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Coherent Structures in Cross Sheared Flows in Weakly Stratified Environments

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Abstract

In this study, direct numerical simulation (DNS) has been carried out to investigate the coherent structures of cross sheared stratified (CSS) flows, in which the flow in the streamwise direction with a basic sheared flow velocity interacts with the flow in the spanwise direction with another basic sheared flow velocity in a stratified environment. One additional major governing parameter for CSS flows is the cross shear ratio ξ, which represents the relative extent of the cross shear stresses in the streamwise and spanwise directions. ξ is varied from 0 to 0.5 in the DNS runs to be in line with the available experiments, with constant Reynolds, bulk Richardson and Prandtl numbers at 1200, 0.025 and 1, respectively. In the typical case of $\xi = 0.5$, the coherent structures of the CSS flow consist of small spanwise eddies along with a dominant streamwise Kelvin-Helmholtz (KH) 'cat eye' eddy. These small spanwise eddies originate from the braid region of a streamwise KH eddy and extend along the streamline to the top of the next streamwise KH eddy, forming unique 'eddy wrap' structures that wrap up the entire KH eddy structures. The results also demonstrate that the transition processes to turbulence in the CSS flow instability and in the classic KH instability are distinctively different. The dependence of the occurrence of 'eddy wrap' structures in CSS flows on ξ is also examined over $0 \leq \xi \leq 0.5$.

Introduction

The coherent structure concept has significantly enhanced the understanding of turbulent flows in contrast to the traditional statistical concepts that are indifferent to the subtle changes in the flow structures. The understanding of coherent structures in different flows also enables effective design and control of many environmental and engineering systems involving turbulent mixing. The prototype of coherent structures in sheared stratified flow is the KH instability. The main features include the main 'cat eye' (roller or billow) streamwise eddy core and the elongated 'braid' (rib) structures that connect two adjacent KH eddies. The study of KH coherent structures is further elaborated by consideration of, *e.g.*, mixing efficiency, secondary instability dynamics, etc.

Typical KH coherent structures have been produced experimentally and numerically based on perturbed parallel sheared (PS) flows, where the initial basic flow velocity only has a single, streamwise component, leading to unidirectional basic sheared flows. However, PS flows are hard to maintain in realistic situations due to the inevitable cross shears from the spanwise direction, due to, *e.g.*, boundary effects from the topography, internal waves, etc. The CSS flows studied here differ from the PS flows in that the CSS flows have comparable streamwise and spanwise velocity components in the basic state and occur in a stratified environment. In addtion to the KH eddy structures, other coherent structures are also observed in CSS flows.

CSS flows occur, *e.g.*, in the upper ocean or the cumulus layer in the atmosphere. In spite of their significance, studies on coherent structures in CSS flows have been scarce. Atsavapranee and Gharib [1] (abbreviated to AG97 hereafter) studied the influence of cross shear stresses on stratified plane mixing layers. By tilting a water tank in the spanwise direction, they produced cross shear stresses as soon as the first streamwise KH billows were captured. With similar appearance to the streamwise KH eddy, spanwise eddies were observed to develop from the braid section of the KH main eddy, but their length scales were one order of magnitude smaller than the main KH eddy. Inspired by AV97, Lin *et al.* [2] (abbreviated to Lin00 hereafter) conducted three-dimensional numerical simulation with the pseudo-spectral method to study cross sheared flows by varying the cross angle between two different shear layers. Their simulations produced the spanwise eddies similar to those observed by AV97.

However, the critical conditions for the occurrence of spanwise eddy structures in CSS flows are still unknown, which motivates the current study. Furthermore, the fragmentary information in AV97 and Lin00 do not provide a comprehensive view of the coherent structures in CSS flows. Despite the observation of spanwise eddies, little attention has been paid to the interactive mechanism between the spanwise and streamwise eddies. Hence, this study will not only reproduce the spanwise eddies of Lin00, but also provide a comprehensive view of the coherent structures in CSS flows.

Numerical Methodology

The governing equations for the DNS simulation of this study are the continuity, Navier-Stokes, and density equations for incompressible flows with the Boussinesq approximation, which are written in the Cartesian coordinates (x, y, z) as follow:

$$\nabla \cdot \mathbf{u} = \mathbf{0} \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \frac{p}{\bar{\rho}} + g\frac{\rho - \bar{\rho}}{\bar{\rho}}\vec{k} + \nu \nabla^2 \mathbf{u} \quad (2)$$

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = \kappa \nabla^2 \rho \quad (3)$$

where $\mathbf{u}$ is the velocity vector, with components u, v, w in x, y, z directions, t is time, p is pressure, ρ is density, ν is the kinematic viscosity, κ is the thermal diffusivity, g is the gravitational acceleration, and $\bar{\rho}$ is a reference density.

The following typical velocity and density profiles of free shear flows are used as the initial basic flow states:

$$\phi_0 = \Delta\phi_0 tanh[\frac{2}{\delta_s}(z - \frac{1}{2}L_z)], \quad (4)$$

where ϕ represents u, v, ρ, the subscript '0' denotes the initial magnitude of a physical property, Δu_0, Δv_0 and $\Delta\rho_0$ are the initial velocities and density changes across the sheared/stratified layer, δ_s is the thickness of the initial sheared/stratified layer, and L_z is the vertical extent of the domain.

To initiate the formation of coherent structures, the following perturbations are imposed on the initial conditions, aiming to excite the primary and secondary instabilities,

$$\phi_{pri} = -0.02\Delta\phi_0 cos(\frac{2\pi x}{L_x}) sech(\frac{2z-L_z}{\delta_s}) tanh(\frac{2z-L_z}{\delta_s}), \quad (5)$$

$$\phi_{sec} = A_\phi \Delta\phi_0 \{1-|tanh[\frac{2}{\delta_s}(z-\frac{1}{2}L_z)]|\} r_\phi(x,y,z), \quad (6)$$

where the subscripts '*pri*' and '*sec*' denote the perturbations for exciting the primary and secondary instabilities, respectively, r_ϕ is a random number between -1 and 1 for perturbations in the ϕ field, and A_ϕ is the amplitude coefficient for the secondary instability perturbations. For u, v, ρ, A_ϕ are selected as 0.1, 0.1, 0.5, respectively. As predicted by Xiao *et al.* [3], the primary instability mode in CSS flows is the stationary mode usually corresponding to vortex structures, therefore the velocity shear will be the only source to excite the primary instability. Accordingly, '*pri*' perturbations are only imposed on the two basic velocity components u and v, not on ρ. Thus, the initial field of a quantity will be the sum of the background profile, the primary perturbation, and the secondary perturbation.

Periodic boundary conditions are set in the horizontal directions. At the top and bottom boundaries, the impermeable condition of $w|_{z=0,L_z} = 0$ is set for w, the zero flux boundary conditions of $(\partial u/\partial z)|_{z=0,L_z} = 0$ and $(\partial v/\partial z)|_{z=0,L_z} = 0$ are set for u and v, and the boundary condition of $\rho|_{z=0} = \bar{\rho} + 0.5\Delta\rho_0$ and $\rho|_{z=L_z} = \bar{\rho}$ is set for density, respectively.

This study aims to investigate the primary coherent structures of the CSS flow, therefore the initial bulk Reynolds number, is selected as 1200, in its common range for laboratory flows. To focus on the cross shear effect, only the initial bulk Richardson number of 0.025, a value representing a weakly stratified environment in which the sheared dynamics dominates, is considered. The Prandtl number of 1 is used for all DNS simulations.

A reliable and widely used numerical code, PUFFIN, developed by one of the authors (MPK) is used to perform DNS. The governing equations (1)-(3) are discretized in space using a finite volume formulation on a uniform, staggered, Cartesian grid. The spatial derivatives are discretised by the 4th-order central finite difference scheme. The ULTRA-flux limiter is applied to the scalar advective terms. The pressure correction approach is used to correct the pressure and velocity fields. The 2nd-order Adams-Bashforth and Crank-Nicolson schemes are used for the time advancement. The CFL number criterion is used to ensure the simulation is stable, with the minimum and maximum limits of 0.3 and 0.4, respectively. The discretised equations are solved by the Gauss-Seidel method. The pressure correction equation is solved by the BICGSTAB solver with modified strongly implicit preconditioner.

The dimensions (L_x, L_y, L_z) of the computational domain are set based on our stability analysis results [3]. L_x is choosen as $(2\pi/\alpha)(\delta_s/2)$ which is one wavelength of the instability mode, where α is the wavenumber corresponding to the most unstable mode. $\alpha = 1/3$ is also selected based on our stability analysis results [3]. L_y and L_z are set to be equal to L_x, the same length scale used in the streamwise and spanwise directions to avoid boundary effects. δ_s is set as 0.1795 m based on the numerical tests of Smyth and Winter [4]. The number of uniformly spaced cells $N_x \times N_y \times N_z$ in x, y and z are $128 \times 128 \times 128$, which has proved to be sufficient to capture the necessary features of the primary coherent structures.

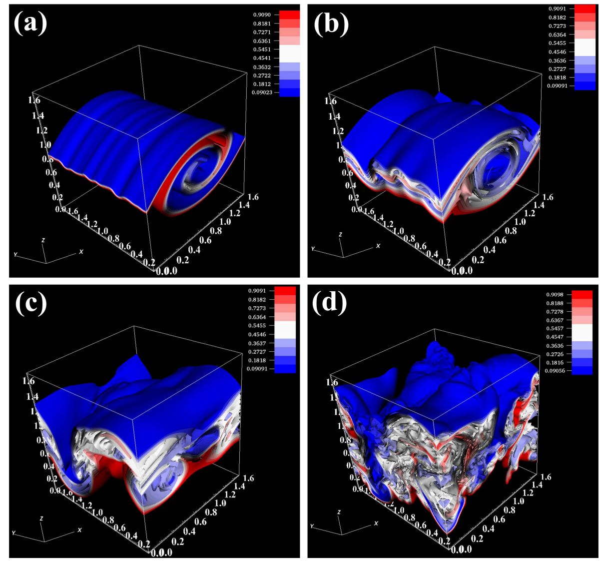

Figure 1. Contours of concentration c for the KH instability in the $\xi = 0.0$ case: (*a*) the primary KH eddy at $t = 1200s$; (*b*) the secondary instability in y at $t = 2000s$; (*c*) the saturation of coherent structures at $t = 2500s$; and (*d*) the collapse and decay into turbulence at $t = 3000s$.

Results

Observations

Two cases, $\xi = 0$ and $\xi = 0.5$ with the same values of other initial parameters, are selected to present the general evolution history of the KH instability and the typical CSS flow instability, respectively. It is found that for CSS flows with $\xi > 0.2$, the spanwise eddy wrap structures are similar to the $\xi = 0.5$ case, so the $\xi = 0.5$ case represents a typical one showing the spanwise eddy wrap structures for the CSS flow instability.

Figure 1 presents the concentration c at four times during the KH instability evolution for the $\xi = 0.0$ case obtained by DNS. The primary KH eddy starts to develop at around 800s until the 'cat eye' structure, with a clear center region and an adjacent braid region, are mostly developed at 1200s. As the two-dimensional primary KH eddy is saturated in the x direction, the streamwise KH eddy tube starts to extend outwards and deflect under the interaction between the streamwise and spanwise vortices. At this stage, the alignment of the streamlines in the y direction is completely destroyed, followed by the onset of the secondary instability in the spanwise direction. The whole coherent structure keeps expanding until the secondary instability structures are also saturated at 2500s. After that, the three-dimensional structures start to collapse into turbulence.

Figure 2 presents the concentration c at four times during the CSS flow instability evolution for the $\xi = 0.5$ case obtained by DNS. In figure 2(*a*), the characteristic length of the central spanwise eddy (close to the diameter) is approximately 0.2 m, which is 10% $\sim$ 12.5% of the KH eddy layer size. This length scale agrees well with the result obtained by AV97. By a visual estimation, the spanwise eddy tube extends across most of the domain in the x direction. Subsequently, such a spanwise eddy tube not only extends along the x direction but also wraps over the central KH eddy tube. Interestingly, as shown in figures 2(*b*) and 2(*c*), the weaker spanwise eddies last longer than the 'cat eye' eddy, in fact even continuing to grow after the 'cat eye' eddy begins to decay. On the other hand, the three-dimensional

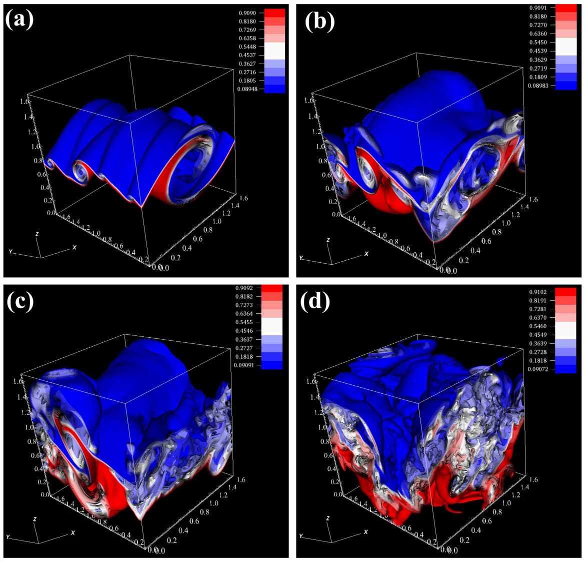

Figure 2. Contours of concentration c for the CSS flow instability in the $\xi = 0.5$ case: (*a*) the primary eddy wrap structure at $t = 1000s$; (*b*) the collapse of the streamwise eddy and the growth of the spanwise eddy at $t = 1380s$; (*c*) the collapse of the streamwise eddies and the entire coherent structure at $t = 1600s$; and (*d*) the decay into turbulence at $t = 2000s$.

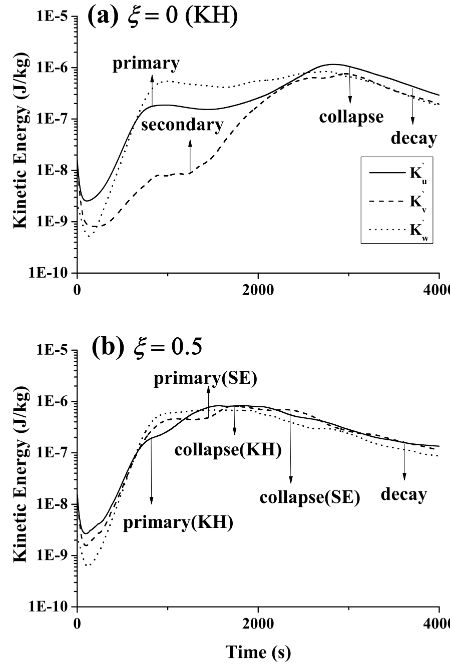

Figure 3. Time series of K'_u, K'_v, and K'_u for (*a*) $\xi = 0$ and (*b*) $\xi = 0.5$. The milestone events are marked, with 'KH' and 'SE' representing the streamwise KH eddy structures and the spanwise eddy structures, respectively.

coherent structures of the $\xi = 0.5$ case start to decay at about t =2000 s, significantly earlier than in the $\xi = 0$ case, which occurs at about 2500 s, indicating a faster transitional process to turbulence of the CSS flow.

To demonstrate the entire transitional process to turbulence more clearly, the evolution history of the perturbation kinetic energy $K'_{\mathbf{u}}$ is presented in figure 3. $K'_{\mathbf{u}}$, whose components are K'_u, K'_v, and K'_w, is defined as,

$$K'_{\mathbf{u}} = \langle \mathbf{u}' \cdot \mathbf{u}'/2 \rangle_{xyz}, \tag{7}$$

where

$$\mathbf{u}'(x, y, z, t) = \mathbf{u} - \langle \mathbf{u} \rangle_{xy},$$

in which the operator $\langle \rangle_{xy}$ denotes the average on the horizontal plane, *i.e.*,

$$\langle f \rangle_{xy} = \frac{1}{L_x L_y} \int_0^{L_y} \int_0^{L_x} f(x, y, z, t) dxdy.$$

For the KH instability, as shown in figure 3(*a*), K'_u, K'_v and K'_w all experience two peaks, which correspond to the developed streamwise KH eddy and spanwise KH instability, respectively. Each peak occurs at almost the same instant for K'_u, K'_v and K'_w. The second peak of K'_u is due to the expansion of the initial interface layer in the z direction, as $L_z = L_x$ used in this study rather than $L_x = 0.5L_z$ used in other studies such as [2]. However, it has no influence on the evolution of the coherent structures. It should be noted that the terms 'primary' and 'secondary' used in the CSS flows in this study have different meanings from that used in the literature on KH instabilities.

Compared to figure 3(*a*) for the $\xi = 0$ case , it is seen from figure 3(*b*) for the $\xi = 0.5$ case that the first peaks of K'_u and K'_v, at $t \approx 800s$ and $t \approx 1000s$, are actually the turning points, and their magnitude difference is much smaller, indicating that the spanwise instability is able to develop into similar 'cat eye' eddy structures, as shown in figure 2(*a*). After the turning point, K'_u keeps increasing with a smaller gradient until its second peak coincides with the 'primary(SE)' at $t \approx 1300s$, then keeps constant. Whereas after its turning point, K'_v becomes nearly constant until the 'primary(SE)' point and after that rises quickly to its second peak at the 'collapse(KH)' point at $t \approx 1600s$. Therefore, the streamwise instability decays at an earlier time ($t = 1300s$) than the spanwise instability ($t = 1600s$). It should be noted that from the 'primary(SE)' point at $t \approx 1300s$ to the 'collapse(KH)' point at $t \approx 1600s$, K'_u and K'_w stay nearly constant, therefore the increasing K'_v during this period implies that the spanwise instability not only continues growing but also absorbs energy from the main KH eddy.

Eddy Wrap Structures and Their Critical Conditions

As predicted by the linear stability analysis [3] and from the above observations, the coherent eddy structures belong to the stationary wave mode which does not propagate. Such stationary features suggest that the coherent structures develop from small vortices, therefore it is appropriate to examine the details of the vortex structures by examining the spanwise vorticity ω_x.

Figure 4 presents the contours of ω_x of the KH and the eddy wrap structures at four different instants. For the KH structures, the plots on the $y = 0.5L_y$ plane, which is the central slice plane at the center of the main KH eddy, were selected. Because of a slight misalignment of the spanwise eddy structures with respect to the y axis, the plots on the plane defined by an origin point at (0, $0.5L_y$, 0) and a normal vector $0.12\vec{i} + 1\vec{j} + 0\vec{k}$, where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the unit vector components in the x, y and z directions, are selected for the eddy wrap structures. For the KH instability, at $t = 820s$, the positive and negative ω_x regions consist of a symmetrical pattern and the periphery of the rolling structures contains only 10% of the maximum ω_x at the center. When the 'cat eye' along with the adjacent braid structures are well developed at $t = 1220s$, most of the negative ω_x regions (cyan color) concentrates at the center, with some parts of the large positive ω_x region (orange color) surrounding the core and the rest located on the peripheral braid regions. Unlike the KH

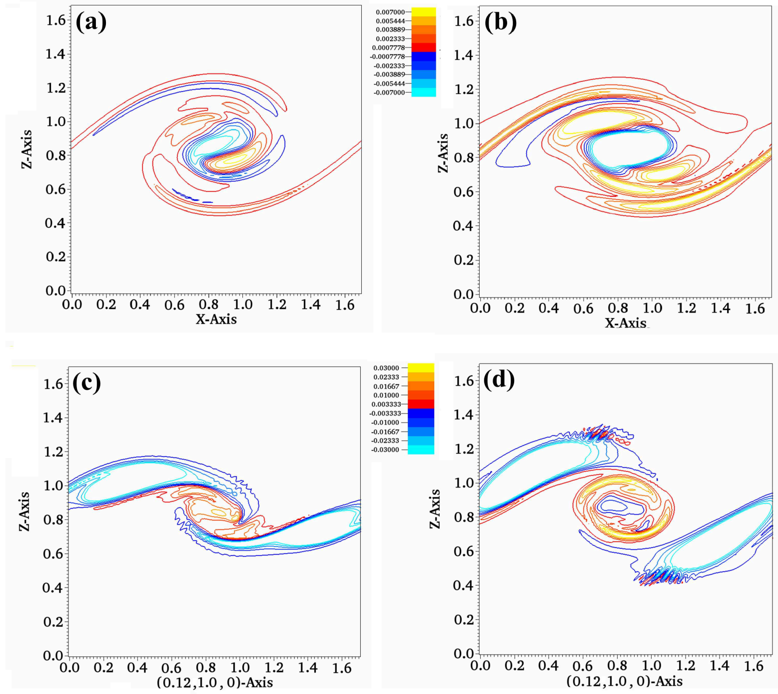

Figure 4. Contours of ω_x for the KH instability at (*a*) $t = 820s$ and (*b*) $t = 1220s$ and for the CSS flow instability at (*c*) $t = 1000s$ and (*d*) $t = 1200s$, respectively. For the KH instability, the slice plane is at $y = 0.5L_y$. For the CSS flow instability, the slice plane is defined by the origin point-normal vector due to a slight misalignment of the spanwise eddy structures with respect to the y axis. The origin point is selected as $(0,0.5L_y,0)$ and the normal vector is $0.12\vec{i}+1\vec{j}+0\vec{k}$ as indicated in (*c*) and (*d*). Cold (hot) color indicates negative (positive) quantity of ω_x.

instability, the CSS flow instability shows a much larger negative ω_x region (cyan color) at the periphery of the KH eddy core and wraps over the entire KH eddy. At $t = 1000s$, the eddy wrap structures are already very clear even when the central 'cat eye' has not yet developed. At t =1200s, when the 'cat eye' eddy is developed, the surrounding spanwise eddy tube grows even larger and entrains the outside fluid at the top and bottom shells. Compared to the KH instability, the 'cat eye' eddy seems to be compressed by the spanwise eddy in the CSS flow instability, and therefore decays much earlier, as indicated by 'collapse' and 'collapse(KH)' in figures 3(*a*) and (*b*).

The critical conditions for the eddy wrap structures are obtained from DNS runs with ξ varying over the range $\xi = 0 \sim 0.5$. For $\xi \geq 0.2$, the spanwise eddy wrap structures are observed to be similar to the $\xi = 0.5$ case. For $\xi < 0.2$, smaller spanwise eddies are observed but are overwhelmed before the streamwise 'cat eye' eddy is developed. Thus, no eddy wrap structures are formed, indicating that the critical ξ for the occurrence of the spanwise eddy wrap is 0.2. Through our observations, the spanwise eddies in the $\xi = 0.1$ case are insufficient to develop into the eddy wrap structures and start to collapse at $t = 1600s$, a time when the KH 'cat eye' eddy is just about to develop. When $\xi < 0.1$, the roll-up is not observed at any wrap-up spots along the spanwise streamlines, indicating that the critical ξ for the spanwise roll-up or eddy is 0.1.

Conclusions

It is shown that in the typical case of $\xi = 0.5$, the coherent structures of the CSS flow consist of small spanwise eddies along with a dominant streamwise KH 'cat eye' eddy. These small spanwise eddies originate from the braid region of a streamwise KH eddy and extend along the streamline to the top of the next streamwise KH eddy, forming unique 'eddy wrap' structures that wrap up the entire KH eddy structures. The evolution history of the perturbation kinetic energy in the $\xi = 0.5$ case shows a distinctively different transitional process to turbulence from that in the classic KH instability case. The critical ξ for the occurrence of the spanwise 'eddy wrap' is found to be 0.2, while for the spanwise roll-up or eddy it is found to be 0.1.

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