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# Direct Numerical Simulation of Confined Wall Plumes

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## Abstract

We present results from the direct numerical simulation (DNS) of a wall attached thermal plume in a confined box. The plume originates from a local line heat source of length, $L$, placed at the bottom left corner of the box. The Reynolds number of the wall plume, based on box height and buoyant velocity scale, is $Re_H = 14530$ and a parametric study is carried out for boxes of two different aspect ratios (ratio of box width to box height) for a particular value of $L$. In the simulation, the plume develops along the vertical side wall while remaining attached to it before spreading across the top wall to form a buoyant fluid layer and eventually moving downwards and filling the whole box. Further, the original filling box model of Baines and Turner [1] is modified to incorporate the wall shear stress and the results from the DNS are compared against it. A reasonable agreement is observed for the volume and momentum fluxes in the quiescent uniform environment and also for the time-dependent buoyancy profile calculated far away from the plume.

## Introduction

Turbulent plumes inside a confined region has received considerable attention because of their wide application in many industrial and geophysical fluid flows, for example, design of buildings (e.g. Hunt *et al.* [4] ), smoke propagation in rooms (e.g. Zukoski [11] ) and in the oceans (e.g. Killworth and Turner [5] ). A turbulent plume arising from a local source of buoyancy in a confined region can lead to stratification of the fluid surrounding the plume, which is described in detail in the filling box problem of Baines and Turner [1] hereinafter referred to as BT. They considered a plume generated at the centre of the bottom boundary within the initially uniform confined environment. The plume rises to the upper boundary and spread towards the sidewalls, and form a density interface between the plume outflow and the ambient fluid. The continuous supply of buoyant fluid causes this density interface to move downwards towards the source. Baines and Turner [1] developed an analytical model for the filling box problem based on the classical plume theory presented by Morton *et al.* [7]. Later, Worster and Huppert [10] extended this model and obtained an analytical expression for the time-dependent density profiles in the filling box. The aspect ratio of the box is an important parameter in the filling box studies. The effect of aspect ratio (i.e. ratio of the radius of a circle with an area equivalent to the cross-sectional area of the tank to the height of the tank, $R/H$) on the filling box process for round plumes was investigated by Barnett [2] both analytically and experimentally. Barnett found that the filling box process occurs only for large aspect ratios ($R/H \geq 1.0$). For moderate aspect ratios ($0.172 < R/H < 1.0$), the plume outflow in the environment is observed as horizontally inhomogeneous and overturning circulation is developed in the environment. In the case of extremely small aspect ratios ($R/H \leq 0.172$), the turbulent plume breaks down due to the interaction with the side walls and the plume no longer reaches the top of the tank.

While all the above studies focus on the plumes generated at the centre of the confined box, investigation of plumes attached to the wall, which is the focus of the present study, has received less attention in the literature. One such study, however in an open environment (not confined), is by Grella and Faeth [3]. They carried out a similarity analysis on wall attached turbulent plumes originated from a line heat source with an assumption of constant skin friction coefficient $c_f$ . However, the development of a turbulent plume in a confined box is different and has primarily two stages: the transient and the asymptotic stage. In the *transient stage*, the plume that develops along the vertical side wall (similar to the open environment), impinges on the top wall thereby forming an interface layer and with time moves downwards to the bottom wall (see figure 1). The region below the interface layer is considered as a uniform environment. In the *asymptotic stage*, the interface layer would have reached the bottom wall, and the temperature at every point in the box increases with time, and all the velocities become statistically steady.

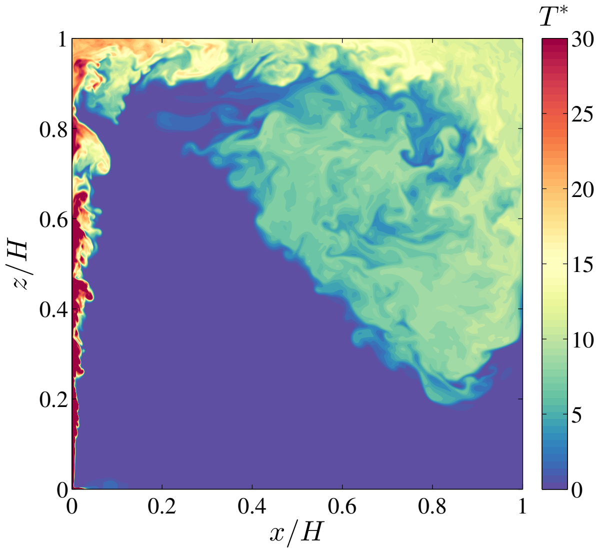


Figure 1: Contour plot of the instantaneous non-dimensionalised temperature ($T^*$) at $y/H = 0.5$ for aspect ratio ($AR$) $= 1.0$.

In this paper, we carry out direct numerical simulations (DNS) of wall attached line plume in confined boxes, and compare the results with a modified version of BT's analytical model. The present study is restricted only to the transient stage and the results from the asymptotic stage are not discussed.

## Theoretical model

We have adapted the original filling box model developed by BT[1] to model the wall attached line plumes in a confined region by including wall effects. The schematic diagram of wall attached plume in a confined region is shown in figure 2, where a line heat source is located at the bottom left corner of the box. The box height and width are $H$ and $R$, respectively. The source generates a buoyancy flux $F_0$ per unit length and zero fluxes of volume and momentum.

We begin by considering the volume, momentum and buoyancy fluxes for a wall attached line plume, which can be derived from the continuity equation, the simplified Reynolds averaged momentum equation with Boussinesq approximation in the vertical direction (the pressure and the fluctuating terms are neglected) and the simplified energy equation (equations 1, 2 and 3, respectively). The mean velocity in the vertical direction $z$ is denoted by $\overline{w}$, and in the $x-$direction is denoted by $\overline{u}$.

$$\frac{\partial \overline{u}}{\partial x}+\frac{\partial \overline{w}}{\partial z}=0, \tag{1}$$

$$\frac{\partial \overline{u}\,\overline{w}}{\partial x}+\frac{\partial \overline{w}\,\overline{w}}{\partial z}+\frac{\partial \overline{u'w'}}{\partial x}+\frac{\partial(\overline{w'^2}-\overline{u'^2})}{\partial z}=\frac{1}{\rho}\frac{\partial \tau_x}{\partial x}$$
$$+\frac{1}{\rho}\frac{\partial \tau_z}{\partial z}+g\beta(\overline{T}-\overline{T}_\infty), \tag{2}$$

$$\frac{\partial \overline{u}\,(\overline{T}-\overline{T}_\infty)}{\partial x}+\frac{\partial \overline{w}\,(\overline{T}-\overline{T}_\infty)}{\partial z}+\frac{\partial \overline{T'u'}}{\partial x}+\frac{\partial \overline{T'w'}}{\partial z}=-\overline{w}\frac{\partial \overline{T}_\infty}{\partial z} \tag{3}$$

Here $\rho$ is the density of the ambient fluid, $g$ is the gravitational acceleration, $\beta$ is the coefficient of thermal expansion, $\overline{T}$ is the mean temperature and $\overline{T}_\infty$ is the mean environmental temperature, which is far away from the plume. Also, $u'$, $w'$ and $T'$ are the fluctuating components of $u$, $w$ and $T$, respectively. Lastly, $\tau_x=\mu\partial\overline{w}/\partial x$ and $\tau_z=\mu\partial\overline{w}/\partial z$ are the shear stresses, where $\mu$ is the dynamic viscosity.

Integration of equation 1 along $x$ gives

$$\frac{\mathrm{d}Q}{\mathrm{d}z}=-u_e, \tag{4}$$

where $Q=\int_0^\infty \overline{w}\,\mathrm{d}x$ is the volume flux and $u_e$ is the entrainment velocity ($u_e=\overline{u}|_{x=\infty}$), which is evaluated far from the plume. Similarly, integration of equation 2 and 3 (ignoring the contribution of the fluctuating terms in the momentum and buoyancy flux) along $x$ gives

$$\frac{\mathrm{d}M}{\mathrm{d}z}=\int_0^\infty g\beta\,(\overline{T}-\overline{T}_\infty)\,\mathrm{d}x-\tau_0,\text{ and} \tag{5}$$

$$\frac{\mathrm{d}F}{\mathrm{d}z}=-Q\frac{\partial\Delta_\infty}{\partial z}. \tag{6}$$

Here $M=\int_0^\infty \overline{w}^2\,\mathrm{d}x$ is the momentum flux, and $\tau_0=c_f\,\frac{1}{2}w_m^2$ is the wall shear stress, where $c_f$ is the skin friction coefficient and $w_m$ is the maximum vertical velocity, $F=\int_0^\infty g\beta\,(\overline{T}-\overline{T}_\infty)\,\overline{w}\,\mathrm{d}x$ is the buoyancy flux and $\partial\Delta_\infty/\partial z$ is the environmental buoyancy gradient, with $\Delta_\infty=g\,\beta\,\overline{T}_\infty$.

Here, the mean vertical velocity($\overline{w}$) and reduced gravity($g\beta(\overline{T}-\overline{T}_\infty)$) are approximated by a half-Gaussian form, i.e. $\overline{w}=w_m\exp(-x^2/b_w^2)$, $g\beta(\overline{T}-\overline{T}_\infty)=\Delta\exp(-x^2/b_T^2)$, where $w_m(z)$ is the maximum vertical velocity, $\Delta(z)$ is the centreline reduced gravity and $b_w$ and $b_T$ are typical plume width associated with the vertical velocity and reduced gravity, respectively. Here, Gaussian profiles of equal width have been assumed for the vertical velocity and reduced gravity fields in the plume (i.e. $b_w=b_T=b$).

As suggested by Morton *et al.* [7], the rate at which fluid is entrained into the plume is taken as proportional to the mean maximum vertical velocity of the plume, $u_e=\alpha w_m$, where $\alpha$ is the entrainment coefficient.

Now we can express the fluxes in terms of a maximum vertical velocity, $w_m(z)$, and reduced gravity, $\Delta(z)$, along with a plume

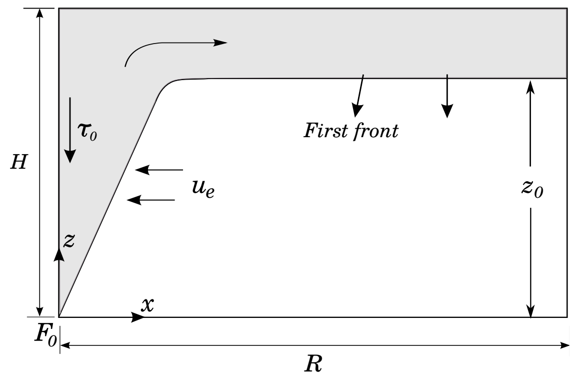


Figure 2: Schematic diagram of filling box model of wall attached line plume. The first front is the interface between buoyant fluid and the ambient fluid. The time-dependant position of the first front position is denoted by $z_0$.

width, $b(z)$, which are defined by,

$$Q=\frac{\sqrt{\pi}\,b\,w_m}{2},\quad M=\frac{\sqrt{\pi/2}\,b\,w_m^2}{2},\quad F=\frac{\sqrt{\pi/2}\,b\,w_m\,\Delta}{2}. \tag{7}$$

In terms of the fluxes, (4) and (5) become

$$\frac{\mathrm{d}Q}{\mathrm{d}z}=\sqrt{2}\,\alpha\frac{M}{Q},\text{ and,} \tag{8}$$

$$\frac{\mathrm{d}M}{\mathrm{d}z}=\frac{FQ}{M}-c_f\frac{M^2}{Q^2}, \tag{9}$$

respectively.

The plume equations described above are reasonable approximations for the line plumes in unconfined environments with $c_f=0$ (e.g. Lee and Emmons [6]; Paillat and Kaminski[9]). In the case of confined plumes, two other equations are considered for describing the environmental flow parameters. The conservation of mass in the filling box can be written as

$$Q=-RU, \tag{10}$$

where $U$ is the downward velocity of the environment, and the development of the buoyancy field in the environment is governed by

$$\frac{\partial\Delta_\infty}{\partial t}=-U\frac{\partial\Delta_\infty}{\partial z}. \tag{11}$$

**Set-up of direct numerical simulations**

In this study, we employ direct numerical simulation (DNS) to solve the equations of mass, momentum and energy conservation within the Boussinesq approximation. The line plume originates from a line heat source of length $L$ and initial width $b_0$ placed along the $y-$ direction at the bottom left corner of the box. The confining box has width $R$ and height $H$ in the $x-$ and $z-$directions, respectively. Here, the gravity acts in the negative $z-$direction, i.e. in the opposite direction to the rising plume.

The flux of temperature per unit area at the wall $f_w\equiv\kappa|\mathrm{d}T/\mathrm{d}z|_w$ ($=q_w/(\rho C_p)$), where $\kappa$ is the thermal diffusivity, $q_w$ is the wall heat output per unit area $(W/m^2)$, $C_p$ is the specific heat at constant pressure and $\rho$ is the reference density of the fluid; the subscript $w$ denotes properties at the bottom wall. For numerical simulation we take a smooth half-Gaussian profile at the wall over a distance of $R$ in $x-$ direction: $f_w=\kappa A_0\exp(-x^2/b_0^2)$, where $A_0$ is the maximum value of $|\mathrm{d}T/\mathrm{d}z|_w$ and $b_0$ is the initial

| $Re_H$ | $R/H$ | $L/H$ | $n_x$ | $n_y$ | $n_z$ | $\Delta x_c/b_0$ | $\Delta y/b_0$ | $\Delta z/b_0$ |
|---|---|---|---|---|---|---|---|---|
| 14530 | 1 | 0.5 | 1024 | 256 | 512 | 0.123 | 0.157 | 0.157 |
| 14530 | 2 | 0.5 | 2048 | 256 | 512 | 0.123 | 0.157 | 0.157 |

Table 1: Simulation parameters of the present cases. The cell grid sizes, $\Delta x_c$, $\Delta y$ and $\Delta z$ are non-dimensionalised by initial plume width $b_0$. The grid spacing in $x-$ direction, $\Delta x_c$ is measured at the centre.

plume width. The buoyancy flux per unit area = $g\beta\times$ (temperature flux per unit area) = $g\beta f_w$. Now, buoyancy flux per unit length (in $y-$direction),

$$F_0 = g\beta\int_0^\infty f_w\,\mathrm{d}x = \sqrt{\pi}g\beta\kappa A_0 b_0/2. \qquad (12)$$

The dimensionless parameters governing the present simulations are Reynolds number:

$$Re_H = F_0^{1/3}H/\nu, \qquad (13)$$

where $\nu$ is the kinematic viscosity and Prandtl number, $Pr = \nu/\kappa$, which is fixed at the value for air: $Pr = 0.71$.

The Reynolds number considered is $Re_H = 14530$ for boxes of two different aspect ratios, $R/H = 1$ and 2 (table 1). BT[1] observed that in order to avoid a large-scale circulation generated by the plume in the confined box, the stabilising buoyancy force in the region of plume outflow at the top of the box have to be larger than the inertial force of the plume. The ratio between these forces depends purely on the geometry of the box, i.e. aspect ratio ($R/H$) in this case, and not on the buoyancy flux or any other flow properties. In their experiments, they concluded that the critical value of the aspect ratio is about one. Therefore, in the present case, the lowest aspect ratio ($R/H$) of the box is set to one.

The bottom, top, left, and right boundaries are no-slip walls. Periodic boundary conditions are imposed on velocities, pressure and temperature in the $y-$direction. We set all initial velocities to zero and add a random perturbation to the temperature field in the entire domain, in order to trigger a transition to turbulence in the rising plume. The magnitude of temperature perturbations ($T(t=0)$) added to the flow is based on $(g\beta T b_0)^{1/2}b_0/\nu = 25.0$, and is kept constant for all simulations.

The grid spacing is uniform in the $y-$ and $z-$directions and a cosine stretching grid is set in the $x-$ direction. The DNS employs a mixed spectral/finite-difference algorithm for the spatial discretisation. While a fully conservative fourth-order, staggered finite-difference scheme is used for the velocity field calculation in the $x-$ and $z-$ directions, a Fourier spectral method is used for that in the $y-$ direction. The QUICK scheme is used to advect the temperature field. The equations are marched using a low-storage third-order RungeKutta scheme. Further details of the numerical technique can be found in Ng *et. al.*[8].

## Comparison of DNS results and theoretical model

### Wall plume in uniform environment

Figure 3 shows of the statistical mean vertical velocity and buoyancy profiles at different $z/H$ locations in a uniform environment for $R/H = 2$ case. The mean profiles are obtained by averaging spatially along the $y-$ direction as well as averaging across the time instances during which the horizontal front travels from $x/H = 0.5$ till $x/H = 2$. In figures 3(a) and (b),

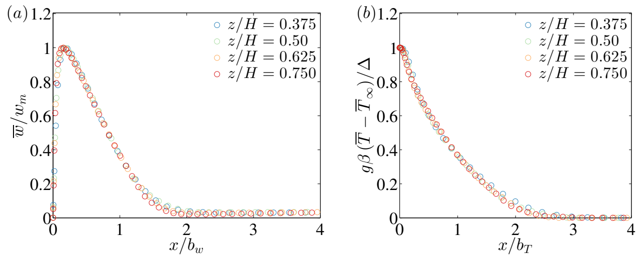


Figure 3: Mean profiles of (a) Vertical velocity and (b) buoyancy at $0.375\leq z/H\leq 0.75$ for the $R/H = 2$ case.

the $x-$ axes are normalised with plume widths $b_w$ and $b_T$, the widths at which the distribution has fallen to $1/e$ of its peak value. In the theoretical model, the mean vertical velocity and buoyancy profiles are assumed to be self-similar with height. It is clear from both figures 3 (a) and (b) that, within the region $0.375\lesssim z/H\lesssim 0.75$, the vertical velocity and buoyancy profiles are self-similar.

Figures 4 (a) and (b) show, respectively, the comparison of mean volume and momentum flux with the theoretical models. In order to find the theoretical fluxes, we solved (8) and (9) numerically with initial conditions of $Q_0 = 0.0$, $M_0 = 0.0$ and $F = F_0 = 1.0$ for $c_f = 0.0$ and $c_f = 0.012$. In the uniform environment, the buoyancy flux $F$ is constant with respect to the height (i.e. $F(z) = F_0$). The value of skin-friction coefficient, $c_f = 0.012$ is obtained from our present DNS data. In the present study, we used an entrainment coefficient $\alpha = 0.06$, which is determined by fitting the theoretical volume flux profile to the DNS data for $c_f = 0.0$. Grella and Faeth [3] have found similar values of entrainment coefficient, which is about 0.067 for wall attached line plumes in an open environment. To compute the integrals over the $x-$ direction of the plume for calculating the fluxes, we defined the upper integration limit as $x/H = 0.2$, which ensures that the vertical environmental velocity is small compared to that of the plume. The mean volume flux profile (figures 4 a) shows good agreement with the theoretical model and the skin-friction coefficient has an insignificant effect on volume flux. But the mean momentum flux (figures 4 b) shows moderate agreement with the model for $c_f = 0.012$. The difference between the model and the DNS data highlights the need for improved models in future.

### Time-dependent environmental buoyancy profile ($\Delta_\infty$)

As the plume hits the top wall, it spreads and advects downwards as a front (cf. figure 2). The front location and the instantaneous buoyancy profile in the environment $\Delta_\infty$ can also be estimated from our model. As such, we solved the differential equations (6), (8), (9) and (11) simultaneously to obtain the time-dependent environmental buoyancy profiles. Euler method is used to solve these equations. In the Euler method, a pure plume solution is given at the source (i.e. $Q_0 = 0.0$, $M_0 = 0.0$ and $F_0 = 1.0$) and integrating (11) over each time step to obtain the behaviour of $\Delta_\infty$. The non-dimensional time step used in the

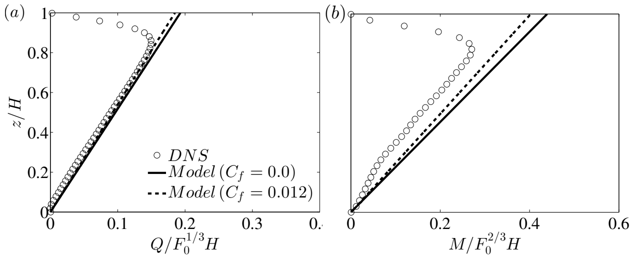


Figure 4: Comparison of mean (a) volume and (b) momentum flux with the theoretical model in a uniform environment. The black circle represents the DNS results , the black solid and dashed lines represent the theoretical model with $c_f = 0.0$ and $c_f = 0.012$, respectively.

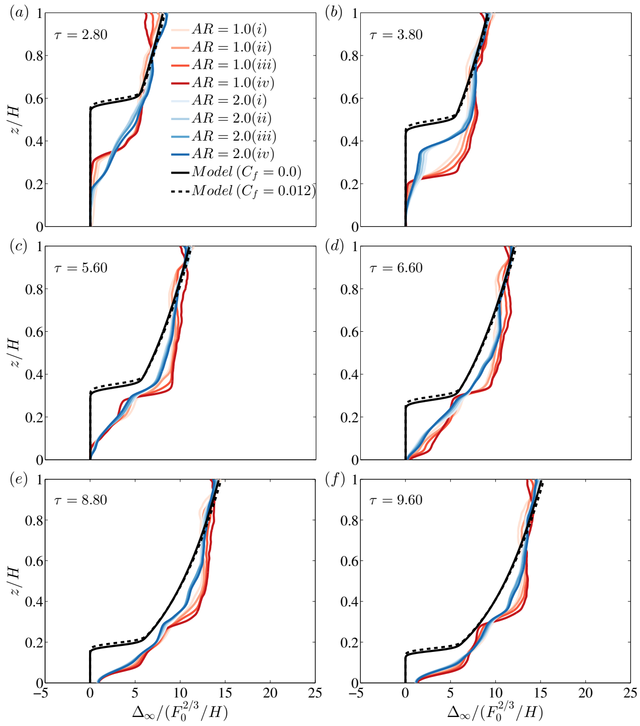


Figure 5: Comparison of the time-dependent environmental buoyancy profile with theoretical model for $AR = 1.0$ and $AR = 2.0$; (a) $\tau = 2.80$; (b) $\tau = 3.8$; (c) $\tau = 5.6$; (d) $\tau = 6.60$; (e) $\tau = 8.8$; and (f) $\tau = 9.6$. The colour gradients of red and blue indicates the starting point of horizontal average for two different aspect ratios; (i) for horizontal average from $x/H = 0.2$, (ii) for $x/H = 0.4$, (iii) for $x/H = 0.6$ and (iv) for $x/H = 0.8$.

computations is $1 \times 10^{-4}$, which satisfies the stability considerations. Here we assume that the time step ($\delta t$) is much smaller than the spatial resolution ($\delta z$), i.e $\delta t \ll \delta z$ and we also assumed that the plume spread quickly to form a horizontal layer at the top of the box ($z = H$), i.e. $\Delta_\infty(H, t + \delta t) = \Delta(H, t)$. The comparison of the time-dependent environmental buoyancy profile with the theoretical model for $AR = 1.0$ and $AR = 2.0$ is shown in figure 5. Here, $\tau$ is the non-dimensionalised time, which is defined as $\tau = t\, {F_0}^{1/3}/R$. We take the time $\tau = 0$ as the moment when the plume first touches the right wall, which is taken as the reference time. The line-averaged buoyancy near the side walls are considered to be environmental buoyancy, which is used here for comparison. In figure 5, the gradients red and blue lines represent the horizontally averaged buoyancy from different $x/H$ locations to the right wall for $AR = 1.0$ and $AR = 2.0$, respectively. The solid black line shows the numerical solution of theoretical model with $c_f = 0.0$ and the dashed line shows the numerical solution with $c_f = 0.012$. The influence of skin friction coefficient on environmental buoyancy is observed to be negligible. Considering the assumptions involved, the model shows reasonable agreement with the DNS results at all times.

## Conclusions

We have analysed the evolution of wall attached turbulent line plumes in a confined region using direct numerical simulations. The Reynolds number of the confined plume based on the box height and buoyant velocity scale is considered to be $Re_H = 14530$, and for boxes of two different aspect ratios, $R/H = 1$ and 2. The results from the DNS are compared against a modified theoretical model based on BT [1], where wall shear stress is incorporated to model the wall-attached plume. The distribution of mean volume and momentum fluxes in the uniform environment is observed to show good agreement with the theoretical model. The time-dependent buoyancy profile showed moderate agreement with the model, and the effect of wall shear stress on environmental buoyancy is found to be small.

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