PROPERTIES OF OCEAN WAVES GENERATED ON CURRENT

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ABSTRACT

Properties of waves produced on ocean and tide currents are investigated from the viewpoint of their steepness and damping on the basis of observed records around Japan.

These analysed results are applied to the criterion for the rolling safety of seagoing ships.

1. INTRODUCTION

The Japan Archipelago is entirely surrounded by steady ocean currents. This geographical environment causes many noticeable effects on the properties of ocean waves. Owing to the existence of stationary currents, wave motion is often very stable and wave height becomes very high in head winds and vanishes rapidly in tail winds. Thus the condition of the sea surface is quite different especially in eddy viscosity because of emulsed foams generated from the rough sea surface.

As a result, in such a confused sea, the ship's safety is very important. The character of its rolling motion is different when there is no current. The author was once engaged in revising the rules and laws concerning marine safety, and has since investigated oceanographical and meteorological data around the coast of Japan, from the naval architectural point of view. It has been found that there are important differences in ocean waves when current exists. This paper summarises the results of lengthy research begun in 1947.

2. Generation of Ocean Waves by Prevailing Winds:

Many distinguished papers concerning the theory of ocean waves have been published by Sverdrup and Munk, Darbyshire, and other scientists in England and U.S.A. These theories are mainly applicable to vast open sea areas such as the Atlantic Ocean or the Pacific Ocean. However, in the case of the coasts of Japan, a monsoon which blows for several weeks with constant velocity and direction seldom occurs. Hence the theory of the above-cited authors is not applicable. Moreover, wind force changes daily, and ocean and tidal currents sweep the coasts quite strongly. Thus we are obliged to establish another theory which expresses more exactly the state of waves around Japan.

The growth of a wave is given in Fig. 1, which shows that wave age (given by $\beta = c/u$ = wave velocity/wind velocity) tends to unity when wind duration tends to infinity. This tendency seems to be quite natural because there is no relative motion between wind and wave in this state. The duration curve in this figure is expressed in the following fundamental equation,

$$\frac{d\,(EC)}{Cdt} = R \tag{1}$$

where E and C are total energy and velocity of wave, R is force generated by wind. Using wave age β and wave steepness δ, the above equation is transformed to the form:

$$\frac{d\,(\beta^5\delta^2U^5)}{dt} = 2\sigma\,\frac{\rho'}{\rho}\,g(1-\beta)^2\,\beta^2\delta^2U^4 \tag{2}$$

where g is gravitational acceleration, σ is essentially the sum of frictional and sheltering coefficients chosen by Sverdrup and Munk, namely $\sigma = 0.013/2 + 0.0026 = 0.0091$ (which is the same value as analysed from the data around the coast of Japan) ρ' and ρ are density of air and sea water. Here, writing H, L and J as height, length and slope of wave respectively

$$E = \frac{1}{8}\rho g H^2 = \frac{\pi^2}{2}\,\frac{\rho}{g}\,\beta^4\delta^2U^4$$

$$R = \sigma\rho'\,(U-C)^2\,J^2C = \pi^2\rho'\,(1-\beta)^2\,\beta\delta^2\,U^3 \tag{3}$$

$$\beta = C/U,\ \delta = H/L;\quad J = \pi H/L,\quad L = 2\pi\,C^2/g$$

The term J^2C is the rate of momentum transmission by Michell's theory for a Stokes wave, and the term $\sigma\rho'\,(U-C)^2$ is the frictional stress caused by the roughness of the sea surface with the correction of relative velocity between wind and wave.

3. Wave Steepness and Wave Age:

The relation between steepness and age of an ocean wave is easily deduced from the viewpoint of mutual exchange of energy between wind and wave. If we assume that κL times the energy of the wind is supplied completely to the wave, the following equation is valid, namely

$$\frac{1}{2}\rho' U^2 \times \kappa L = \frac{1}{8}\rho g H^2. \tag{4}$$

κ is an absorption coefficient and is almost equal to unity, which means that wind energy in one wave length corresponds to total wave energy. The above equation leads directly to the next relation:

$$\beta\delta = \sqrt{\frac{2}{\pi}\,\frac{\rho'}{\rho}\,\kappa} \tag{5}$$

Assuming that $\kappa = 1$ and putting $\rho' = 0.001205$ grm/cm^3, $\rho = 1.025$ grm/cm^3, the numerical value of the right-hand side becomes 0.02736, which gives a curve passing through the middle part of observed data in Fig. 2. In this figure the steepness curve for the significant value is drawn as $1/10\pi$ of the right-hand side value, namely

$$\beta\delta = 0.03183 \tag{6}$$

which means $\kappa = 1.353$.

According to the results derived by Darbyshire $\delta\beta = 0.163/\sqrt{U(kt)}$, this gives $\kappa = 18.26/U$(m/sec), so that for the above value $U = 13.49$ m/sec.

On the other hand, wind velocities for these data occur most frequently in the vicinity of $10\sim15$ m/sec, so we may accept the validity of the above-cited view of energy exchange.

4. Waves Generated by Monsoons:

Now returning to Fig. 1, when we apply the previous relation between age and steepness of wave, the fundamental equation becomes

$$3\beta^2(1-\beta)^{-2}\frac{d\beta}{dt} = D_o\frac{g}{U},\quad D_o \equiv 2\sigma\frac{\rho'}{\rho} = 0.00002140 \tag{7}$$

or, after integrating and using the condition that $\beta = 0$ when $t = 0$,

$$3\left[\frac{1}{1-\beta} + 2\log(1-\beta) + (\beta-1)\right] = D_o\frac{gt}{U}. \tag{8}$$

Data in the range of $gt/U=10^3 \sim 10^4$ are obtained at inland seas and gulfs.
The region of $10^4 \sim 10^5$ corresponds to seas at coastal areas, and for $10^5 \sim 10^6$ data from swells on the open sea surface are included. This curve is similar to that of duration-graphs drawn by Sverdrup and Munk, Bretschneider, etc. In the waters near Japan, the most frequently occurring wave age is near unity. On the other hand, according to Darbyshire's result, the relation T(sec) = 0.25 x 3/2 U(kt) holds, which indicates that $\beta = g/2\pi$. T/U = 1.137. Thus the condition that velocities of wind and wave are equal seems to be the most stable one.

5. Waves Generated by Moving Storms:

When the direction and velocity of the wind change continually, such as in travelling or developing storms (this is the usual weather condition around Japan), we should consider another theory for predicting waves. The following equation holds at every instant, with a varying wind, because of the interaction of the sea surface with the wind.

$$\frac{d(\beta^3 U^5)}{dt} = D_o g(1-\beta)^2 U^4 \qquad (9)$$

or

$$\frac{d\beta}{dt} = \frac{1}{3\beta^2 U}\left[D_o g(1-\beta)^2 - 5\beta\frac{dU}{dt}\right] \qquad (10)$$

This equation can be integrated graphically using isoclinic curves in the plane (β, t) for a given value of U(t) at every stage of storm. Now, let us consider the maximum value of the height or period of a wave in the vicinity of the storm centre.

Before the centre arrives, i.e. before the maximum wind velocity is reached, wind velocity increases almost steadily so that wave age keeps a stationary value. This condition is expressed as:

$$\frac{d\beta}{dt} = 0 \quad \therefore \quad \frac{(1-\beta)^2}{\beta^3} = \frac{5}{D_o g}\frac{dU}{dt} \qquad (11)$$

so that wind velocity gradient determines wave age.
As du/dt = 1~3 m/sec/hour, then from this relation, β = 0.3~0.6, which is in the vicinity of the state of maximum steepness introduced by Sverdrup and Munk, namely: $\beta = 0.4$, $\delta = 0.1$. Taking (for brevity) mean values of observed data as $\beta = 0.5027$, we get the maximum height and period of wave in the centre of storm:

$$T(sec) = 0.3210U(m/sec.), \quad H = 0.01025\, U^2 (m/sec) \qquad (12)$$

6. Waves Generated by Tides and Ocean Currents:

Ocean waves are generated not only by wind, but also by currents. Especially in Japan, where there are many narrow channels and straits, we can see these very high waves on the sea surface everywhere. Of course, this kind of wave occurs even in calm weather, but when the wind is directly against the current, sharp, regular and violent waves with breaking crests form, which may cause serious accidents to ships. In general, drift currents after storms have passed away may have the same effect also.

The steepness of this current wave sometimes exceeds 1/10, so that there may be higher transmission of wave energy than that explained by the Stokes wave theory. According to the observed data shown in Fig. 3, the appropriate speed should be J V, which is proved by the theory of drift motion of a ship swaying amongst waves. The fundamental equation becomes:

$$\frac{dE}{dt} = R \qquad (13)$$

in which

$$E = \frac{1}{8}\rho g H^2, \qquad R = \sigma\rho' V^2 \;\; J\,V \qquad (14)$$

Here V is the velocity of the current, and other notations are as before. Then we obtain

$$\frac{dH}{dt} = D_o V \qquad (15)$$

so that under the assumption that tide current varies as $C = C\max \sin\omega t$, integration of the result leads to MaxH = $2D_o C\max/\omega$. Taking roughly $\omega = 2\pi/12 \times 3600$, the numerical value of the above relation becomes Max. H(m)=0.1514 Vmax(kt), which is indicated in the figure Fig. 4 drawn as a straight line.

Next, we consider the waves produced on an ocean current under the action of wind. As is seen from the observed results, wave steepness usually reaches more than $1/\pi$ that of a cycloidal wave, and the relation of energy contribution from current to wave is

$$\frac{1}{8}\rho g H^2 = \frac{1}{2\pi} L \times \frac{1}{2}\rho W^2 \qquad (16)$$

where W is ocean current velocity. The factor $1/2\pi$ is the same sort of factor as (used before) and so chosen that the wave steepness has the value of $1/\pi_\kappa$. Then combining the relation of energy exchange between wind and wave, we obtain

$$\frac{1}{8}\rho g H^2 = L \left| \left\{ \frac{1}{2}\rho' U^2 + \cos\phi \right| \cos\phi \right| . \frac{1}{2\pi}\rho W^2 \right\} \qquad (17)$$

in which is difference of direction of current and wind. As is seen in this relation, if the velocity is only a few knots, wave height may increase with a head wind or decrease with a tail wind by a factor of up to 2.

Table 1 shows the observed sea state at a lighthouse where the warm equator current sweeps eastwards. Maximum wave height is then increased in an east opposing wind. Referring again to Fig. 2, all data above the significant curve are produced by a head wind, and data below are observed in a tail wind.

TABLE 1

Frequency of Sea State for Eastern and Western Wind Direction (Left W, Right E).

Grade of Sea State	Grade of Wind Force: 0 (W)	0 (E)	1 (W)	1 (E)	2 (W)	2 (E)	3 (W)	3 (E)	4 (W)	4 (E)	5 (W)	5 (E)	6 (W)	6 (E)	Total (W)	Total (E)
0																
1	1	1	6	16	1	3									8	20
2		1	17	53	16	31	3	4		1					36	90
3			14	15	25	34	15	26		4					54	79
4			7	4	29	12	37	22	16	6		2			89	46
5			1	3	6	3	33	16	23	17	6			1	69	40
6					3		6	5	17	2	14	2	1	2	41	11
Total	1	2	45	91	80	83	94	73	56	30	20	4	1	3	297	286

7. Decay of Sea State:

The existence of ocean currents have an important effect on the process of decay of waves. From the daily record of the sea state, superposing all dates when maximum values occur, and overlapping the further records for every proper long term, then taking the mean value at every date, we can obtain the average process of decay.

This phenomenon is generally expressed in the following equation

$$\frac{dS}{dt} + \lambda S = F(t) \qquad (18)$$

F(t) is the term of effect of wind force on sea state. S is deviation of sea state from mean value and λ is damping coefficient. Through the above mathematical treatment we obtain

$$\text{Mean } S = \text{Mean } S_o\, e^{-\lambda t} \qquad (19)$$

using

$$\text{Mean } F(t) = 0 \qquad (20)$$

Table 2 shows the frequency distribution of λ around Japan. Here the left end corresponds to isolated islands in the Pacific Ocean and the right end to the inland sea. Accordingly, owing to the ocean current, the restoring time from storm to calm sea surface is reduced. The Japanese Islands experience westerly winds after storms pass and, since the current direction is eastwards, this natural environment smooths rough seas in a short time.

TABLE 2

Frequency of Decay Coefficient of Sea State

Damping Coefficient of Decay of Sea State λ	0.5	1.0	1.5	2.0	2.5	3.0
Frequency (Total Number = 54)	2	19	17	14	1	1

8. Safety of a Ship on Stormy Seas:

As has been stated, seas and swells around Japan seem to be somewhat higher than in other areas of open sea, especially where there are currents. Actually, the total number of accidents of vessels exceeds one thousand a year due to heavy seas and strong winds. For the purpose of preventing these accidents, the author considered the limiting ability of a ship to voyage safely, considering oceanographical and meteorological data.

The equation of the rolling motion of a ship can be written as follows :

$$I_s \frac{d^2\theta}{dt^2} \pm K_s \left(\frac{d\theta}{dt}\right)^2 + MgG_z = MgG_m \gamma_s J \cos\omega_s t - \frac{1}{2}\rho' CAU^2_{max} a \qquad (21)$$

in which θ is the roll angle, I the moment of inertia, K the damping coefficient. Mg is displacement, G_z is restoring lever, $G_m = \left.\frac{dG_z}{d\theta}\right|_{\theta=o}$, = metacentric height, J is maximum wave slope and $2\pi/\omega_s = \tau_s$ is resonant wave period. γ_s is coefficient

Write $G_{za}\theta_r = GU^2$ (28)

and $\frac{1}{2}G_m\theta_o^2 = HU$ (29)

Then (27) becomes $GU^2 + HU - S_d = o$ (30)

where (31)

$$G = \frac{1}{2}\rho' C_a Aa(U_{max}/U)^2\theta_r/Mg = 0.000002653A(m^2)a(m)\theta_r(deg\gamma)/Mg(ton)$$

in which $\frac{1}{2}\rho' C_a = 0.000076(ton/m^3)$

and $H = \frac{1}{2}G_m\frac{\theta_o^2}{U}$ (not wave height)

$$= \frac{1}{2}G_m\frac{\pi\gamma_s}{2N_s}\frac{J}{U} = 0.03513G_m(m)/\tau_s(sec). \quad (32)$$

in which, assuming that $\gamma_s = 0.8$, $N_s(deg^{-1}) = 0.02$, so that $\theta_s(deg) = 360/\beta$, and

$$\beta = g\tau_s/2\pi U, \ g=9.80665m/sec^2.$$

The critical wind velocity that a ship endures is then expressed by a root of the quadratic equation (30), i.e.

$$U = \frac{1}{2G}\left[\sqrt{H^2 + 4GS_d} - H\right] \quad (33)$$

The frequency distribution of the linearised damping factor is shown in Table 3.

TABLE 3

Frequency of Damping Coefficient Analysed from Records of Rolling of a Ship. In this case, Equation of Motion is Expressed as $\theta + k\theta + \omega\theta = 0$

Linearized Damping Coefficient k	0.1	0.2	0.3	0.4	0.5
Frequency (Total Number = 17)	7	8	1	1	

At the left end is the data obtain from the readings on rough seas with a current, and the right end corresponds to that of still waters. This indicates that the sea area where a current flows, or where drift prevails after a storm, should be the most dangerous, since heavy resonant rolling occurs due to the joint action of decreases of damping factor and increase of wave height. In fact, the positions where ships were capsized are distributed over the narrow zone of Equator Current.

of effective slope, ρ' is density of air, c is drag coefficient, A is exposed area and a is the height between the centres of wind and water pressure respectively, above and below the load water line.

The GZ curve is already known for the given load condition. First, we consider the action of the wind. U_{max} is the peak gust with appropriate duration and the relation between U_{max} and mean velocity U is seen in Fig. 5, where the straight line shows the significant gustiness as

$$U_{max}^2/U^2 = 2 \quad (22)$$

This gust causes steady heel to the lee side, given by

$$G_{za} = \frac{1}{2}\rho' C_a AaU_{max}^2/Mg. \quad (23)$$

Thus we can easily guess that the whole area of the GZ curve above the straight line may correspond to the reserved ability for restoring the ship. This area is expressed approximately as $S_d - G_{za}\theta_r$, where $S_d = \int_o^{\theta_r} G_z\, d\theta$ is the dynamical lever and θ_r is the maximum range of stability where $G_z(\theta_r) = o$.

Next, we consider the action of a wave. The fundamental equation is readily transformed to

$$\ddot{\theta} + \frac{3}{4}N_s\dot{\theta}^2 + \omega_s^2\theta = \omega_s^2 J\cos\omega_s t \quad (24)$$

$$\frac{3}{4}N_s = K_s/I_s, \ \omega_s^2 = MgG_m/I_s \quad (25)$$

where $2\pi/\omega_s = \tau_s$ is the natural period of rolling.

Assuming $\theta = \theta_o\cos(\omega_s t + \varepsilon)$ and multiplying both sides of the equation and integrating over one swing, we obtain the relation in the resonance state

$$\theta_o^2 = \frac{\pi}{2N_s}\gamma_s J \quad (26)$$

The corresponding area can be expressed as

$$\int_o^{\theta_o} G_z\, d\theta \doteq \frac{1}{2}G_m\theta_o^2,$$

so that equating the two values of this area we obtain the following equilibrium condition,

$$S_d - G_{za}\theta_r = \frac{1}{2}G_m\theta_o^2 \quad (27)$$

Now, let us further simplify this equation for the criterion of safety of a ship.

Table 4 shows the estimated maximum wind velocity for two famous accidents caused by typhoons, which shows that the above method seems to be useful for the purpose of judging the safety of a ship.

TABLE 4

Wind Velocity of Typhoons which Capsized Passenger Ships

Name of Capsized Ship	Name & Date of Typhoon	Estimated Velocity	Observed Velocity & Name of Station
Aoba-maru	Della, VI 20 21, 1949	15.2m/sec.	22.0m/sec. Bofu 47-763
Toya-maru	Marie, IX 26 27, 1954	33.8m/sec.	36.1m/sec. Esashi 47-428

Thus in Japanese waters, roughly speaking, for the limit of stability to be reached U must exceed 15m/sec. for coastal ships, and 20m/sec. for ocean-going ships. If a ship is anchored in a harbour or at the centre of a storm, thus eliminating the effect of waves, danger velocity is calculated from $U = \sqrt{S_d/G}$, which exceeds 25m/sec for coastal ships and 35m/sec for ocean-going ships for limit of stability. The former values correspond to the prevailing wind, and the latter to the maximum velocity of the storm for lows or typhoons.

9. Conclusion:

The influence of tide, ocean, and drift currents on the character of waves is very complicated, especially because the increase of wave height by the action of above-cited factors causes tips of wave crests to be cut and blown down. The surface layer of the sea then encloses foam and its eddy viscosity changes. The analytical considerations of the mechanism of propagation of waves across the current or around the centre of storm whould be continued further, though the growth theories of decay of oceanic waves seem to have been completed up to the present. The author expresses his thanks for the help of a number of members of the Meteorological Agency and Maritime Safety Agency and Institute of Meteorology, and Dr. S. Inoue of the Kyushu University, and daily assistance of Miss K. Kawakatsu of his Department.

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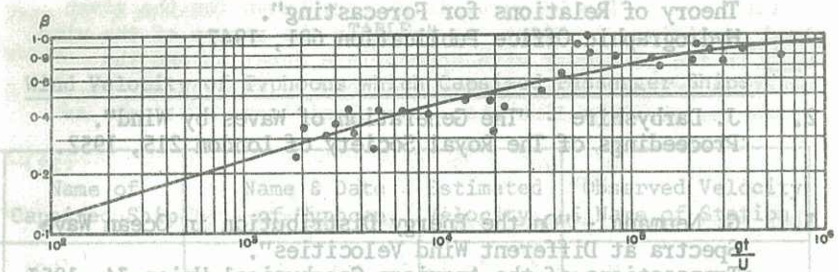

Fig.1 Growth of Wave Age by Duration of Monsoon

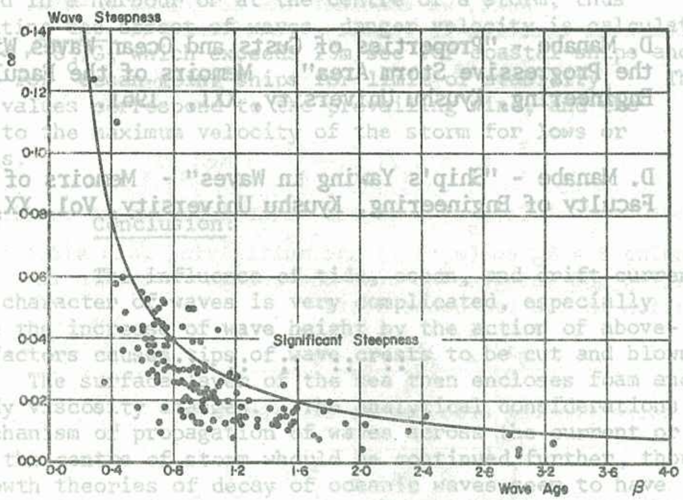

Fig.2 Steepness and Age of Wave

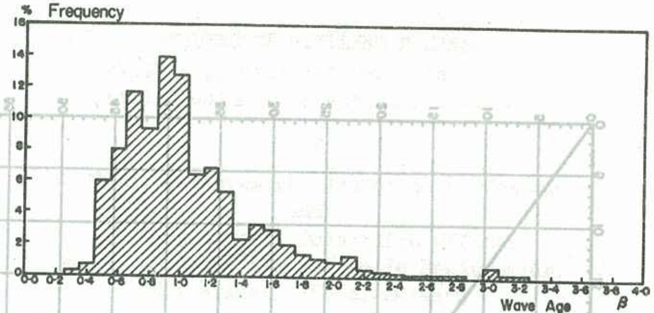

Fig.3 Frequency Distribution of Wave Age

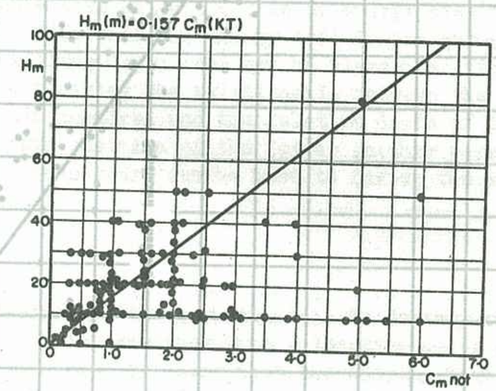

Fig.4 Wave Height due to Tide Current

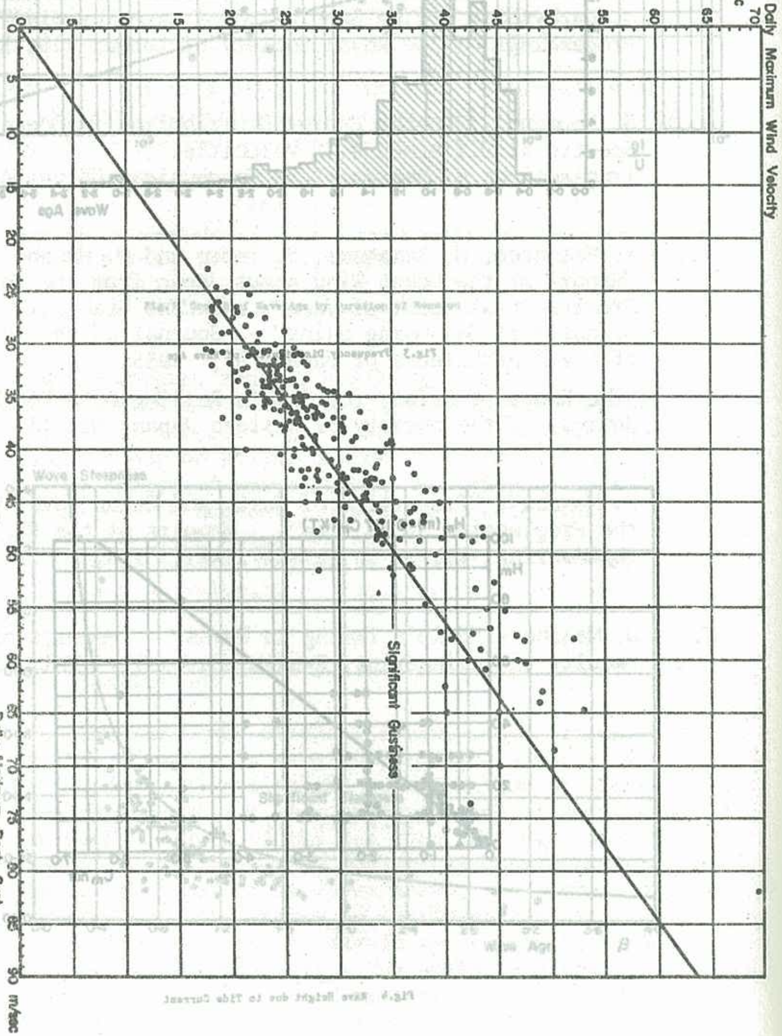

Fig.5 Peak Gust for Maximum Wind Velocity

STUDIES ON STILLING BASINS
(With a special reference to the jet diffusion type stilling basins)

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Abstract: When a high velocity submerged jet issues into a standing pool of water its energy gets dissipated along its course. This phenomenon of energy dissipation is useful for the design of stilling basins under hydraulic structures. This paper deals with certain investigations conducted under different conditions of flow in our laboratories here.

In all the studies it has been noted that there exists a critical section within which the rate of energy dissipation is predominant and after this the energy left in the stream is quite small and the stilling basin need not be elaborate after this section. A chart giving the relationship between the length of the stilling basin required and the relative depth of the stilling basin with relative position of the jet as another parameter has been proposed. This chart can be used to fir up the stilling basin length.

Introduction:

Stilling basins and the appurtanances to destroy the energy of flowing water are being designed from a longtime using different methods, the main necessity being that the eddying that is prevalent in the high energy in coming flow results in high impact pressures on the bed of the stilling basin resulting in the scouring of bed and retrogression of levels which undermines the structure and result in the failure of the same if proper protective measures are not provided. In particular the case of the jet diffusion in a stilling basin occurs when the flood water discharges in the form of jets through high head sluices, syphons etc. under high tail water conditions. Such a jet ejected out with a high efflux energy