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Turbulent Energy Scale Budget Equation Applied to the Centreline of a Free, Round, Turbulent Jet and the Effect of a Passive Ring

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Abstract

A novel extension to the similarity-based form of the transport equation for the second-order velocity structure function of $\langle(\delta q)^2\rangle$ along the jet centreline has been obtained. This new self-similar equation has the desirable benefit of requiring less extensive measurements to calculate the inhomogeneous (decay and production) terms of the transport equation. The validity of this equation is first investigated via cross-wire measurements. Then, this equation is used as a energy scale budget equation to quantify the effect of initial conditions on all scales of a round free jet flow on its centreline. In the current study, the initial conditions of the jet are changed by using a passive grid positioned inside the shear layer near to the exit. It is found that initial conditions affect the virtual origin of the jet as well as the power-law exponent of turbulent kinetic energy. The effect of initial conditions on the turbulent energy scale budget equation is restricted to the inhomogeneous large-scale terms of the transport equation, while the diffusion term remains unaffected.

Introduction

Kolomogorov [1] in 1941 derived an important exact relation between the second- and third-order moments of the longitudinal velocity increment (known as Kolmogorov's 4/5 law) from the Navier-Stokes equations assuming a very high Reynolds, homogeneity and isotropy as

$$-\langle(\delta u)^3\rangle + 6\nu\frac{\mathrm{d}}{\mathrm{d}r}\langle(\delta u)^2\rangle = \frac{4}{5}\langle\varepsilon\rangle r\,, \qquad (1)$$

where $\delta u \equiv u(x+r) - u(x)$ is the longitudinal velocity increments (for the streamwise velocity component u), r is the distance between two points considered along the x direction and ν is the kinematic viscosity. Here, $\langle\varepsilon\rangle$ is the mean dissipation rate of turbulent kinetic energy defined as

$$\langle\varepsilon\rangle = \frac{1}{2}\nu\left\langle\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2\right\rangle. \qquad (2)$$

Equation (1) implies that at a scale r, the dissipation of turbulent kinetic energy is the sum of turbulent advection (first term left-hand side in (1)) and molecular diffusion (second term left-hand side in (1)).

However, the assumption that the Reynolds number should be very large is obviously not realized in practical flows (encountered in the laboratory conditions). Typical values of Taylor Reynolds number, Re_λ, in laboratory experiments such as gird turbulence and jet flows are of the order of 10^2 - 10^3, while a proper inertial range is unlikely to be established for $Re_\lambda \leq 10^4$ (e.g., [2], [3]). For such low Re_λ, the effect of the initial conditions may persist in space or time. This possibility has important implications when experimental data are used to either test theories or apply turbulent models to engineering flows. Therefore, one may expect that (1) cannot to be balanced for low Re_λ flows. As such, Danaila et al. [4] derived a energy scale budget equation (for the total turbulent energy structure function $\langle(\delta q)^2\rangle$ $(= \langle(\delta u)^2\rangle + \langle(\delta v)^2\rangle + \langle(\delta w)^2\rangle)$) along the centreline of a turbulent round jet, viz.

$$-\langle(\delta u)(\delta q)^2\rangle + 2\nu\frac{\mathrm{d}}{\mathrm{d}r}\langle(\delta q)^2\rangle - \frac{U_c}{r^2}\int_0^r s^2\frac{\partial}{\partial x}\langle(\delta q)^2\rangle\mathrm{d}s$$
$$-2\frac{\partial U_c}{\partial x}\frac{1}{r^2}\int_0^r s^2(\langle(\delta u)^2\rangle - \langle(\delta v)^2\rangle)\mathrm{d}s = \frac{4}{3}\langle\varepsilon\rangle r\,. \qquad (3)$$

Here, U_c is the local mean streamwise velocity along the centreline, and s is a dummy separation variable. The third term on the LHS of (3) reflects the inhomogeneity due to the streamwise decay of $\langle q^2\rangle$, which was introduced as a consequence of low Re. The forth term on LHS of this equation represents the role of the production.

However, in order to compute the inhomogeneous decay term in (3), $\langle(\delta q)^2\rangle$ has to be measured at different streamwise locations, which involves significant uncertainties associated with the numerical differentiation of the data [5]. Therefore, the main goal of the current work is to apply a novel similarity analysis and introduce a self-similar form to (3). A particularly useful feature of the this analysis is that it can reduce some of the difficulties involved in the calculation of the $\partial/\partial x$ terms (production and decay terms). As will be demonstrated, the terms in energy scale budget equation can be studied if $\langle(\delta q)^2\rangle$ and mean velocity (U_c) only are measured at a single point, rather than several points along the axis. The other goal of the current work is to study the effect of initial conditions on different energy terms in a round jet. The role of initial conditions on different classes of turbulent flows (e.g. grid turbulence, jet, wake) is now well accepted and has been confirmed in many experimental investigations. An effective way of changing the initial conditions in turbulent jet flows is to use passive objects near the exit. Sadeghi and Pollard [6] studied the effect of placing a thin square ring inside the shear layer of a round free jet. It was found that the stable vortex pairing of the shear layer mode completely disappeared when the ring was introduced into the shear layer. Eliminating the shear layer mode also affected the characteristic length scales in the development region of the jet. In this paper, the effect of introducing a passive ring on different terms of the transport equation is investigated. Here, a self-similar based form of the transport equation is first introduced and validated. Then this equation is used as a tool to identify scales affected by introducing the ring.

Similarity of energy structure function in turbulent jets

The concept of similarity, or self-preservation, which assumes the flow scales with single velocity and length scales, has been an important analysis tool in turbulence research. Following the same procedure in [4], an equilibrium similarity has been developed for the transport equation of the second-order energy structure function of $\langle(\delta q)^2\rangle$ along the centreline of a round turbulent jet (3). The equilibrium similarity forms of the second- and third- order structure functions of u, v and q are given by

$$f(r/\lambda) = \langle(\delta q)^2\rangle / \langle q^2\rangle, \tag{4}$$

$$e(r/\lambda) = \langle(\delta u)^2\rangle / \langle u^2\rangle, \tag{5}$$

$$h(r/\lambda) = \langle(\delta v)^2\rangle / \langle v^2\rangle \tag{6}$$

and

$$g(r/\lambda) = -\langle(\delta u)(\delta q)^2\rangle / (3^{-1/2} Re_{\lambda}^{-1} \langle q^2\rangle^{3/2}), \tag{7}$$

respectively. Here, g is the normalized third-order structure function and f, e and h are the normalized second-order structure functions. Assuming axisymmetry, $\langle q^2\rangle = \langle u^2\rangle + 2\langle v^2\rangle$. It should be noted that the accuracy of this assumption has been confirmed in both on and off the centreline of round jets [7]. The general definitions of Taylor microscale and Taylor microscale Reynolds number are

$$\lambda^2 = 5\nu \frac{\langle q^2\rangle}{\langle\varepsilon\rangle}, \tag{8}$$

and

$$Re_{\lambda} = \frac{\langle q^2\rangle^{1/2}\lambda}{3^{1/2}\nu}, \tag{9}$$

respectively [8].

One possible equilibrium similarity solution of Equation (3) is a power-law of the form

$$\langle q^2\rangle = A(x - x_0)^m, \tag{10}$$

where x_0 is the virtual origin, m is the power-law exponent and A is a constant of proportionality. The same power-law behavior is also suggested for $\langle u^2\rangle$ and $\langle v^2\rangle$ as

$$\langle u^2\rangle = A_1(x - x_0)^m, \langle v^2\rangle = A_2(x - x_0)^m. \tag{11}$$

The virtual origin follows from the variation of the mean velocity along the centreline, viz.

$$U_c = C/(x - x_0), \tag{12}$$

where C is a constant. For the region near the axisymmetric jet centreline, the kinetic energy budget equation is approximated as

$$\langle\varepsilon\rangle = C\left[\frac{-(A_1 + 2A_2)m}{2} + (A_1 - A_2)\right](x - x_0)^{m-2}. \tag{13}$$

The similarity form of (3) follows after substituting (4)-(13) into (3), viz.

$$g + 2\frac{\mathrm{d}f}{\mathrm{d}(r/\lambda)} + 10(c_1 + 2c_2)\frac{\Gamma_1}{(r/\lambda)^2} - 10m(c_1 + 2c_2)\frac{\Gamma_2}{(r/\lambda)^2} + 20c_1\frac{\Gamma_3}{(r/\lambda)^2} - 20c_2\frac{\Gamma_4}{(r/\lambda)^2} = \frac{20}{3}(r/\lambda), \tag{14}$$

where Γ_1, Γ_2, Γ_3 and Γ_4 are given by

$$\Gamma_1 = \int_0^{r/\lambda} \left(\frac{s}{\lambda}\right)^3 \frac{\mathrm{d}f}{\mathrm{d}(r/\lambda)} \mathrm{d}\left(\frac{s}{\lambda}\right), \quad \Gamma_2 = \int_0^{r/\lambda} \left(\frac{s}{\lambda}\right)^2 f \,\mathrm{d}\left(\frac{s}{\lambda}\right),$$

$$\Gamma_3 = \int_0^{r/\lambda} \left(\frac{s}{\lambda}\right)^2 e \,\mathrm{d}\left(\frac{s}{\lambda}\right), \quad \Gamma_4 = \int_0^{r/\lambda} \left(\frac{s}{\lambda}\right)^2 h \,\mathrm{d}\left(\frac{s}{\lambda}\right).$$

Here, $c_1 = \frac{A_1}{-Am+2(A_1-A_2)}$ and $c_2 = \frac{A_2}{-Am+2(A_1-A_2)}$.

Dividing by $(20/3)r/\lambda$, (14) can be rewritten symbolistically as

$$A^* + B^* + D^* + P^* = C^*, \tag{15}$$

where A^* is the turbulent advection term (the first term in (14)), B^* is the diffusion term (the second term in (14)), D^* is the inhomogeneous decay term along streamwise direction x (the sum of third and forth terms in (14)), P^* is the production term (the sum of fifth and sixth terms in (14)) and C^* is the balance of all other terms. The accuracy of this new equation is first verified by using cross-wire data. Then, the effect of introducing a passive ring into the jet shear layer on each term of the transport equation is investigated.

Experimental details

An air jet was generated using a fan mounted on anti-vibration pads. The air then exits a settling chamber via a round duct to the inlet of a smoothly contracting axisymmetric nozzle with exit diameter D = 73.6 mm. A wire ring, with square cross-section, of side $h = 1.5$ mm, and outer diameter $D_{wire} = 71.6$ mm was placed at a stand-off distance (to the ring leading edge) downstream of the jet nozzle exit plane $x/D = 0.03$. The ring was supported by three prongs (1.5 mm square, and length 2.2 mm) located at 120 degrees intervals. More details about the current experimental setup can be found in [3] and [6]. The experiments were carried out at the exit Reynolds number of $Re_D = 50,000$, where Re_D is calculated based on the jet exit mean velocity (U_j=10.65 m/s) and the nozzle exit diameter. The measurements were performed for $10 \leq x/D \leq 20$, where x is the downstream location, along the jet centreline. Measurements of the turbulence statistics were obtained using a stationary cross-wire probe. The wires were made of of 2.5 micron diameter tungsten wire with a 0.5 mm sensing length. The cross-wire was calibrated using a look-up table, with calibration angles within the range ± 40, in intervals of 10°. The signals were low-pass filtered at a cut-off frequency f_c, which was selected based on the onset of electronic noise and close to the Kolmogorov frequency, $f_k \equiv U/2\pi\eta$, where $\eta \equiv \nu^{3/4}/\langle\varepsilon\rangle^{1/4}$. The measurements were taken with a sampling frequency of $f_s \geq 2f_c$.

Basic characteristics

The axial mean velocity along the jet centreline is presented in Figure 1. For a self-similar jet, the centreline velocity variation is given by

$$\frac{U_j}{U_c} = \frac{1}{B}\left(\frac{x - x_0}{D}\right). \tag{16}$$

A least-squares fit to the data gives the mean velocity decay constant of $B = 6.6$ and the virtual origin of $x_0 = -1.69D$ for the unmodified jet (without the ring), and $B = 6.3$ and the virtual origin of $x_0 = 0.47D$ for the modified jet (with the ring). This confirms the significant effect of initial conditions on the mean decay constant and virtual origin in jet flows. A few other basic

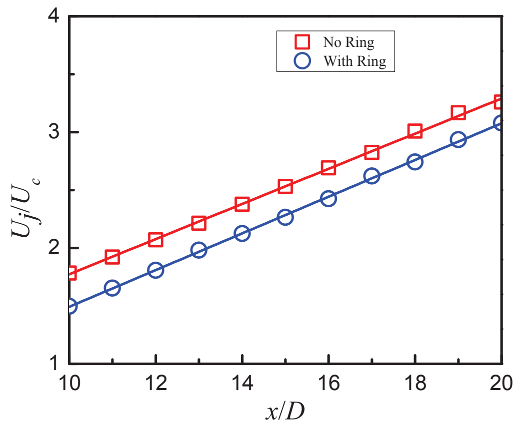

Figure 1: Axial decay of the mean velocity along the centreline (U_j=10.65 m/s). Solid line is the least squares fit to the data.

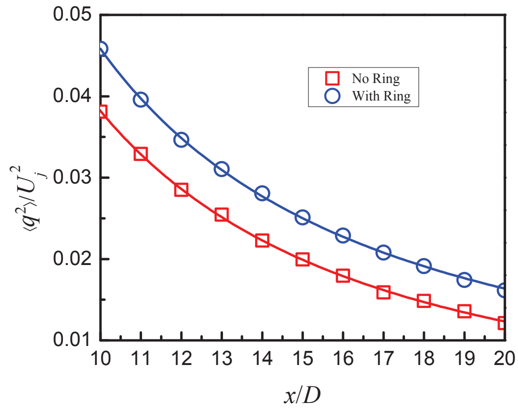

Figure 2: Streamwise variation of $\langle q^2\rangle$ along the centreline. The solid lines are the least squares fits to the data.

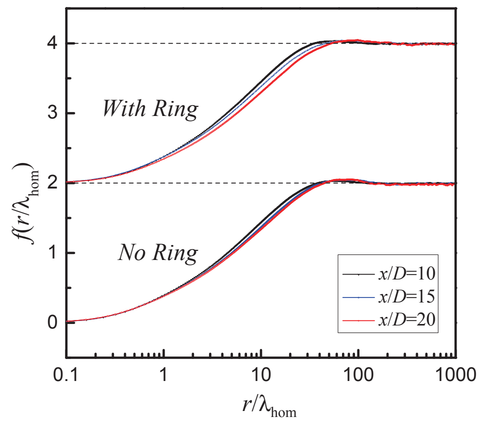

Figure 3: Distributions of $f(r/\lambda_{hom})$ at three axial locations of $x/D = 10, 15, 20$. Structure functions for with the ring case have been shifted successively (offset 2) with respect to the lower one. Each horizontal dashed line is 2.

quantities measured at three selected axial locations for both with and without the ring cases are summarized in Table 1 for reference. Here, $\langle\varepsilon\rangle_{hom}$ is

$$\langle\varepsilon\rangle_{hom} = 3\nu\left[\left\langle\left(\frac{\partial u}{\partial x}\right)^2\right\rangle + 2\left\langle\left(\frac{\partial v}{\partial x}\right)^2\right\rangle\right]. \tag{17}$$

λ_{hom} is calculated by replacing $\langle\varepsilon\rangle_{hom}$ into (8). Re_λ is obtained from (9) using λ_{hom}.

Similarity solutions

The streamwise variation of $\langle q^2\rangle$, measured along the jet centreline and normalised by U_j^2, for both with and without the ring cases are shown in Figure 2. A curve fit was applied to the data using the virtual origin of $x_0 = -1.69D$ for the unmodified jet and $x_0 = 0.47D$ for the modified jet. It was found that $\langle q^2\rangle$ follows closely a power-law with exponent $m = -1.83$ for the unmodified jet and $m = -1.44$ for the modified jet with the ring. This confirms the validity of (10) for jet flows regardless of the nature of initial conditions. The effect of initial conditions can be observed in the value of power-law exponent (m). Distributions of $f(r/\lambda)$ measured at the three locations considered here ($x/D = 10$, 15 and 20) are shown for both jets in Figure 3. λ_{hom} is used since the assumptions employed for its estimate are less restrictive than the alternatives estimates given the available measurements. The second-order structure functions of q are found to collapse approximately at each streamwise location.

Energy scale budget equation

Validation of the energy scale budget equation

First, in order to illustrate the validity of (14), the term $g(r/\lambda)$ is calculated from this equation (14) using the corresponding power-law exponents m and the decay rates A_1 and A_2 at $x/D = 15$ (identified as g_c) and compared with the measured profile of $g(r/\lambda)$ (denoted by g_m) in Figure 4 for both modified and unmodified jets. A relatively good agreement (within $\pm 12\%$, similar to results from grid turbulence experiments) is found between g_m and g_c for both cases. Note that the normalised third-order structure functions are divided using r/λ so that their maximum peaks can be compared with the onset of the inertial range. It can be observed that the asymptotic value of 20/3, which represents the onset of the inertial range for a high Reynolds number, is significantly higher than the maximum measured and calculated g. Sadeghi et al. [3] showed that a proper inertial range is unlikely to be established along the jet axis unless a very high Reynolds number of $Re_\lambda = 10^4$ can be reached.

The effect of the initial conditions on the budget terms

Equation (15), which is a normalised form of (14), is used as a scale-by-scale budget to quantify the effect of initial conditions on all scales of a round free jet flow on its centreline. The scale-by-scale budget terms, measured at $x/D = 15$ on the centreline and $Re_D = 50,000$, are given in Figure 5. This figure demonstrates that (15) is adequately satisfied by the experimental data ($0.88 \le C^* \le 1.12$) for nearly all scales of the jet flows on the centreline. For both jets, at small separations, the diffusion term B^* dominates, while at large separations, the decay term D^* and the production term B^* are the dominant terms. The advection term A^* goes to zero at both very small and large separations, while its maximum is located at nearly the same location for both modified and unmodified jets ($r \simeq 0.8\lambda$). Direct comparison of each term for the modified and unmodified jet indicates that B^* is nearly unaffected by the use of the ring. The impact of the initial condition can be observed mainly on the large scales for P^* and D^*. The magnitude of A^* is higher for the modified jet, which can be related to the increase in turbulent Reynolds number.

Unmodified jet (no ring)					Modified jet (with ring)				
x/D	$\langle\varepsilon\rangle_{hom}$ (m^2s^{-3})	λ_{hom} (mm)	Re_λ	η (mm)	x/D	$\langle\varepsilon\rangle_{hom}$ (m^2s^{-3})	λ_{hom} (mm)	Re_λ	η (mm)
10	33.9	3.15	241	0.11	10	42	3.11	262	0.098
15	9.30	4.35	241	0.15	15	13.7	4.06	254	0.13
20	3.40	5.62	243	0.19	20	5.89	4.98	252	0.16

Table 1: A few basic parameters at three downstream locations along the jet centreline for both modified and unmodified jets.

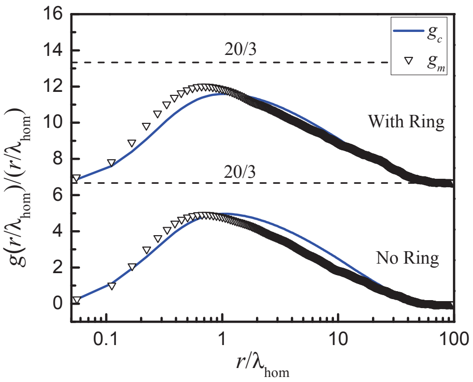

Figure 4: Comparison between measured (triangles) and calculated (solid line) distributions of g divided by r/λ_{hom}(at $x/D = 15$). Structure functions for with the ring case have been shifted successively (offset 20/3) with respect to the lower one. Dashed line is 20/3.

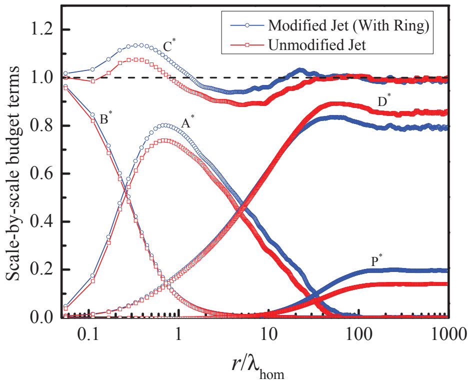

Figure 5: Terms in (15) at $x/D = 15$ and $Re_D = 50,000$ for both modified (blue circles) and unmodified (red squares) jets. A^* is the advection term, B^* is the diffusion term, D^* is the decay term, P^* is the production term, C^* is the sum of all other terms.

Conclusion

The effect of initial conditions has been studied in a around jet flow on its centreline with the help of the transport equation for the second-order velocity structure function of $\langle(\delta q)^2\rangle$. To achieve this, a fine ring, with square cross-section, was designed and placed very close to the jet exit ($x/D = 0.03$). We have first developed a similarity-based form of the transport equation for the second-order velocity structure functions of $\langle(\delta q)^2\rangle$. This new self-similar equation has the desirable benefit of requiring less extensive measurements to calculate the inhomogeneous (decay and production) terms of the transport equation. It was found that the self-similar form of the transport equations yield to a solution where the turbulent kinetic energy decays following a power-law of the form $\langle q^2\rangle \propto (x - x_0)^m$ along the centreline. Experiments were performed to verify the similarity solutions and investigate the effect of initial conditions. It was found that power-law decay regions exist over the present range of measurements for $\langle q^2\rangle$ with substantially different exponents, m, for the modified and unmodified jets. It was shown that the distributions of $\langle(\delta q)^2\rangle$, when normalised by $\langle q^2\rangle$ and λ, satisfied similarity to a close approximation over all range of scales for both unmodified and modified jets. Allowing for experimental uncertainty, the calculated and measured distributions of the normalised third-order structure functions were found in a good satisfactory in (14). The energy scale budget equation of (14) has been used to study quantitatively the effect of initial conditions on the different scales of the jet flow on its centreline. The impact of initial conditions was mainly observed on the inhomogeneity large-scale terms, however, the diffusion term remained nearly unchanged.

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