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An Investigation of Particulate Flow Behaviour and Sedimentation in Curved Channel using Euler-Euler Approach

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Abstract

Fluid flow through curved duct has been extensively studied for a wide range of applications with a key base of heat transfer and mixing enhancement. Such motivation has provided a fairly comprehensive knowledge of physic and numerical modelling addressing intrinsic vortices structure promoting mixing and momentum transfer. This study extends this area to a multi-phase flow case study where liquid-granular phase interaction stands for a commonly seen scenario of bed erosion in a duct flow. Advances in numerical modelling and a wide range of sub-models, specified for momentum exchange, turbulence structure and granular phase behaviour, are examined to achieve a fairly valid model. An Eulerian model is used alongside with k-ε turbulence model and granular phase is modelled using a gas dynamic analogy. Results demonstrate a pivotal quantitative and qualitative role of secondary flow in curved duct where dimensionless helicity function is used for study of fluid flow structure associated with boundary reshaping and phase rearrangement.

Introduction

Slurry flow in open and confined channels is commonly seen in a wide range of environmental and industrial scenarios, such as river bed erosion and sedimentation, mining processes, particle classifications and power generation. In these types of flow, the suspension and settling rate of solid particles are essentially dependent on drag and lift forces and, fundamentally determine the processes involving erosion and sedimentation. The behaviour of these crucial flow field parameters emanates through fluid rheology that is changed with particle concentration. Particle-particle fluid interactions are other influential parameters that lead to the definition of frictional and pressure artificial viscosity in a granular continuous model.

Slurry flow in a curved channel is mechanistically more complex due to the centrifugal body forces generated by channel curvature that act on suspended solid particles in addition to inherent drag and lift forces. In curved channels, centrifugal action develops secondary flow and unique vortex structure identified as Dean vortices. Whilst the secondary flow induces a helical fluid motion in axial fluid flow through curved channels, the centrifugal forces will affect variously on solid (dispersed) and fluid (continuum) phases in slurry flow and, cause uneven secondary vortex structures. Consequently, the phase distribution in slurry flow in curved channels exhibits unique particle flow behaviour compared to straight conventional ducts. A prime example of this is the river bed erosion where the sedimentation process predominantly occurs in the vicinity of the inner curved riverbank.

Dispersed-continuum phase for internal and external flow has been investigated chiefly by using the Euler-Lagrange and Euler-Euler for a wide range of applications. The mathematical model, developed be Kobayashi et al.[1], may be addressed as one of the pioneer analyses of sedimentation and erosion in a duct flow. They considered fluid-particle as well as particle-particle interaction, in bed vicinity, and investigated vertical velocity and concentration profiles. Their analytical model, was validated against an experimental set of data for plane bed, suggested a valuable fundamental suggestions of drag and lift coefficient for such an interactive domains. Liang et al.[2] developed a 2-dimensoinal finite difference model to simulate scour underneath a pipe line. Using a more realistic geometry and considering crucial influence of vortices on sedimentation process, both suspended load and bed load were modelled. Zhao and Fernando [3] carried out a similar case study for scour around a cylindrical pipe using Euler-Euler two phase coupled model. They numerically modelled fluid-particle and particle-particle interaction and the interface between sand and water was specified using a threshold volume fraction of sand, and the evolution of the bed's shape were studied in detail. Onate et al. [4] presented some advance formulation of the particle finite element method (PFEM) for solving complex fluid–structure interaction problems and in particular extended PFEM for the analysis of bed erosion via mesh generation and development of an algorithm for free surface flows. The suggested erosion model was based on the frictional work at the bed surface originated by the shear stresses in the fluid. A Lagrangian coupling two-phase flow model was utilized by Zanganeh et al. [5] to simulating scour processes beneath a marine pipeline with respect to the sediment and fluid phase interactions. They applied Smooth Particle Hydrodynamic (SPH) method to both dispersed and continuum phases where assumed them as non-Newtonian and Newtonian phase respectively. Yeganeh et al.[6], applied Distinct Element Method (DEM) to simulate bed-load transport. This model assumed the load-bed transport as dynamically interdependent motion of individual sediment particle under flow motion. The concept of vertical momentum transfer is exploited to describe features of concurrently present of saltation and sheet-flow layers. They concluded that the vertical motion of particles is not significantly active in sheet-flow in layer and the particle momentum is preserved. Yeganeh et al. [7], in a later work, developed an Euler-Euler model for investigation of flow-bed interaction. Using Navier-stocks equation for both fluid and sediment phase, they coupled phases through drag and lift forces. Chandratilleke et al.[8, 9] have widely investigated flow in curved channel and structure of Dean vortices in curved channels for both single and two-phase flow domains. They introduced a non-dimensional Helicity function for investigation of vortices structure in single-phase flows [8] and successfully extended that for multi-phase flow domain where this scalar is non-dimensionalized for each phase using different reference values.

For investigating the secondary flow influence on bed erosion, this paper presents a numerical study whereby a model based on Eulerian method (continuum-granular phases) is developed and used for examination of bed reformation in a rectangular curved channel. Transient phases interfacial shape and erosion rate across the channel are investigated where interaction between water (continuum phase) and sand (granular phase) is simulated.

Numerical Modelling

Euler-Euler approach has been used which treats continuum and dispersed phases as two separate fluids where Navier-Stocks equations are solved for both phases to model their behaviour. Since both phases are numerically present at the same place in all times, key parameter of volume fraction represents relative amount of each phase. Assuming no phase change (i.e. no mass transfer), Continuity equation for each phase is defined as:

$$\frac{1}{\rho_{rq}}\left[\frac{\partial}{\partial t}(\alpha_k \rho_k) + \nabla.(\alpha_k \rho_k \vec{v}_k)\right] = 0 \qquad (1)$$

Fluid phase momentum equation is derived considering a Reynolds stress function generated with a RANS model and extra source of turbulence fluctuation accounting for dispersed particle influence. In the Eulerian frame, where f, s represent fluid and solid phase respectively, momentum equation for fluid phase is:

$$\frac{\partial}{\partial t}(\alpha_f \rho_f \vec{v}_f) + \nabla.(\alpha_f \rho_f \vec{v}_f \vec{v}_f) = -\alpha_f \nabla p + \nabla.\bar{\bar{\tau}} + \alpha_f \rho_f \vec{g} + k_{fs}(\vec{v}_s - \vec{v}_f) \qquad (2)$$

In simulating the granular phase using Navier-Stocks equation, an analogy is made between random particle motions, as a result of particle-particle collision, and thermal motion of molecules in gases [10] where elasticity of granular phase is additionally taken into account. On the basis of such, momentum equation for solid phase is:

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \vec{v}_s) + \nabla.(\alpha_s \rho_s \vec{v}_s \vec{v}_s) = -\alpha_s \nabla p - \nabla p_s + \nabla.\bar{\bar{\tau}}_s + \alpha_s \rho_s \vec{g} + k_{fs}(\vec{v}_f - \vec{v}_s) \qquad (3)$$

A key parameter, in coupling fluid and solid phases, is interfacial momentum exchange coefficient which is denoted in equation 2 and 3 as k_{fs}. Gidaspow et al. [11] formulation, appropriate for $\alpha_f \le 0.8$ (i.e. dense bed), is applied as follow:

$$k_{fs} = 150\frac{\alpha_s(1-\alpha_f)\mu_f}{\alpha_f d_s^2} + 1.75\frac{\rho_f \alpha_s \left|\vec{v}_s - \vec{v}_f\right|}{d_s} \qquad (4)$$

Dispersed Turbulence Model:

<u>Turbulence in fluid phase</u>

This turbulence model is a continuous closure which accounts for influence of dispersed particles and predicts turbulent quantities using Tchen theory [12,13] of dispersion of discrete particles by homogenous turbulence. Transport equations of $k-\varepsilon$ standard model, for fluid phase, are modified as:

$$\frac{\partial}{\partial t}(\alpha_f \rho_f k_f) + \nabla.(\alpha_f \rho_f \vec{U}_f k_f) = \nabla.[\alpha_f \frac{\mu_{t,f}}{\sigma_k}\nabla k_f] + \alpha_f G_{k,f} - \alpha_f \rho_f \varepsilon_f + \alpha_f \rho_f \Pi_{k,f} \qquad (5\text{-a})$$

$$\frac{\partial}{\partial t}(\alpha_f \rho_f \varepsilon_f) + \nabla.(\alpha_f \rho_f \vec{U}_f \varepsilon_f) = \nabla.[\alpha_f \frac{\mu_{t,f}}{\sigma_\varepsilon}\nabla \varepsilon_f] + \alpha_f \frac{\varepsilon_f}{k_f} + C_{1\varepsilon} G_{k,f} - C_{2\varepsilon}\rho_f \varepsilon_f + \alpha_f \rho_f \Pi_{\varepsilon,f} \qquad (5\text{-b})$$

where $\vec{U}_f$ is phase weighted velocity and $G_{k,f}$ is production of turbulent kinetic energy. Additionally, $\Pi_{k,f}, \Pi_{\varepsilon,f}$ represent influence of dispersed phase on continuous phase:

$$\Pi_{k,f} = \frac{k_{fs}}{\alpha_f \rho_f}(k'_{sf} - 2k_f + \vec{v}_{sf}.\vec{v}_{dr}) \qquad (6\text{-a})$$

and according to Elghobashi and Abou-Arab [17]:

$$\Pi_{\varepsilon,f} = C_{3\varepsilon}\frac{\varepsilon_f}{k_f}\Pi_{k,f}, \quad C_{3\varepsilon} = 1.2 \qquad (6\text{-b})$$

The granular temperature which represents kinetic energy of particle random motion in solid phase is another fundamental equation for this Euler-Euler scheme. This transport equation accounts for energy generation, dissipation and exchange as

$$\frac{3}{2}[\frac{\partial}{\partial t}(\rho_s \alpha_s \Theta_s) + \nabla.(\rho_s \alpha_s \vec{v}_s \Theta_s)] = (-\rho_s \bar{\bar{I}}_s + \bar{\bar{\tau}}_s):\nabla \vec{v}_s + \nabla.(k_{\Theta s}\nabla\Theta_s) - \gamma_{\Theta s} + \phi_{fs} \qquad (7)$$

Where

$(-\rho_s \bar{\bar{I}}_s + \bar{\bar{\tau}}_s):\nabla \vec{v}_s$ The generation of energy by solid stress tensor

$k_{\Theta s}\nabla\Theta_s$ Diffusion of energy

$\gamma_{\Theta s}$ The collision dissipation of energy

ϕ_{fs} The energy exchange between phases

Diffusion coefficient which is a fundamental modelling parameter is suggested by Gidaspow et al.[11] as:

$$k_{\Theta s} = \frac{150 d_s \rho_s \sqrt{\Theta_s \pi}}{384(1+e_{ss})g_{0,ss}}[1 + \frac{6}{5}\alpha_s g_{0,ss}(1+e_{ss})]^2 + 2 d_s \rho_s \alpha_s^2 (1+e_{ss}) g_{0,ss}\sqrt{\frac{\Theta_s}{\pi}} \qquad (8)$$

Collisional dissipation of energy

$$\gamma_{\Theta m} = \frac{12(1-e_{ss}^2)g_{0,ss}}{d_s\sqrt{\pi}}\rho_s \alpha_s^2 \Theta^{3/2} \qquad (9)$$

Energy exchange coefficient

$$\phi_{fs} = 3k_{fs}\Theta_s \qquad (10)$$

General form of boundary condition for granular temperature on the wall

$$q_s = \frac{\pi}{6}\sqrt{3}\phi_{fs}\frac{\alpha_s}{\alpha_{s,\max}}\rho_s g_{0,ss}\sqrt{\Theta_s}\vec{U}_{s,\|}.\vec{U}_{s,\|} - \frac{\pi}{4}\sqrt{3}\frac{\alpha_s}{\alpha_{s,\max}}(1-e_{sw}^2)\rho_s g_{0,ss}\Theta_s^{3/2} \qquad (11)$$

<u>Model setup, initial and boundary conditions</u>

As the numerical scheme was explained, a turbulent Euler-Euler two-phase model is developed to simulate fluid flow and erosion/sedimentation behaviour of a sand bed ($\rho_s = 4300 kg/m^3, d_s = 150\mu m$), exposed with water flow ($\rho_f = 1000 kg/m^3, \mu_f = 0.001 N.m/s$) in a curved closed duct. The duct has a uniform cross section of rectangular with aspect ratio (i.e. height/width) of 4 and in the curved section a fixed curvature ratio ($\gamma_c = D_h / R_{Curvature}$). As it is demonstrated in figure.1, the sand bed (defined as a granular phase) occupies one fourth of channel and rest of volume is filled with water. Upstream inlet velocity, which is applied with a fully developed profile, is set for liquid (continuous) phase inlet section and solid boundaries (i.e. wall) are defined for the bed area at both inlet and outlet sections. This boundary condition arrangement is necessary to prevent fluidization and/or abrupt removal of sand bed as a result of direct upstream flow. This means fully developed velocity profile is just applied on the upper fluid section and similarly pressure outlet boundary condition is just for fluid section where bed volume will be confined with wall boundary condition at the end of geometry.

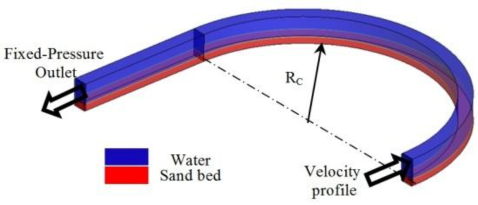

Figure.1. Applied geometry and initial configuration of phases

Granular viscosity of secondary phase is set as suggested by Gidaspow et al. [14] and angle of internal friction between particles is fixed at 30 degrees. Although the packing limit value is defined as 0.7, for numerical stability purposes, volume fraction of second phase is initialized as 0.95. This allows a valid and stable development of liquid phase flow and by monitoring volume fraction throughout granular phase, result from initial time steps are neglected until achieving packing limit of 0.7. Phase coupled SIMPLE algorithm is implemented for pressure-velocity scheme where all the equation except momentum (second order) are discretised using first order upwind. The model is strictly sensitive against relaxation factors which should be optimized especially for turbulence equations as a single mixture turbulent model is shared for both phases. For the purpose of grid independency analysis, velocity field of both phases and volume fraction scalar have been examined against increasing of grid number along height and width of cross section as well as trough channel length. Accordingly, fully hexagonal grid with 900,000 cells (30×60×500) has been chosen as an optimized mesh and then mesh inflation toward walls was set to keep y+ value in range with k-ε standard wall function requirement (i.e. 30< y+<300)

Results and Discussions

Features and structure of secondary flow, as the lateral component of fluid flow in curved channel, have been discussed in literature for closed and open channels. Secondary flow is known to be generated as a result of interaction between viscous forces (in stagnation area) directed inward curvature and inertial forces (induced by centrifugal force) directed outward curvature. By increasing flow rate or centrifugal force, such instability potentially could be repeated within each secondary cell forming extra vortices known as Dean vortices. Shear forces at the boundary of fluid is a key parameter, mainly determining shape and position of secondary flow cells. This is the main difference between closed and open channels distinguishing flow pattern and vortices structure. In the current case, liquid phase is in momentum exchange with a granular phase where velocity field has lower order of magnitude. Unique momentum field of granular phase makes it different from fluid phase or solid medium. According to physics of granular phase and developed model (quantified with packing limit), non-zero fraction of liquid is expected in the bed area which makes different type of boundary as compared to open channel or solid boundary conditions. Moreover, the granular phase is deformed transitionally which lead to moving boundaries and accordingly unsteady vortices structure. Following discussions are based on a single flow rate, where transient behaviour of case is investigated in dimensionless time intervals. A dimensional analysis based on influential geometrical and flow parameters has been carried out and dimensionless time is defined as:

$$t^* = t\left(\frac{\bar{V}_{in}}{D_h}\right)\left(\frac{H_{Channel}}{H_{Bed}}\right)\gamma_c \tag{12}$$

Helicity function is a well-established mathematical representation of helix-like motion in fluid dynamic which is used here for visualization of vortices structure. Considering Eulerian frame of equations, helicity is defined with reference of phase velocity field for liquid phase separately.

Discarding results from initial time steps, (according to initializing policy) bed shape and flow structure are investigated on the cross section located at the end of curved section ($\theta_{curvature}=180°$). A certain cross section, perpendicular to curvature, is necessary to be set for investigation of lateral influence of secondary vortices since complex helical motion of fluid throughout curved section is combination of axial and secondary flow. Particles are expected to be suspended owing to drag and turbulence drift forces through liquid phase and correspondingly water should penetrate the granular phase and through bulk of particles. Ensuing particle behaviour and sedimentation characteristics are briefly illustrated in Figure.2, for mentioned cross section in six dimensionless time steps. As it could be read from coloured contour legend, volume fraction of granular phase has upper and below limits of 0.7 (representing bed) and 0.1 (representing water-particle mixture) respectively which refers discussion regarding suspension and penetration of phases. Dimensionless Helicity values are demonstrated using line-contours where solid and dashed lines distinguish clock-wise and counter clock-wise direction of rotation respectively. The red colour in these contours is used to identify centre of vortices as area of higher helical intensity. This helps to discuss repositioning and strength of vortices as flow pattern keeps changing. Figure.2.a shows the first reported situation of flow field where boundary of phases is observed to be horizontal. Assuming liquid area on top of bed, as a fairly rectangular shape, structure of secondary flow is expected to be symmetrical vortices dividing flow area into two sections. Nevertheless, this area is observed to have higher helicity value at top (next to solid wall) and expanded helical structure with lower intensity at the bottom next to bed area (i.e. granular phase).

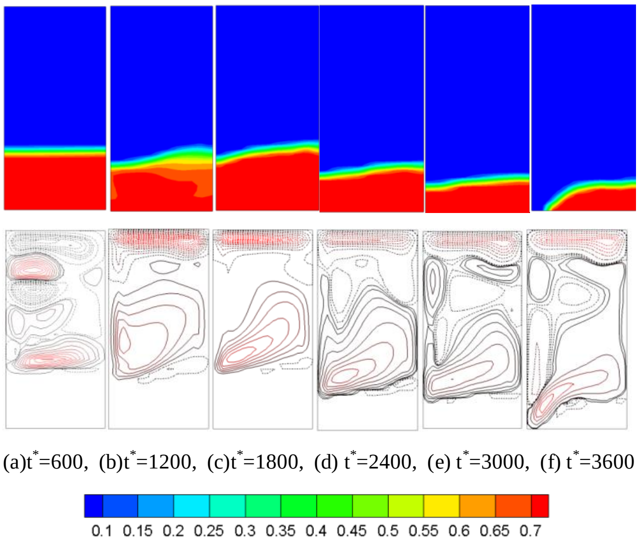

Figure.2. Volume fraction granular phase and Helicity function of fluid area

As importance of shear forces on strength and positioning of secondary flow cells was discussed, such differences in upper and lower secondary flow cells could be associated with higher shear force at the solid boundary as compared to granular phase boundaries. The fact that the lower cell centre is located toward outer wall, determines reformation of bed as it could be found from figure.2.b, showing erosion initiates from outer wall. Deformation of bed, as the bellow boundary of liquid zone, would significantly influence flow structure. Centre of upper secondary flow cell is pushed toward top wall of duct and the entire cell is compacted whereas the below cell is expanded and centre of which is pushed toward outer wall more. Such erosion trend continues while the centre of below cell approaches not only toward outer wall but also below wall as the beds gets eroded and granular phase is becoming thinner.

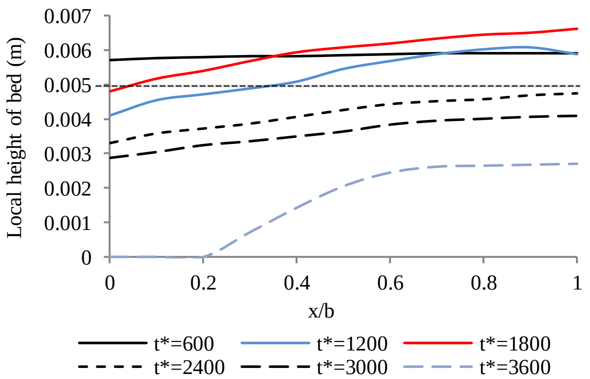

Fig.3. Position of phases boundary at the end of curvature

Boundary between phases is quantitatively defined as iso-surface volume fraction of 0.5 as depicted in figure.3 for various cases. Albeit, for all cases, erosion/sedimentation regime is qualitatively the same (higher rate at outer wall), for two early time steps, sedimentation is observed as determining process. Considering the fact that the cross section is located at the end of cross section, eroded particles from upstream initially saturate the mixture and influence of flow at this cross section would be limited to pushing particles toward inside wall. Accordingly from $t^* = 600$ to $t^* = 1800$, sedimentation is determining phenomenon whereas it turns to be erosion for the rest of reported intervals. In the next time intervals, from $t^* = 2400$ to $t^* = 3000$, height of bed successively decreases in the same uneven pattern as could be seen from figure.2 & figure.3.

Conclusion

Taking flow and phase fraction regime into account a combination of sub-models for drag and turbulent drift force and a modified formulation of interfacial momentum exchange coefficient for non-dilute mixture were applied to model. Performance of model is briefly studied for a single flow rate and three different curvature ratios to understand cross sectional and volumetric erosion regime in the system. An uneven erosion/sedimentation pattern is observed in cross sectional view where secondary flow was accounted for such. Progressive reconfiguration of bed boundaries, mutually, affects vortices structure and positioning. Unlike solid or liquid boundaries, shear stress at boundaries of granular phase is seen to be unique, leading to an unsteady secondary flow structure. Quantitative rate of erosion for the same flow rate was examined against various curvature ratios which explained the transitional mechanism of erosion in curved ducts and unveiled pivotal role of secondary flow intensity as an extra flow feature promoting erosion potential.

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