THE NON-DIMENSIONAL REPRESENTATION OF PROFILES OF GRADUALLY VARIED FLOW

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ABSTRACT

Generalised profiles, in non-dimensional form, are presented for type M and type S water surface profiles of gradually varied flow in wide channels. The form of each type is seen to depend upon the Froude number of the uniform flow.

INTRODUCTION

It has been shown (1) that a pattern of similarity exists for M1-type profiles in rectangular channels of given width, and that these profiles can be represented on a non-dimensional plot. For a given maximum-depth to normal-depth ratio, all profiles plot in an ordered series according to their uniform flow Froude numbers, irrespective of their actual bed slopes, Manning n values and normal flow depths.

The generalised profiles in the non-dimensional presentation provide a ready means of investigating the effects of changes in the channel and flow characteristics on the form of the water surface profiles; and they provide the practising engineer with an approximate guide to the dimensions of profiles for particular cases, thus obviating the need for tedious computation in preliminary studies.

In this paper, the non-dimensional approach is extended to cover the M2, M3, S1, S2 and S3 profiles.

DEVELOPMENT OF PROFILES

The differential equation for gradually varied flow profiles in very wide channels, based on the Manning formula, is

$$\frac{dy}{dx} = S_o \frac{1 - \left(\frac{y_o}{y}\right)^{10/3}}{1 - \left(\frac{y_c}{y}\right)^3} \qquad 1.$$

where $\frac{dy}{dx}$ = water surface slope relative to the bed

S_o = bed slope, measured as +ve downwards

y_o = normal depth

y_c = critical depth

y = flow depth

x = distance along the channel

For cases in which the uniform flow Froude number is less than 3.0, the bed slope is normally flatter than 3 per cent and negligible errors are involved if y is measured vertically instead of normal to the bed and x is measured horizontally rather than parallel to the channel bed.

If the profile computations are made by computing length increments, δx, corresponding to assumed depth increments, δy, the computational equation corresponding to Equation 1 is

$$\delta x = \frac{\delta y}{S_o} \cdot \frac{1 - \frac{y_c^3}{y_1^2 y_2^2 \bar{y}}}{1 - \left(\frac{y_o}{\bar{y}}\right)^{10/3}} \qquad 2.$$

where $\bar{y}$ = average of the depths, y_1 and y_2, at the ends of the incremental length, δx; and $\delta y = y_2 - y_1$.

In order to obtain non-dimensional profiles, reference lengths for the vertical and horizontal measures are necessary. The obvious vertical reference length is the normal flow depth, y_o. For the horizontal reference length, L_o, a measure suitable for M and S type profiles is the distance over which the fall in bed level equals the depth, y_o. (This was not the reference length adopted in the paper on M1 curves (1). It happens to be one-half of that length. The M1 curves reproduced in the present paper are based on L_o as defined above).

$$\therefore \quad L_o = \frac{y_o}{S_o} \qquad 3.$$

The computational equation for the non-dimensional profiles is therefore

$$\frac{\delta x}{L_o} = \frac{\delta y}{y_o} \cdot \frac{1 - \frac{y_c^3}{y_1^2 y_2^2 \bar{y}}}{1 - \left(\frac{y_o}{\bar{y}}\right)^{10/3}} \qquad 4.$$

For infinitesimal increments, this is equivalent to

$$\frac{\delta x}{L_o} = \frac{\delta y}{y_o} \cdot \frac{1 - \left(\frac{y_c}{\bar{y}}\right)^3}{1 - \left(\frac{y_o}{\bar{y}}\right)^{10/3}} \qquad 5.$$

Since $\left(\frac{y_c}{y_o}\right)^3 = F_o^2$ where F_o is the uniform flow Froude number,

and $\bar{y} = y_o + \Delta y$ where Δy is the sum of the increments, δy, to the reach under consideration,

$$\frac{\delta x}{L_o} = \frac{\delta y}{y_o} \cdot \frac{1 - F_o^2 \left(\frac{1}{1 + \frac{\Delta y}{y_o}}\right)^3}{1 - \left(\frac{1}{1 + \frac{\Delta y}{y_o}}\right)^{10/3}}$$

i.e. $$\frac{\delta x}{L_o} = \frac{\delta y}{y_o} f\left(F_o, \frac{\Delta y}{y_o}\right) \qquad 6.$$

Evidently the non-dimensional profile, plotted as $\frac{\Delta y}{y_o}$ v. $\frac{x}{L_o}$ is simply a function of F_o.

Equation 4 was programmed for the IBM 1620 computer, to produce the profiles for a range of F_o values, for the M2 and M3 and the S type curves. The curves were plotted with $\Delta y/y_o$ as a function of x/L_o for various values of F_o. On these plots, Δy is the difference in actual depth at x from the depth at the control section. This control section depth is the critical depth, for the M2, M3, S1 and S2 curves; for the S3 curve, Δy is measured from the bed, although the curve was computed from a control depth of 0.1 y_o and extrapolated to locate the theoretical point of intersection with the bed. For the previously computed M1 curve, the control depth was arbitrarily chosen.

In the non-dimensional plots, the water surface slopes in the regions of uniform flow are all 45°, allowing for the horizontal and vertical scale differences.

THE PROFILES

(i) The M1 Profile. This profile occurs when steady flow in a mild sloping channel is checked by a downstream control, which increases the depth above normal depth. It is convex upwards, being asymptotic to the normal uniform flow surface in the upstream region and approaching the horizontal at the downstream end (Fig. 1, insert).

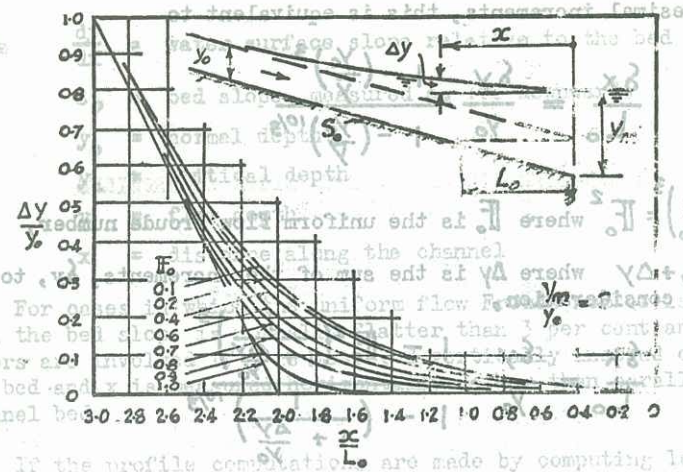

FIG. 1 - NON-DIMENSIONAL M1 PROFILES
IN WIDE CHANNELS, FOR $y_m/y_o = 3$

For the presentation of these profiles it was necessary to adopt an arbitrary value of y_m/y_o. The value of 3 was selected. For other values, the profiles retain their shape but the scale of the abscissa changes, since x is measured from a different origin, while L_o is unaffected by the y_m/y_o ratio.

It is clear that, on the non-dimensional plot, the height of the curve increases with decrease in F_o; and, since

$$F_o = \frac{V_o}{\sqrt{g y_o}} = \frac{1.49}{\sqrt{g}\, n}\, y_o^{1/6}\, S_o^{1/2}$$

the effects of changes in n, y_o and S_o on the profile follow from a consideration of their effects on F_o.

(ii) The M2 and M3 Profiles. In mild sloping channels, with the depth between normal and critical, the profile is an M2 curve, convex upwards, as in flow towards a brink. If downstream conditions permit it, the profile passes through critical depth near the brink and this is taken as the control section for the upper profiles in Fig. 2. Theoretically, the profiles cross the critical depth line with the water surface vertical at this section. In practice, owing to effects of local curvilinear flow not allowed for by the ordinary profile theory, this does not occur and a slight error, which is generally ignored, results in the profile computations.

If the depth is less than critical, as in rapid flow from beneath a sluice gate into a mild channel, an M3 curve results. The lower profiles in Fig. 2 are M3 curves. From a theoretical viewpoint, the M2 and M3 profiles, for a given value of F_o, form the two parts of one curve. For this reason, as well as for convenience in presentation, they are presented on the one diagram.

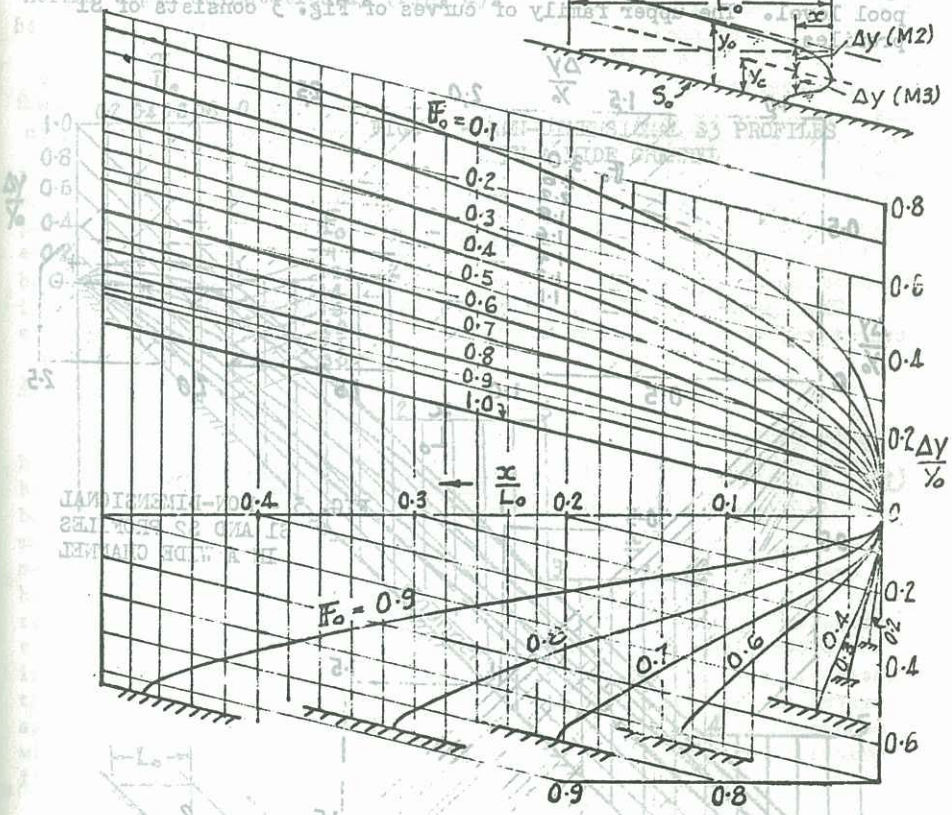

FIG. 2 - NON-DIMENSIONAL M2 AND M3 PROFILES
IN A WIDE CHANNEL

As a matter of academic interest, it may be noted that the M3 curves each exhibit a point of contraflexure near the bed, approaching the bed so as to intersect it vertically, according to computations based on the Manning formula.

(iii) The S1 and S2 Profiles. In steep channels, that is, those in which a given discharge would normally flow at depths less than critical, an S1 profile results if the flow depth exceeds critical depth. The S1 profiles have been computed from the critical depth section, although, in practice, they commence at the downstream end of a hydraulic jump, at a depth greater than critical, and extend, convex upwards, towards a horizontal asymptote represented by the downstream pool level. The upper family of curves of Fig. 3 consists of S1 profiles.

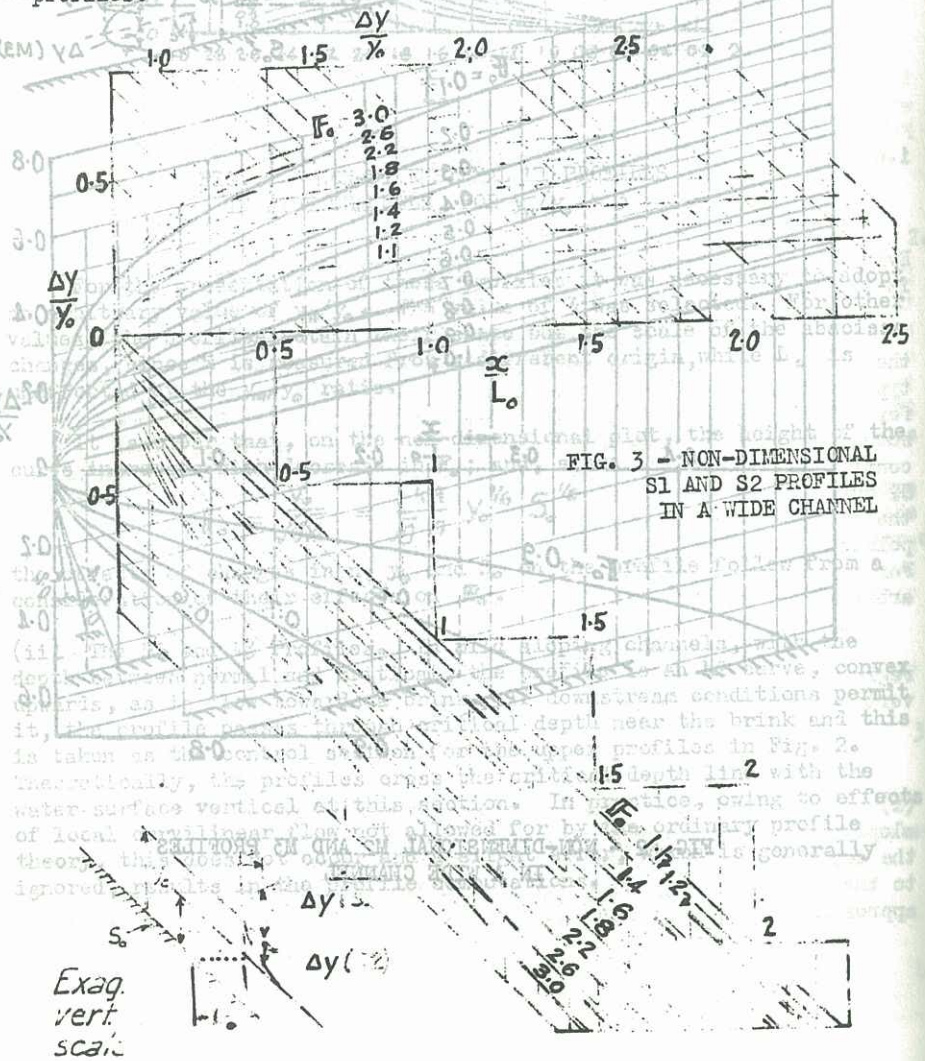

FIG. 3 - NON-DIMENSIONAL S1 AND S2 PROFILES IN A WIDE CHANNEL

If the flow depth is between the normal and critical depths, as when flow at greater than normal depth at the channel inlet accelerates along the channel, the "draw-down" profile is an S2 curve. The lower curves in Fig. 3 are of this type and they have been computed as commencing at the critical depth section. For a given F_o value, the S1 and S2 profiles are the two limbs of the one curve.

(iv) The S3 Profiles. In steep channels, flow at depths less than normal results in an S3 profile which is convex upwards. Theoretically, if the Manning formula is used, it approaches the bed vertically at the upstream end. Downstream, it approaches the line of normal depth asymptotically (Fig. 4).

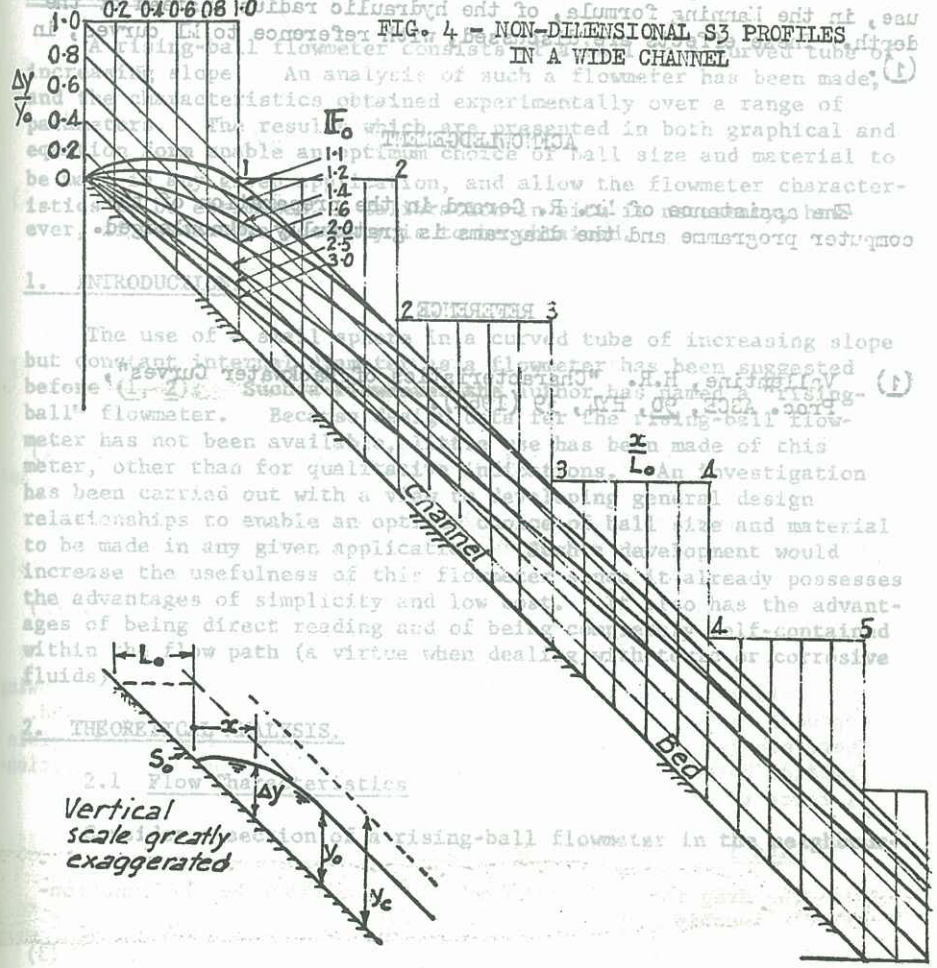

FIG. 4 - NON-DIMENSIONAL S3 PROFILES IN A WIDE CHANNEL

The critical slope profiles, known as type C profiles are limiting cases of the M and S profiles, with F equal to unity.

CONCLUSION

The non-dimensional representation of the water surface profiles provides a clear picture of the manner in which the profiles depend upon the uniform flow Froude number, and hence on the channel and flow characteristics. It serves also to give an approximate idea of the dimensions of profiles, and this may be of value in preliminary investigations of practical cases of gradually varied flow in open channels. It is to be noted that the profiles presented refer only to wide channels. In non-wide channels, the resistance to flow exerted by the side walls must be taken into account by the use, in the Manning formula, of the hydraulic radius instead of the depth. These effects are discussed, with reference to M1 curves, in (1).

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REFERENCE

(1) Vallentine, H.R. "Characteristics of Backwater Curves", Proc. ASCE, 90, HY4, 39 (1964).

THE CHARACTERISTICS OF THE RISING-BALL FLOW METER

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ABSTRACT.

A rising-ball flowmeter consists of a ball in a curved tube of increasing slope. An analysis of such a flowmeter has been made, and the characteristics obtained experimentally over a range of parameters. The results which are presented in both graphical and equation form enable an optimum choice of ball size and material to be made in any given application, and allow the flowmeter characteristics to be estimated. Calibration in situ is necessary, however, if reasonable accuracy is to be obtained.

1. INTRODUCTION.

The use of a small sphere in a curved tube of increasing slope but constant internal diameter as a flowmeter has been suggested before (1, 2). Such a flowmeter the author has named a "rising-ball" flowmeter. Because design data for the rising-ball flowmeter has not been available, little use has been made of this meter, other than for qualitative indications. An investigation has been carried out with a view to developing general design relationships to enable an optimum choice of ball size and material to be made in any given application. Such a development would increase the usefulness of this flowmeter since it already possesses the advantages of simplicity and low cost. It also has the advantages of being direct reading and of being completely self-contained within the flow path (a virtue when dealing with toxic or corrosive fluids).

2. THEORETICAL ANALYSIS.

2.1 Flow Characteristics

Consider a section of a rising-ball flowmeter in the neighbour-