as those of Childs and Collis-George (Ref. 2), Millington and Quirk (Ref. 7) or Brutsaert (Ref. 8) where empirical formulae are used to calculate the $K$-$\theta$ relation from the $h$-$\theta$ relation. Even when these latter methods are modified by fitting constants as suggested by Jackson et al (Ref. 9) they do not have the inherent advantage of comparing the predicted outflow with the experimentally determined values. This paper studies the feasibility of the proposed method in detail using two sands and also discusses questions arising in the analysis, such as the effect of not using a dynamically determined moisture characteristic and the sensitivity of the method to small changes in the geometry of the $K$-$\theta$ curve.

Basic to the whole approach in this paper is the confidence that the numerical analysis used predicts accurately the pressure head and water content changes occurring in a one-dimensional gravity drainage system. This confidence is based on extensive testing of the analysis against several sets of available data as given by Whisler and Watson (Ref. 6).

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