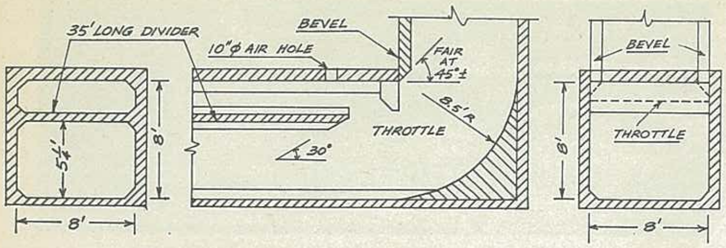

Fig. 6.—Details of Divider—Curve and Throttle.
(Model Scale: 1 cm. to 1 ft.)

It may be seen from the test results that satisfactory operation of the spillweir and culvert was achieved when—

1. A sharp-edged throttle was installed at the base of the drop chute 6.0 ft. above the bottom of the culvert.
2. The base of the drop chute was faired to a radius of 8.5 ft.
3. A 10-in. vent pipe was let into the roof of the culvert near the throttle.
4. A horizontal divider, 35 ft. in length, was installed in the culvert just downstream from the throttle which effectively prevented the flow from filling the culvert and thus ensuring open channel flow at all times.

The authors are convinced that the solution to the problem of the maintenance of open channel flow in the culvert as offered by these model tests is conservative. They are however, concerned with their inability to duplicate, properly, air entrainment in drop chute spillweir when air entrainment is not the only governing factor. The use, as in this model, of additional air added artificially is perhaps begging the question. However, at this time, a modest research project on the subject of duplicating air entrainment in drop chute spillweir models is underway at Singapore Polytechnic.

ACKNOWLEDGMENT

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Non-Dimensional H_2 Profiles in Horizontal Prismatic Channels

BY R. M. ADVANI, M. M. DANDEKAR and J. K. VIRMANI*

Summary.—A non-dimensional form of the equation for the surface profile of H_2 type in horizontal prismatic channels of triangular, rectangular and trapezoidal shapes, is developed. It is analysed with the help of a computer for different shape factors of the channels, and the results are presented in graphical form. It is seen that the effect of increase of side slope z is to flatten the profile of the curve, while the effect of increase of the depth to bed-width ratio is to make the profile steeper.

LIST OF SYMBOLS

The symbols are defined where they first appear in the paper and are also given below in alphabetical order for convenience of reference.—

- A Cross-sectional area of flow.
- b Channel bed-width.
- C The integration constant.
- E Specific energy of flow.
- K_c Conveyance at critical depth.
- K Conveyance of the channel $= \dfrac{Q}{S_f^{\frac{1}{2}}}$.
- L_b Brink length.
- $L = \dfrac{y_c}{S_c}$ Critical length for the channel.
- $M = 2\dfrac{\log Z}{\log y}$ Hydraulic exponent for the critical flow.
- $N = 2\dfrac{\log K}{\log y}$ Hydraulic exponent for the normal flow.
- P Wetted perimeter.
- Q Discharge.
- R Hydraulic mean depth.
- S Bed-slope of the channel.
- S_c Critical slope for the channel.
- S_f Friction slope.
- T Width at the water surface.
- V Mean velocity of flow.
- x Length along the channel, measured positive in a direction opposite to that of the flow.
- $X = \dfrac{x}{L_c}$ Non-dimensional length factor.
- y Depth of flow.
- y_c Critical depth.
- $Y = \dfrac{y}{y_c}$ Non-dimensional depth factor.
- Z Section factor of the channel $= \sqrt{\dfrac{A^3}{T}}$.
- Z_c Section factor at critical depth.
- z Side slope in trapezoidal channels. (z horizontal to 1 vertical).

INTRODUCTION

The importance of the presentation of surface profiles of flow in open channels in a non-dimensional form has been appreciated more and more in recent years (Refs. 1 and 2). This representation helps to bring out the characteristic features of the surface flow profiles in a vivid manner. In this paper, a general non-dimensional equation for the surface profiles in horizontal channels is proposed, and its solution is presented for the triangular, rectangular and trapezoidal channels with various shape factors y/b and different side slopes z. The results for H_2 profiles are shown graphically for all these channels and then compared.

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BASIC THEORY

The basic differential equation for gradually varied flow surface profiles in horizontal prismatic channels is:

$$\frac{dy}{dx} = \frac{S_f}{1 - \left(\frac{Z_c}{Z}\right)^2} \quad \text{.........................(1)}$$

In the above equation, y is the depth of flow, x is the length measured positively in a direction opposite to that of the flow, Z_c is the section factor at the critical depth and Z is the section factor at depth y. It may be noted that $S_f\,(= dE/dx)$ is the rate of slope of the energy line and is positive for the assumed positive direction of x.

The following established relations for channel flow are also well known.

$$S_f = \frac{Q^2}{K^2} \quad \text{.............................(2)}$$

$$S_c = \frac{Q^2}{K_c^2} \quad \text{.............................(3)}$$

$$K^2 \propto y^N \quad \text{.............................(4)}$$

$$Z^2 \propto y^M \quad \text{.............................(5)}$$

$$Z_c^2 \propto y_c^M \quad \text{.............................(6)}$$

In Eqs. (2) to (6), y_c and S_c are the critical depth and the critical slope respectively, Q is the discharge, and M and N are the hydraulic exponents for the critical flow and the normal flow conditions respectively. The values of these exponents change very gradually for the rectangular and the trapezoidal channels. It is assumed that, for integration purposes, these values can be taken as approximately invariant. Substituting the values from Eqs. (2) to (6) in Eq. (1), the following form of the resulting equation is obtained (Ref. 3):

$$\frac{dy}{dx} = \frac{S_c\left(\frac{K_c}{K}\right)^2}{1 - \left(\frac{y_c}{y}\right)^M} = \frac{S_c\left(\frac{y_c}{y}\right)^N}{1 - \left(\frac{y_c}{y}\right)^M} \quad \text{...............(7)}$$

On integration, this gives

$$\frac{S_c}{y_c}x + c = \frac{\left(\frac{y}{y_c}\right)^{N+1}}{N+1} - \frac{\left(\frac{y}{y_c}\right)^{N-M+1}}{N-M+1} \quad \text{..............(8)}$$

where C is the constant of integration.

Now, if a characteristic length L_c for the channel is introduced so that

$$L_c = \frac{y_c}{S_c} \quad \text{..................................(9)}$$

then, Eq. (8) can be written in the non-dimensional form as

$$\frac{x}{L_c} + c = \frac{\left(\frac{y}{y_c}\right)^{N+1}}{N+1} - \frac{\left(\frac{y}{y_c}\right)^{N-M+1}}{N-M+1} \quad \text{...........(10)}$$

From definition sketch, (Fig. 1), when $y = y_c$, $x = 0$

$$\therefore \quad c = -\frac{M}{(N+1)(N-M+1)} \quad \text{.....................(11)}$$

If $x/L_c = X$ and $y/y_c = Y$, then Eq. (10) takes the following non-dimensional form:

$$X = \frac{M}{(N+1)(N-M+1)} + \frac{Y^{N+1}}{N+1} - \frac{Y^{N-M+1}}{N-M+1} \quad \text{......(12)}$$

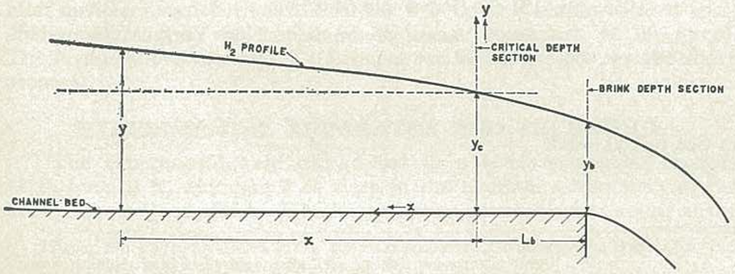

Fig. 1.—Definition Sketch.

Eq. (12) is a two-parameter equation in X and Y for known values of M and N, and can be solved on substituting proper values of M and N for any given channel.

VALUES OF M AND N

For triangular, rectangular and trapezoidal channels, the following formulae for M and N are established:

$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right) \quad \text{.......................(13)}$$

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right) \quad \text{.......................(14)}$$

In the above equations, T is the water surface width, P is the wetted perimeter and R is the hydraulic mean depth. These equations are solved for different values of shape factor y/b for rectangular and trapezoidal channels and the result obtained is given in Table I below:

TABLE I

S No.	Shape factor y/b	Triangular channel		Rectangular channel		Trapezoidal channel $z = 0.50$		Trapezoidal channel $z = 1.00$		Trapezoidal channel $z = 2.00$	
		N	M	N	M	N	M	N	M	N	M
1.	0.05	—	—	3.20	3.00	3.35	3.05	3.35	3.20	3.40	3.10
2.	0.10	—	—	3.10	3.00	3.25	3.05	3.35	3.10	3.45	3.20
3.	0.20	—	—	2.95	3.00	3.25	3.10	3.40	3.20	3.65	3.40
4.	0.50	—	—	2.65	3.00	3.30	3.25	3.65	3.50	4.05	3.80
5.	1.00	—	—	2.45	3.00	3.55	3.50	4.00	3.70	4.45	4.20
6.	∞	5.33	5.00	—	—	—	—	—	—	—	—

In the above table, b is the bed-width and z is the side slope of trapezoidal channels (z horizontal to 1 vertical). For any channel, corresponding to a certain shape factor, the values of M and N can be read from Table I and substituted in Eq. (12); the equation can then be solved. A series of such equations for rectangular channels as also trapezoidal channels with side slopes of 0.50, 1.0, 1.50 and 2.0 were programmed for a computer solution. The resulting solution is presented here in a graphical form, Figs. 2 and 3.

CONCLUSION

(1) From Fig. 2, it is seen that as the side slope z increases the profile of the curve becomes flatter and flatter. Thus for the same y/b ratio, a rectangular channel yields a steeper profile than a trapezoidal channel, and in trapezoidal channels the profile becomes flatter as the side slope z increases.

Fig. 3 shows 10 curves, 4 for rectangular channels, 5 for trapezoidal channels and one for a triangular channel. As will be seen from the figure, the profile for the triangular channel is the flattest. The four profile curves for rectangular channel indicate that the profile becomes steeper as y/b increases. The profiles for the trapezoidal channel are bounded on one side by the triangular channel, and on the other side by the rectangular channel. These curves indicate a composite effect of the triangular area and the rectangular area; from the plotted curves it is seen that the effect of the triangular portion is major so that, even for increasing values of y/b, the curves tend to become flatter, the lower limit being that of a triangular channel, i.e., for $y/b \to \infty$.

(2) Fig. 2 can be used to find numerical solutions to any problem. For this, y_c and L_c have to be calculated for a given value of the discharge before the curves can be used. In a problem of calculating the length of the profile from the brink, it may be noted that the length correction L_b has to be separately added in the computations (see Fig. 1). Further, for any given reach of the channel, an average value of the shape factor y/b has to be adopted for the calculations.

(3) Work towards a further generalisation of the method is in progress.

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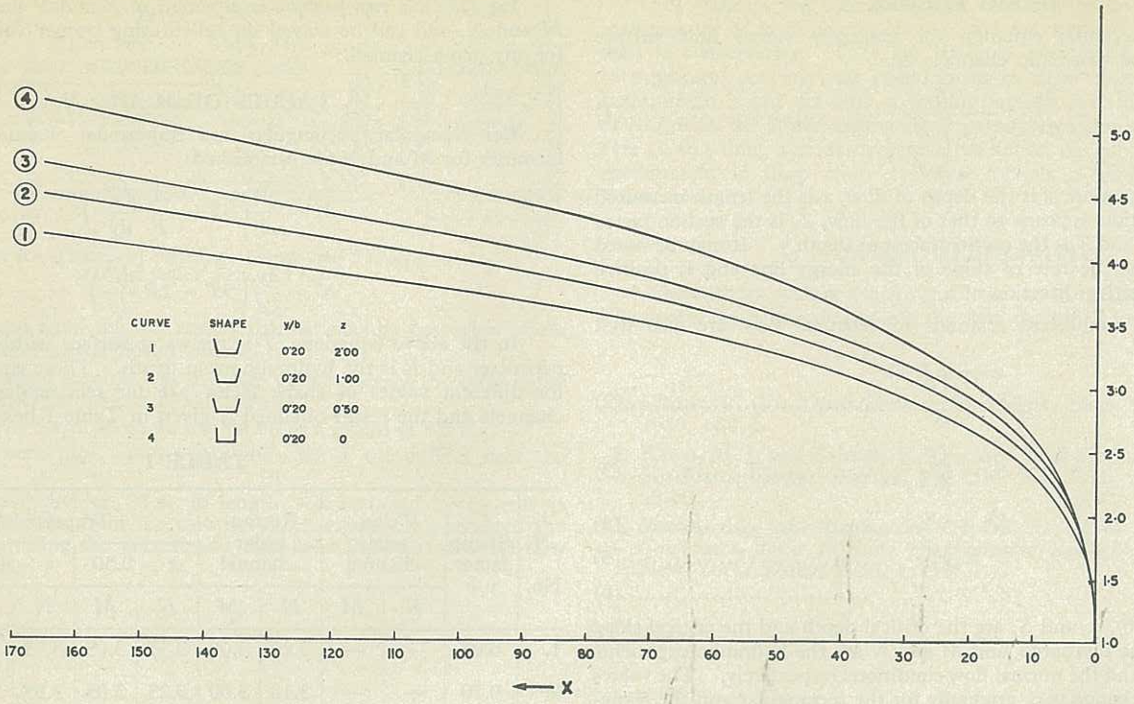

Fig. 2.—Effect of Slope on H_2 Profiles.

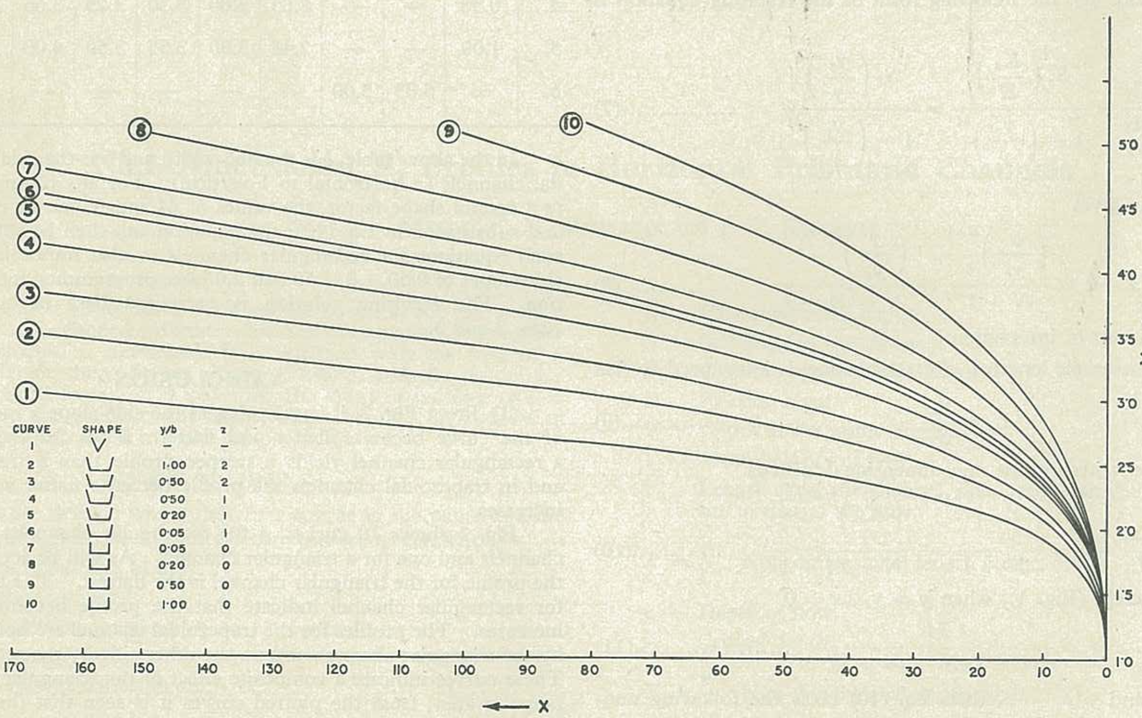

Fig. 3.—Effect of Shape Factor and Slope on H_2 Profiles.

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Authors' Addendum : See p. 250.—*Ed.*