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LATERAL EFFLUX FROM FLOW ALONG A WALL INTO A SUCTION-SLOT*

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ABSTRACT

The lateral efflux from flow along a wall into a single suction-slot in the wall has been studied both theoretically and experimentally. In particular, the contraction-coefficient defined as the ratio of the thickness of the jet in the slot to the slot-width has been computed by using conformal mapping for the free-stream velocity to the jet-velocity ratio varying from zero to one. Experimental results obtained in a subsonic wind tunnel agree surprisingly well with the theory.

INTRODUCTION

There is a fairly widespread interest in the determination of the characteristics of flow past a wall with suction-slots. For example, suction-slots are used in the paper-making process to withdraw water from a mixture of pulp and water carried by a horizontal wire-screen over a plane surface [1]. The idea of using suction-slots as the boundary-layer control device is not new and has been discussed extensively elsewhere [2]. In this paper, the lateral efflux from flow along a wall with a single suction-slot oriented perpendicular to the direction of the free-stream has been determined by using the method of conformal mapping. Experimental results were also obtained in a subsonic wind tunnel in order to verify the theory.

The problem of lateral efflux from flow in a two-dimensional conduit through a slot was investigated by McNown and Hsu [3] and later by Taliyev [4]. Taliyev's computed jet contraction-coefficients correspond well with his measurements. It is to be noted, however, that the result obtained in this study cannot be derived from either the work of McNown or that of Taliyev by taking the limit as the depth goes to infinity.

ANALYSIS

In the analysis, the flow is assumed to be two-dimensional, steady, and irrotational. The effects of gravity and viscosity are assumed to be negligible. The physical configuration of the flow past a suction-slot is shown in Fig. 1 a, in which U and V_j represent respectively the velocity of the free-stream and the jet velocity. The transformation planes used

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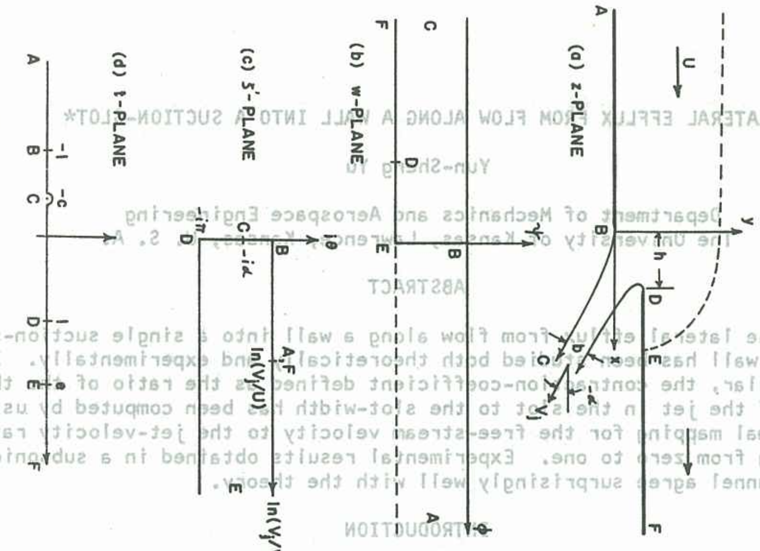

Figure 1. Transformation Planes

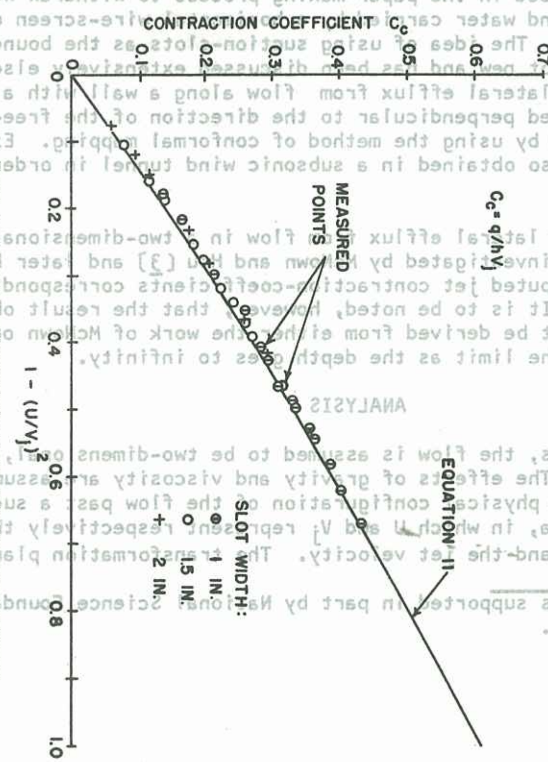

Figure 2. Comparison of Theoretical and Measured Contraction Coefficient

in the conformal mapping are: the complex potential w-plane, Fig. 1 b, the hodograph ζ'-plane, Fig. 1 c, and the t-plane, Fig. 1 d.

Using Schwartz-Christoffel theorem [5], one can transform the flow in the w-plane onto the upper half t-plane by

$$\frac{dw}{dt} = \frac{-b(t-e)V_j}{\pi(e+c)(t+c)} \tag{1}$$

in which b is thickness of the jet at infinity; e and c are constants to be determined. Similarly, one can transform the flow in the ζ'-plane onto the upper half t-plane by

$$\cosh \zeta' = \frac{(et-1)}{(t-e)} \tag{2}$$

where $\zeta' = \ln(V/U) + i\theta$; θ is angle of the velocity vector making with the positive x-axis; V is magnitude of the velocity vector.

Equation 2 can be written as

$$\left(V_j \frac{dz}{dw}\right)^2 + \frac{2(te-1)V_j}{(t-e)} \frac{dz}{dw} + 1 = 0. \tag{3}$$

As $t \to \infty$, $\frac{dz}{dw} = -\frac{1}{U}$ and Eq. 3 becomes

$$e = \frac{1}{2}\left(m + \frac{1}{m}\right) \tag{4}$$

in which $m = U/V_j$. Along the free streamlines BC and CD, $\zeta' = i\theta$; at C, $t = -c$ and $\theta = -\alpha$; thus from Eq. 3, one obtains

$$\cos \alpha = \frac{ce+1}{c+e}. \tag{5}$$

Solving Eq. 3 for dz/dw, taking the positive sign of the square root, and using Eq. 1, one has

$$dz = \frac{b\left[te - 1 + \sqrt{(e^2-1)(t^2-1)}\right]}{\pi(e+c)(t+c)} dt. \tag{6}$$

Equation 6 can be integrated along the contour from B to D shown in Fig. 1 d. Thus,

$$z_D = \frac{b}{\pi(e+c)} \int_{-1}^{1} \frac{te - 1 + \sqrt{(e^2-1)(t^2-1)}}{t+c} dt \tag{7}$$

$$= \frac{b}{\pi(e+c)} \left(P\int + \int_0 \right) \frac{te-1+\sqrt{(e^2-1)(t^2-1)}}{t+c} dt$$

in which the first integral on the right-hand side of Eq. 7 is the Cauchy's principal value defined by

$$\lim_{\epsilon\to 0}\left\{\left(\int_{-1}^{-c-\epsilon}+\int_{-c+\epsilon}^{1}\right)\frac{te-1+\sqrt{(e^2-1)(t^2-1)}}{t+c}\,dt\right\}.$$

Integration of Eq. 7 yields

$$\frac{z_D}{b}=\frac{1}{\pi(e+c)}\left\{2e-(1+ec)\left[i\pi+\ln\frac{1+c}{1-c}\right]+\pi\sqrt{e^2-1}\left[ic+\sqrt{1-c^2}\right]\right\}. \tag{8}$$

The real and imaginary parts of Eq. 7 are respectively

$$\frac{x_D}{b}=\frac{1}{\pi(e+c)}\left\{2e-(1+ec)\ln\frac{1+c}{1-c}+\pi\sqrt{(e^2-1)(1-c^2)}\right\} \tag{9}$$

and

$$\frac{y_D}{b}=\frac{c\sqrt{e^2-1}-(1+ec)}{e+c} \tag{10}$$

Equations 4, 5, 9, and 10 relate the jet angle α, the jet thickness b, the velocity ratio V_j/U, and the relative positions of the two edges of the slot.

If D and B are in the same plane, i.e.; $y_D = 0$, Eq. 9 becomes

$$c=-e-\sqrt{e^2-1}.$$

Consequently, $c = -U/V_j$ and $\cos\alpha = U/V_j$. If $x_D = h$, then the contraction coefficient defined by $C_c = b/h$ is

$$C_c=\frac{\pi(1-m^2)}{2\left\{1+m^2-\frac{m(1-m^2)}{2}\ln\frac{1-m}{1+m}+\frac{\pi}{2}(1-m^2)^{3/2}\right\}}. \tag{11}$$

In a special case with U = 0, Eq. 11 gives $C_c = \pi/(\pi+2)$ which is the well known result for flow into a two-dimensional orifice [6]. A plot of Eq. 11 is shown in Fig. 2 where the absissa is the dimensionless pressure coefficient $2\Delta p/\rho U^2 = 1-m^2$ derived from the Bernoulli-equation. Obviously, Eqs. 9 and 10 can also be used to compute similar results for cases when B and D are not in the same plane.

EXPERIMENTS

Apparatus

Experiments were made in a subsonic wind tunnel of the Department of Mechanics and Aerospace Engineering of the University of Kansas to verify the theoretical results obtained in the preceding section. The circular test section of the wind tunnel is 5 ft in diameter and 10 ft long. Two different values of the free-stream velocity namely, 23.3 fps and 33.8 fps, were used during the experiments.

The apparatus consists of a plywood board, a blower and a duct system. The board was made of two 1/2-in. plywood plates each 4-1/2 ft wide and 2-1/2 ft long lap-jointed to form a plane surface of 4-1/2 ft by 5 ft. The plate was fastened to the tunnel wall by bolts at its four corners and supported by wooden frame works. The surface of the board was treated to keep it smooth and its leading edge was rounded to reduce flow separation.

A slot 1-1/2 ft long and 3 in. wide was cut along the joint in one board. The width of the slot was made adjustable by using different sizes of machined aluminum plates, with sharp leading edge, attached to the board downstream from the slot. Three different slot-widths, namely, 1 in., 1.5 in. and 2 in., were tested. A suction box, 18 in. long, 5 in. wide, and 6 in. deep, was attached to the board beneath the slot. Four pressure taps were installed on the upstream wall of the suction box for measuring the suction pressure. A transition section 10 inches long connected the suction box to a 5-inch pipe. The other end of the pipe was attached to the suction-side of a 1 H.P. blower. The exhausted air from the blower was returned to the wind tunnel through a duct.

The discharge through the blower was controlled by a butterfly valve in the exhaust pipe and measured by a calibrated 2-inch orifice meter located in the 5-inch pipe upstream from the blower. The suction box and the pipe system were sealed air-tight. Wire screens were used at the junction of the suction-box and the transition section to reduce large eddies. Straightening tubes of 1/2-inch diameter were installed in the 5-inch pipe 3 feet upstream from the orifice.

The pressure in the suction-box was measured with a micromanometer which can record to one thousandth of an inch of water head. The free-stream velocity was measured with a pitot tube connected to a zero-displacement manometer which can be read to one thousandth of an inch of kerosene.

Results

The contraction coefficients defined in the previous section can also be expressed as $C_c = q/hV_j$, where q is discharged per foot into the slot. In the experiments, the free-stream velocity U, the discharge q, and the suction-pressure in the slot were measured. From the measurements and the Bernoulli equation with no energy loss, one can compute V_j and consequently, C_c and U/V_j. Measurements showed that the free-stream velocity along the wall was fairly uniform and that the pressure readings from the four pressure taps in the suction-box were close. Therefore, the average pressure was used in computing the jet velocity. The results within the experimental range for three different slot-widths are plotted in Fig. 2 for comparison.

COMPARISON OF THEORY AND EXPERIMENT

Figure 2 shows that the computed contraction coefficients agree surprisingly well with the measurements. In fact all of the measured points fall within about five percent above the theoretical curve. The results clearly indicate that for these experiments the effect of viscosity on the lateral flow is small. Indeed, the Reynolds numbers defined by the free-stream velocity, the length of the upstream plate, and the kinematic viscosity of air, are 3.4×10^5 and 5.0×10^5, respectively, for the two free-stream velocities. For these cases it is very likely that the boundary-layer flow would still be laminar. The corresponding displacement-thicknesses, if the slot were absent, would be 0.089 in. and 0.073 in. [7] which are less than one-tenth of the width of the narrowest slot. The effect of the boundary-layer would be expected to increase the lateral flow, other conditions remaining unchanged.

CONCLUSION

The surprisingly good agreement between the computed and the measured contraction coefficient for flow along a wall with a single suction-slot leads one to conclude that the theoretical curve shown in Fig. 2 can be used to compute the lateral efflux if the slot-width, the suction-pressure, and the free-stream velocity are known.

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THERMAL BOUNDARY LAYERS IN LAMINAR AND TURBULENT FLOW

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SYNOPSIS

This paper describes a rapid method for the numerical solution of a class of nonlinear high order differential equations with two point asymptotic boundary conditions.

The method replaces the asymptotic boundary condition with an assumed condition at the other boundary, thus substituting an initial value problem for the boundary value problem. The assumed condition is progressively and systematically corrected until the asymptotic condition is obtained. The method is applied to the incompressible steady boundary layer equations. Turbulent flow is included by the use of an eddy viscosity. The purpose of the investigation is to obtain a computational procedure which will enable the solution of the equations to be compared with experimental observations, thus testing the validity of hypotheses regarding the variation of eddy viscosity in the boundary layer. The method is at present limited to those flows whose governing equations can be reduced to ordinary differential equations. For constant fluid properties, the procedure can obviously be extended to the solution of the energy equation (for laminar or turbulent flow), once the relevant momentum equation has been solved.

Introduction

A problem which is currently attracting some attention is the influence of turbulence on the transfer of heat by forced convection into the boundary layer on a flat plate. One approach seeks to describe the phenomena in terms of eddy coefficients of momentum and heat transfer, which, in general, will vary both across and along the boundary layer and which may not be equal to each other. It would be the aim of the approach to obtain formulations for these coefficients as functions of, say, position, pressure gradient, Reynolds number and Prandtl number.