An Experimental Water Channel Analogue to Diabatic Gas Flow

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Summary.—The existence of an analogy between spatially varied open channel inviscid liquid flow and one-dimensional inviscid diabatic gas flow was proposed by Beaton et al (Ref. 1). The validity of the analogy between a water channel and the analogue ideal gas is discussed here.

A water channel was built with provision for controlled injection or extraction through the channel bottom surface. Five experiments were carried out with water injection and one with water extraction. All were in the supercritical regime.

It was found that the analogy equations proposed earlier were confirmed within the following restrictions. Boundary layer effects in the channel should be included when liquid is injected. At $F = 1$ the theoretical analogy relationships contain a singularity and experimental results diverge from them when F approaches unity. The analogue gas is an ideal gas having specific heat ratio $\gamma = 2$.

LIST OF SYMBOLS

A	Cross-sectional area of flow.
b	Channel width.
F	Froude number $F = u/(gh)^{\frac{1}{2}}$.
h	Liquid depth.
i	Gas enthalpy.
M	Mach number.
n	Channel friction constant.
Q	Channel flow rate.
Q_i	Injected or extracted flow rate.
q	Injected liquid velocity.
r_h	Hydraulic radius.
S_b	Slope of channel bed.
S_f	Gradient of energy loss (liquid).
u	Liquid velocity in channel.
x	Distance along axis of channel.
α	Energy coefficient.
β	Momentum coefficient.
γ	Specific heat ratio.

Superscript

$'$	Denotes datum conditions at $x = 0$.

INTRODUCTION

The direct analogy between spatially varied open channel liquid flow and the diabatic flow of an ideal gas was examined by Beaton et al (Ref. 1). It was shown that a kinematic analogy exists between F and M provided the analogue gas has a specific heat ratio $\gamma = 2$. A pressure analogy between h^2 and p and an enthalpy analogy between h and i were also indicated. The mathematical analogue is an open channel containing a stream of ideal inviscid liquid so arranged that liquid is added or removed continuously along the bottom surface. The liquid surface height gradient is given by

$$dh/dx = [2F^2/(F^2 - 1)](q/u) \quad \text{.....................(1)}$$

if liquid is added, and by

$$dh/dx = [F^2/(F^2 - 1)](q/u) \quad \text{.......................(2)}$$

if liquid is removed.

It was shown that the pressure analogy is valid if the surface gradient is that given by Eq. (1) and therefore the channel provides a direct pressure analogue to the case of heat addition and to the heat extraction case if allowance is made for the slope predicted by Eq. (2) being 0.5 of that given by Eq. (1). For validity of the enthalpy analogy the liquid surface gradient must be

$$dh/dx = [(2F^2 - 1)/(2F^2 - 2)](q/u) \quad \text{..................(3)}$$

The coefficient of (q/u) in Eqs. (1), (2) and (3) is shown on Fig. 1 as a function of F which indicates that at high Froude numbers the channel closely approximates a direct analogue to the case of heat removal and to the heat addition case if allowance is made for the fact that the measured gradients will be approximately twice that required from Eq. (3).

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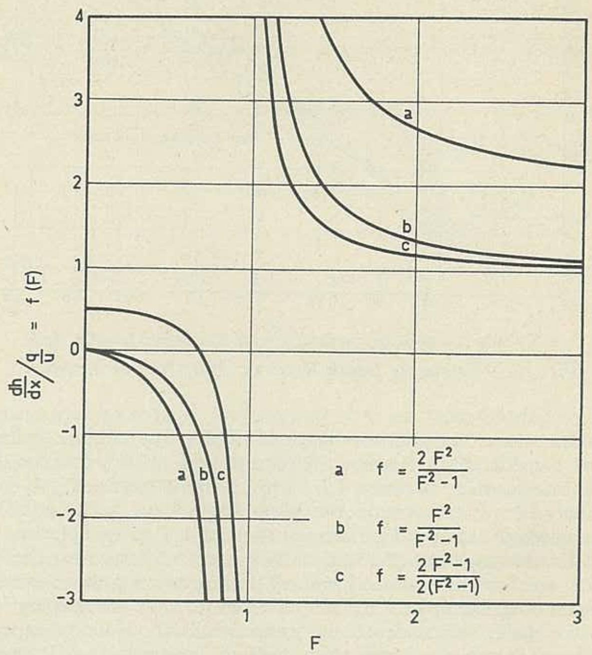

Fig. 1.—Coefficient of q/u as a Function of Froude Number.

An experiment was devised to check these predictions using a water channel in which injection or extraction may be effected through slots in the channel bed. Experiments involving measurement of water surface height were made under controlled conditions of injection or extraction and measured values of surface gradient are presented and compared with those given by the ideal liquid Eqs. (1) and (2).

No experiments were performed on a real gas. It is to be expected, however, that if experiments on a real liquid establish the existence of an analogy between the real liquid and the ideal one-dimensional inviscid diabatic gas flow the analogy can be extended to those real gas flow cases in which viscous effects are not dominant or significant. The usefulness of the analogy is not discussed here. Nevertheless hydraulic analogies have found a place in situations which are difficult to analyse theoretically and in which qualitative comparisons arising from parametric changes yield useful results. Examples are supersonic inlet diffusers and supersonic parachutes.

THE WATER CHANNEL COMPARED WITH THE IDEAL LIQUID MODEL

In the ideal case (Fig. 2(*a*)) liquid addition (or removal) is assumed to be continuous along the bed but in the water channel it takes place through separate transverse slots and the velocity q of Eqs. (1) and (2) is replaced by $\bar{q}$ (Fig. 2(*b*)). Provided a large number of closely spaced slots occur along the test section, surface height and slope will closely approximate that produced by continuous injection.

Surface height is shown as a vertical measurement. Because of the construction of the apparatus h was measured normal to the bed. Maximum slope was 3° and the corresponding error was therefore insignificant.

In the ideal liquid model, velocity is assumed to be uniform over any cross section but in the water stream this will not be so because of the presence of the boundary layer on the channel side and bottom surfaces. The channel was so constructed that at any section Q, h and hence mean velocity $\bar{u}$ may readily be measured. If $\bar{u}$ is used in energy or momentum equations allowance for non-uniform velocity distribution may be made using the energy correction factor,

$$\alpha = \int_A u^3\, dA / \bar{u}^3 A \quad \text{.............................(4)}$$

or the momentum correction factor,

$$\beta = \int_A u^2\, dA / \bar{u}^2 A. \quad \text{..........................(5)}$$

The magnitude of α and β was unknown and it was considered advisable to determine their values at selected stations representative of the experimental flow conditions in at least one case involving water addition.

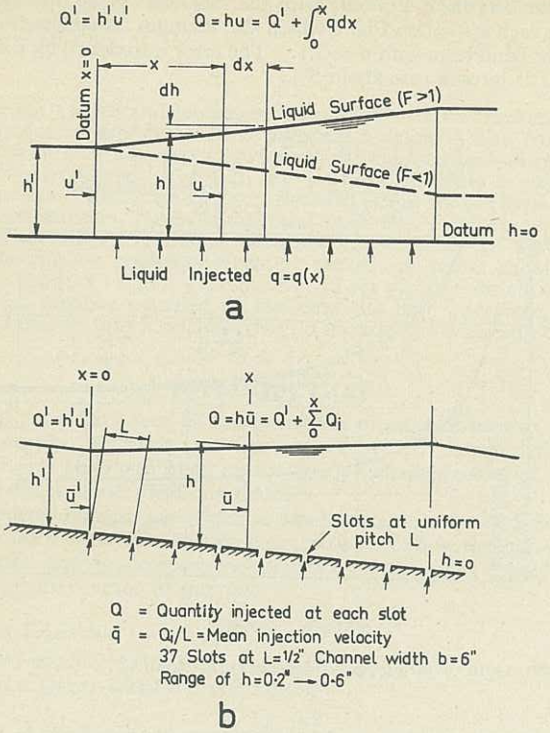

Fig. 2.—(*a*) *The Ideal Liquid Model.*
(*b*) *The Water Channel.*

In the experiment involving removal of liquid it was assumed that the boundary layer on the bottom surface was removed and that the contribution to non-uniformity of the side wall boundary layer was small since $h \ll b$ (Fig. 2(*b*)). Therefore the liquid may be considered to flow with substantially uniform velocity at any cross section and the assumption $\alpha = \beta = 1$ was made.

In the water channel there is loss of specific energy due to wall friction. In the case where no injection or extraction occurs and the bed slope is small it may be shown that when friction loss is taken into account the gradient of surface height is given by

$$(dh/dx)_f = (S_f - S_b)/(\alpha\bar{F}^2 - 1) \quad \ldots\ldots\ldots\ldots(6)$$

In the experimental channel S_b was measured directly and $\bar{F}$ was calculated using $\bar{u}$. The surface slope due to friction S_f was calculated using Manning's formula:

$$S_f = \bar{u}^2 n/2.220\, r_h^{4/3} \quad \ldots\ldots\ldots\ldots(7)$$

The surface roughness coefficient n was obtained experimentally.

If water is added to the main stream further energy loss takes place due to mixing and this cannot easily be calculated since the nature of the mixing process is not known. However, the momentum principle may be applied to a simplified model of zero bed slope to show that, if friction is neglected, the gradient of water surface due to injection alone is

$$(dh/dx)_i = [2\bar{F}^2/(\bar{F}^2 - 1/\beta)](\bar{q}/\bar{u}) \quad \ldots\ldots\ldots\ldots(8)$$

In this paper the combined effect of water injection and friction loss on the water surface gradient was found by addition of Eqs. (6) and (8) thus.—

$$dh/dx = (dh/dx)_i + (dh/dx)_f$$

$$= [2\bar{F}^2/(\bar{F}^2 - 1/\beta)](\bar{q}/\bar{u}) + [(S_f - S_b)/(\alpha\bar{F}^2 - 1)] \quad \ldots(9)$$

In the case where water is removed from the main stream there is no mixing. Consequently the only energy loss is that due to surface friction. By consideration of the energy equation the surface gradient may be shown to be

$$dh/dx = [\bar{F}^2/(\bar{F}^2 - 1/\alpha)](\bar{q}/\bar{u}) + [(S_f - S_b)/(\alpha\bar{F}^2 - 1)] \quad \ldots(10)$$

The friction loss at the side walls is small compared with that on the bottom surface. If the boundary layer on the bottom surface is removed at each extraction slot the friction term may be neglected. If as a further simplification the channel is horizontal and α is assumed to be unity, Eq. (10) reduces to that for the ideal liquid (Eq. (2)).

EXPERIMENTAL APPARATUS AND PROCEDURE

The water analogue was an open channel constructed of perspex and mounted on a steel baseplate shown in semi-diagrammatic form in Fig. 3. Water from a constant head supply tank was admitted through a control valve to an inlet section from which it passed via an inlet nozzle and control gate into the main channel. The channel is of uniform 6-in. width, 4 ft. 6 in. long and has a settling section, a test or analogue section and an exit section each 18 in. long. The test section has 37 transverse slots each 0.030 in. wide machined across the full width of the channel at 0.5-in. pitch. Beneath the test section an injection box containing 37 separate compartments 6 in. deep and extending over the full channel width was fitted. Each compartment communicates with one slot. Water to be injected through any slot passes from an inlet manifold through a 0.25-in. control valve and a measuring orifice into the associated inlet compartment and thence through the slot into the main stream. The injection quantity Q_i is determined from the pressure differential across the previously calibrated orifice. A separate control valve and orifice is provided for each slot. Pressure differential was measured using individual water column manometers.

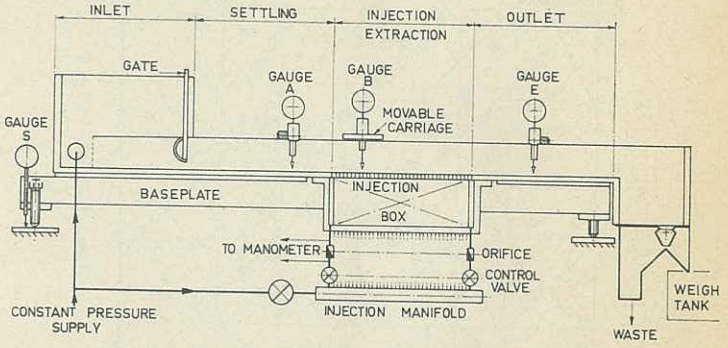

Fig. 3.—*Arrangement of Experimental Channel.*

Flow rate at exit from the channel was measured using a weightank and an electronic timer.

Water surface height was measured using needle point gauges mounted in vertically adjustable holders attached to commercial dial indicators.

Fixed height gauges A and E (Fig. 3) were used to set the bed horizontal taking the surface of a quantity of water in the channel as a reference. The baseplate slope was adjusted by means of a screwjack at the inlet end and bed slope S_b was obtained from dial gauge S (Fig. 3).

During surface profile measurements water surface height h was measured directly using a gauge mounted on a carriage movable in both longitudinal and transverse directions. The injection slots extend across the channel from wall to wall and the test section is in effect a series of 0.5-in. strips. Some variation of height occurred from strip to strip. Considering the bed surface just upstream of the first slot as a datum, the maximum variation along the centre-line of the channel was 0.005 in. and the maximum step between any two adjacent strips was 0.001 in.

Velocity distribution was measured at selected cross sections using a total head tube mounted in a holder attached to the movable carriage. The tube had an inside diameter of 0.026 in. and wall thickness of 0.010 in. Total pressure was measured using a previously calibrated transducer. Static pressure was obtained from surface height h and the height of the tube axis above the bed.

The experimental programme included three groups of surface profile measurements, two with liquid injection and one with extraction. All were carried out in the supercritical regime. In each case the channel slope was set so that the selected flow condition occurred at the inlet to the analogue section. The flow rate Q was checked using the weightank with no injection or extraction. This value remained constant when injecting or extracting and was taken as Q' in all cases. Injection or extraction quantities Q_i were then adjusted to the same desired value in all slots and $Q' + \Sigma Q_i$ was checked using the weightank. At any section along the channel the flow rate Q may be obtained by addition of Q' and ΣQ_i up to that section (Fig. 2(*b*)).

Surface height was measured at the centre-line of the channel at stations 0.25 in. upstream of each slot. A curve of surface height against axial distance was drawn from which values of h and dh/dx were measured at those stations selected for evaluation of the validity of Eqs. (9) and (10).

Eqs. (9) and (10) may be re-arranged into a form convenient for comparison with the ideal liquid on a graph of the form of Fig. 1, namely

$$\left\{dh/dx - \frac{S_f - S_b}{\alpha\bar{F}^2 - 1}\right\}\Big/(\bar{q}/\bar{u}) = \frac{2\bar{F}^2}{\bar{F}^2 - 1/\beta} \quad \text{...........(11)}$$

$$\left\{dh/dx - \frac{S_f - S_b}{\alpha\bar{F}^2 - 1}\right\}\Big/(\bar{q}/\bar{u}) = \frac{\bar{F}^2}{\bar{F}^2 - 1/\alpha} \quad \text{...........(12)}$$

Values of $\bar{F}$ and $\bar{u}$ were calculated from measured Q and h, $\bar{q}$ was obtained from Q_i at the appropriate slot, dh/dx was measured directly and where applicable S_f was calculated using Eq. (7).

Values of h, Q, $\bar{u}$ and $\bar{F}$ at each station selected for evaluation of Eqs. (11) or (12) are shown on Fig. 4, for both injection and extraction cases. Values immediately upstream of the test section are marked " inlet " with respective experiment numbers.

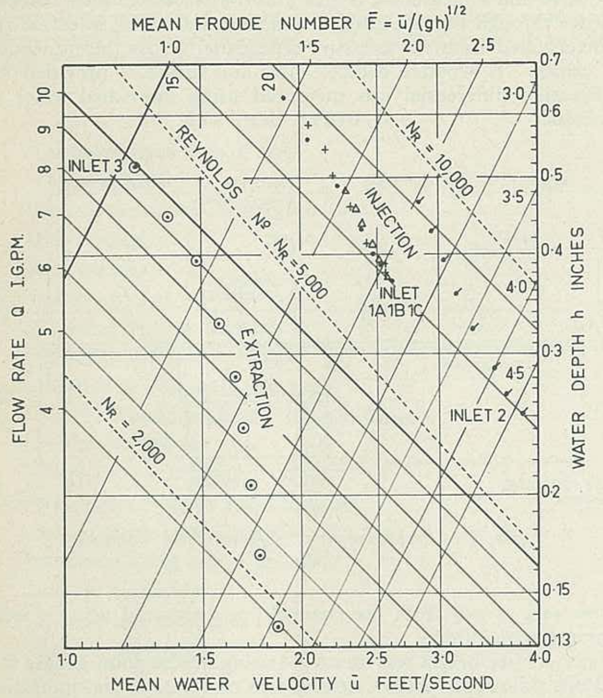

Fig. 4.—Range of Flow Properties Covered by Experiments.

In all experiments the water surface profile was measured with equal quantity passed through each slot, i.e., mean injection or extraction velocity $\bar{q}$ was uniform over the test section.

For the first group of experiments with injection the inlet Froude number was 2.6 and bed slope was 0.0283. The profile was measured with three different rates of injection. The first 30 slots were used, values of $\bar{q}$ being 0.0247, 0.0187 and 0.0108 ft./sec. These are referred to as Experiments 1A, 1B and 1C, respectively. The left hand term of Eq. (11) was evaluated for seven representative stations with the assumption $\alpha = 1$. These values are shown on Fig. 5 with the curve representing the coefficient of (q/u) of Eq. (1) for comparison.

Experiment 2 involved one surface profile measurement at inlet Froude number of 4.68. S_b was 0.0565 and the first 35 slots were used with flow adjusted to give a mean injection velocity $\bar{q} = 0.0283$ ft./sec. The left hand term of Eq. (11) was evaluated for seven stations with the assumption $\alpha = 1$. To determine the significance of the error resulting from this assumption the cross-sectional distribution of velocity was measured at four selected stations at distance x downstream from the first slot. The calculated values of α and β were:

x inches	0.75	4.75	9.75	14.75
α	1.112	1.163	1.144	1.140
β	1.046	1.072	1.069	1.072

The left hand term of Eq. (11) with the measured value of α included is shown for each station on Fig. 5 which also includes for comparison a curve of the right hand term with $\beta = 1.1$. The error introduced by the assumption $\alpha = 1$ is in each case about 5%.

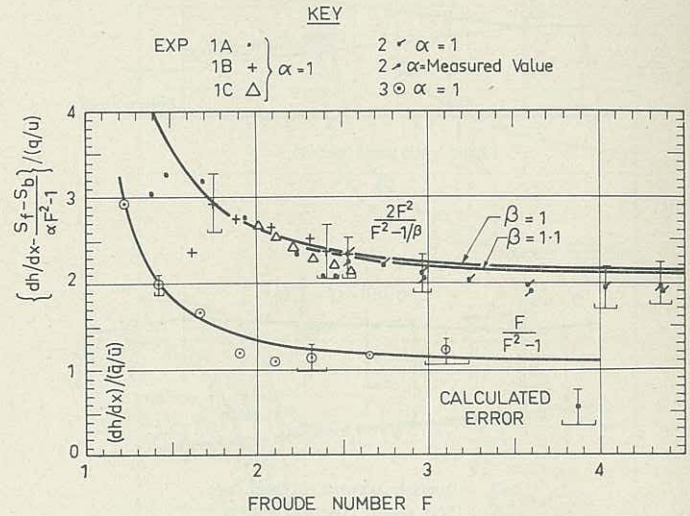

Fig. 5.—Experimental Results.

Experiment 3 refers to the surface measurements made with water extraction. The bed was horizontal and the inlet Froude number was 1.042. Water was removed through the first 35 slots with a mean extraction velocity $\bar{q} = 0.0222$ ft./sec. It was assumed that there was no energy loss because of boundary layer removal. The left hand term of Eq. (12) was calculated with $S_f = S_b = 0$ and $\alpha = 1$ for each of eight selected stations and values obtained are shown on Fig. 5 which also includes, for comparison, a curve showing the coefficient of (q/u) in Eq. (2).

At selected stations the overall error of the results on Fig. 5 was evaluated from the relevant calibration and measurement errors associated with the individual quantities involved. These were calculated maximum errors and it is considered that in most cases the error is less than the extreme values of the limits shown on Fig. 5.

CONCLUSION

A series of experiments has shown that a water channel with controlled injection of extraction may be used as an analogue to the diabatic flow along a constant-area duct of an ideal gas having a specific heat ratio $\gamma = 2$.

Experiments were conducted in the supercritical regime only, the maximum Froude number being 4.68. The analogy is invalid at Froude numbers close to unity.

Energy loss due to channel surface friction has an effect on the water surface slope. It was found that this must be allowed for in the case of water addition but in the extraction case the effect is not significant.

Although the transverse velocity distribution in the water was not uniform, the influence on surface height gradient was found to be small.

ACKNOWLEDGMENT

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Reference

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