This would, of course, be expected to be close to the actual measured wire size. Once the experimental value is established, the same size could be used for later runs where surface viscosity is present. It is believed that, in this way, a more realistic estimation of subsurface tractive effects may be made than has previously been possible. By using the experimental rather than the theoretical equivalent wire size, ring peculiarities as well as some of the approximation in the derivations would be expected to be accounted for.

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REFERENCES

1. W. E. Ewers and R. A. Sack, "The Surface Viscosity of Solution Films," Australian Journal of Chemistry, Vol. 17, 1954.

2. M. Joly, "Surface Viscosity," Recent Progress in Surface Science (Edited by J. F. Danielli, K. G. A. Pankhurst, and A. C. Riddiford), New York, Academic Press, 1964.

STEADY SEEPAGE FLOW ABOVE A SLOPING IMPERMEABLE SUBLAYER

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ABSTRACT

The problem of seepage flow in a saturated layer, bounded below by a sloping impermeable sublayer, with the upper surface free and replenished by infiltration, can be treated by a modified form of the Dupuit-Forchheimer assumption in which the streamlines are assumed parallel to the sloping sublayer. Calculated values of the slope of the phreatic surface at the highest point of the flow region and far downstream are found to be in agreement with exact values obtained by consideration of the flow hodograph. By contrast, the results obtained using the classical Dupuit-Forchheimer assumption that streamlines are horizontal (Werner 1957, Schmid and Luthin 1964) exhibit increasing discrepancies as the slope increases. A simple method exists for checking the accuracy of the modified form of Dupuit-Forchheimer assumption at intermediate points by comparison with the hodograph.

Both the exact and approximate solutions indicate a change of form of the steady-flow solution at a certain critical rate of replenishment; in most practical cases the rates are sub-critical. Now, for the subcritical or critical rate, the hodograph is found to be a curvilinear triangle with interior angles A, B, C obeying the inequalities.

$$0 < A, B, C < \pi \;;\; 0 < B + C, C + A, A + B < \pi \qquad (1)$$

Under the conditions (1) it was shown by H.A. Schwartz (see Poole (1936)) that the function

$$S\,(A/\pi\,,\; B/\pi\,,\; C/\pi\;;\;\lambda\,) = \zeta \qquad (2)$$

maps the upper half of the complex λ-plane onto a curvilinear triangle in the ζ-plane, with interior angles A, B, C. A necessary condition for (2) to possess a one-valued inverse function (an automorphic function) is that A/π, B/π, etc. should be either zero or the reciprocals of integers.

In general the Schwartzian function is expressed as the ratio of two linearly-independent solutions of the hypergeometric equation. However, for the special case where the interior angles are 0, 0, 0, this reduces to the ratio of two elliptic integrals

$$\zeta = K'(\lambda) / K(\lambda) \qquad (3)$$

and the inverse, $\lambda(i\ \zeta)$, equal to the square of the elliptic modulus, is an elliptic modular function. This case has been used by Hamel (1934) and Muskat (1937) in the exact solution for flow through a dam with vertical faces. The function also occurs in certain problems involving salt-water intrusion into coastal aquifers (Henry 1959), since the hodograph has the same triangular form.

Exact solutions using Schwartz's transformation have been compared with approximate results based upon the modified Dupuit-Forchheimer assumption in a special case, viz., saturated seepage flow over an impermeable base of slope 30° to the horizontal with the rate of replenishment equal to the critical value (0.0718 times the hydraulic conductivity). The typical hodograph is a curvilinear triangle with interior angles ($\pi/2, \pi/3, 0$), for which the Schwartzian function reduces to the elliptic modular function

$$J = \frac{4}{27} \frac{(1 - \lambda + \lambda^2)^3}{(\lambda^2 - \lambda)^2} \qquad (4)$$

This case leads to three different physical problems.

(a) Flow over a slope into a horizontal slit drain. The width of drain effective in removing water is equal to about 4.0 per cent of the downslope distance, while the maximum depth of water (measured normal to the impermeable layer) is about 9.65 per cent and occurs at 72 per cent of the distance downslope. The approximate solution obtained using the extended form of Dupuit-Forchheimer assumption is in excellent agreement with the exact result except near the slit drain.

(b) Flow with finite depth at the highest point. The Dupuit-Forchheimer assumption is asymptotically correct at large distances downstream, but fails as the highest point on the phreatic surface is approached, where the surface is horizontal.

(c) Seepage surface parallel to the impermeable layer (the permeable overburden problem). The depth of saturated flow increases with downslope distance, until the phreatic surface intersects the seepage surface at a downslope distance of 7.3 times the overburden depth. Flow across the seepage surface (an inflow) decreases rapidly with further distance downslope. The modified Dupuit-Forchheimer assumption leads to an excellent prediction of the shape of phreatic surface, but treats the seepage flow as zero.

References:

Hamel, G. — Uber Grundwasserstromung. Zeitschr. angew. Math. u. Mech. 14, 129-157, 1934.

Henry, H.R. — Salt intrusion into fresh-water aquifers. J. Geophys. Res. 64, 1911-1919, 1959.

Muskat, M. — The flow of homogeneous fluids through porous media. 763 pp. McGraw-Hill, New York, 1937.

Poole, E.G.C. — Introduction to the theory of linear differential equations. Oxford, 1936.

Schmid, P. and Luthin, J. — The drainage of sloping lands. J. Geophys. Res. 69, 1525-1529, 1964

Warner, P. Wilh. — Some problems in non-artesian ground-water flow. Trans. Amer. Geophys. Un. 38, 511-518, 1957.

Wooding, R.A. and Chapman, T.D. — On ground-water flow over a sloping impermeable layer. I. Application of the Dupuit-Forchheimer assumption. J. Geophys. Res. (submitted).

Wooding, R.A. — On ground-water flow over a sloping impermeable layer. II. Exact solutions by conformal mapping. J. Geophys. Res. (submitted).