Wavelet Multiresolution Analysis on Wake Structure of a Yawed Square Cylinder

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Abstract

The effect of yaw angle \(\alpha\) on wake characteristics behind a yawed square cylinder was examined at a Reynolds number (Re) of 3600. Velocity and vorticity fluctuations were measured using a one-dimensional hot-wire vorticity probe. It is found that the large-scale structures contribute the most to the streamwise and transverse velocity variances, as well as Reynolds shear stress, despite the reduction as \(\alpha\) increases. The most significant contribution to the spanwise vorticity variance comes from the small-scale structures. The independence principle (IP) for vortex shedding is also validated for \(\alpha \leq 40^\circ\).

Introduction

With increasing use of slender structures of square cross-section, the study of the wake of a square cylinder is of theoretical and practical significance. When the cylinder is yawed in a flow at an angle \(\alpha\) (the angle between the flow direction and the plane which is perpendicular to the cylinder axis), the flow velocity in the axial direction may not be ignored and the vortex shedding characteristics may be different from a cylinder in a cross-flow. For a yawed circular cylinder, it has been found that the Strouhal number (St) and the drag coefficient, normalized by the velocity component perpendicular to the cylinder, are the same as those when the structure encounters a normal incidence flow. This is variously known as the Independence Principle (IP). Whereas the validity of IP and the flow structures of a yawed circular cylinder at different Reynolds numbers have been reported previously [8,12], our knowledge about the vortical structures of a yawed square cylinder wake is very limited [6].

A number of techniques have been developed previously to investigate the large-scale structures in turbulent wakes, such as the phase-averaged method [7], window average gradient (WAG) method [1] and the vorticity-based technique [4]. However, apart from the large-scale structures, the relatively small-scale structures, such as the secondary vortices [10], as well as the intermediate structures such as the longitudinal rib-like vortices [3], may also be of significance in the wake dynamics. Given the fact that the techniques mentioned above cannot provide details from various scales of turbulent structures, wavelet multi-resolution analysis has been introduced and proven to be effective in extracting information on the turbulent structures of the cylinder wake [3]. This method allows the measured time-frequency signals to be analysed in time-domain, frequency-domain or a combination of both. The present study aims to investigate the validity of IP for different cylinder yaw angles as well as the features of different scales of turbulence. By examining the power spectra, velocity and vorticity variances at different wavelet levels for different yaw angles, the features of the large-, intermediate- and small-scale turbulent structures in the square cylinder wake can be quantified.

Experiment details

Sketches of the one-dimensional hot-wire vorticity probe are given in figure 1. It consists of two parallel wires straddled by an X-wire. The two parallel wires are used to measure the streamwise velocity \(u\) at two locations separated by a distance \(\Delta y (=1.5 \text{ mm})\) and the X-wire is used to measure the streamwise velocity \(u\) and transverse velocity \(v\) (or \(w\) after a 90\(^\circ\) rotation). The separation \(\Delta z\) between the two inclined wires in the X-wire is 1.6 mm. With these measured velocity components, the spanwise vorticity \(\omega_x\) (or \(\omega_z\) after a 90\(^\circ\) rotation) can be calculated. This probe has been used satisfactorily to measure the spanwise and transverse vorticity components in a wake flow [2]. Measurements were conducted at a constant free stream velocity \((U_0)\) of 4.27 m/s and at \(x/d = 10\) (\(x\) is the distance downstream of the cylinder and \(d = 12.7\) mm is the side length of the cylinder). The corresponding Reynolds number (Re) was about 3600. Wollaston (Pt-10% Rh) wires of 5 \(\mu\)m in diameter and about 1 mm in working length were operated within a constant-temperature circuit at an over heat ratio of 1.5. Signals from the circuits were offset, amplified and then digitized using a 16bit A/D converter at a sampling frequency of 10400 Hz. The sampling duration was 30 s.

Due to the imperfect spatial resolution of the probe in measuring the velocity derivatives, using the method proposed by [2], the spatial resolution of the vorticity probe in measuring the rms values for vorticity components can be corrected if local isotropy is assumed. It has been checked that when \(\alpha\) increases from 0° to 45°, the spatial resolution of the probe improves as the Kolmogorov length scale \(\eta\) on the wake centreline increases from 0.11 mm to 0.13 mm. The Kolmogorov length scale is defined as \(\eta = (\tau/\epsilon)^{1/4}\) where \(\tau/\epsilon\) is the mean energy dissipation rate which can be estimated using Taylor’s hypothesis. It is estimated that the root-mean-square values of the spanwise and transverse vorticity components have been underestimated by about 28%–22% for \(\alpha = 0°\) to 45°. Unfortunately, correction for spatial resolution cannot be performed in the present study due to the lack of local isotropy at \(x/d = 10\).

The wavelet analysis technique includes two parts, namely, the wavelet transform and the wavelet multi-resolution analysis. The former decomposes the signal into different wavelet levels to provide a useful depiction of the signal in both time and frequency, and the latter uses the discrete wavelet transform and its inverse transform to convert a signal into the sum of a number of wavelet components at different levels. One-dimensional discrete wavelet transform is a linear and orthogonal transform. Details of the wavelet method can be found in [7,10].
The transverse velocity $v$ is normal to the cylinder axis (oriented on the large scale structures). The effect will cause the peak on each spectrum at all yaw angles, which corresponds to the occurrence of the large-scale Kármán vortex structures. It has been checked that the three-dimensional effect has been enhanced as reflected by the increase in the spanwise velocity $W$ when cylinder yaw angle increases (results are not shown here). This effect will cause the dispersion of vortex shedding intensity. As a result, the peak on the energy spectra becomes lower and wider as $\alpha$ increases. The shedding frequencies $f_0$, determined by the location of the peak are 44.0 Hz, 43.5 Hz, 41.4 Hz and 36.1 Hz for $\alpha = 0^\circ$, 15$^\circ$, 30$^\circ$ and 45$^\circ$, respectively. These shedding frequencies $f_0$, representing the organized large-scale structures, will therefore be used as the basic frequencies in the wavelet analysis. After normalisation by the velocity component normal to the cylinder axis, as implied by the IP, a Strouhal number $St_k = f_d U_{\infty}$, where $U_{\infty}$ is the velocity component normal to the cylinder axis) of are 0.131, 0.134, 0.142 and 0.152 for $\alpha = 0^\circ$, 15$^\circ$, 30$^\circ$ and 45$^\circ$, respectively, can be obtained. The above result represents a 16% deviation from the IP at $\alpha = 45^\circ$, indicating that IP will not be applicable at this yaw angle.

The velocity and vorticity signals are decomposed into 19 orthonormal wavelet levels. The higher wavelet levels correspond to lower frequency bands or large-scale structures while lower wavelet levels correspond to higher frequency bands or small-scale structures. Only results for wavelet levels 1–9 will be shown here due to the negligible contributions from levels higher than 9. Specifically, wavelet levels $i = 1$–3 represent the small-scale-structures; $i = 4$–6 represent the intermediate-scale structures; $i = 7$ represents the organized large-scale structures located at the vortex shedding frequency band; and $i = 8$ and 9 represent the large-scale structures [9]. The spectra of $v$ at wavelet levels 1–9 for $\alpha = 0^\circ$–45$^\circ$ at $y/d = 0.5$ are shown in figure 2. The spectrum of each wavelet component shows a pronounced peak and spreads over a range of frequencies. The central frequency at each wavelet level can be identified from the most pronounced peak of each spectrum. These central frequencies, together with the frequency bandwidths for all wavelet levels are listed in table 1. Once the dominant frequency $f_0$ is set, other central frequencies at different wavelet levels are found to be about a multiple of $f_0$. Generally the central frequency at the $i$-th level is about half of that at (i-1) level due to the characteristics of wavelet basis applied in the present study.

$$\langle \beta^2 \rangle = \frac{1}{\delta} \sum_{i=1}^{N} \beta_i^2$$

where $k$ is the total number of measured data points. Since the wavelet components at a central frequency lower than 0.35$f_0$ (wavelet levels higher than 9) are not physically important, only the distributions for wavelet levels 1–9 at different yaw angles are shown in the present study.

For velocity variances $\langle u_i^2 \rangle$ in figure 3, the distributions at all wavelet levels are symmetrical about the wake centreline ($y/d = 0$). At $\alpha = 0^\circ$, the component at wavelet level 7, corresponding to the organized large-scale structures, makes the most contribution to $\langle u_i^2 \rangle (24\%)$ at the peak point, followed by that at levels 8 (23\%) and 6 (14\%). Structures at these three levels contain the most energy. This result agrees well with that for a circular cylinder wake where the large-scale structures at 0.5$f_0$ and $f_0$ together make the largest contribution (43\%) to the Reynolds normal stresses [11]. The distributions at levels 7 ($f_0$) and 8 (0.75$f_0$) show double peaks at $y/d = \pm 1.0$. The contributions from both organized large-scale structures decrease significantly from 47\% to 31\% with the increase of the yaw angle from 0$^\circ$ to 45$^\circ$. Though the magnitude of the contributions from the intermediate-scale structures (levels 4–6) as well as the small-scale structures (levels 1–3) keeps constant for all angles, the contribution ratio of the large-scale structures actually increases considering the decrease of the measured time-averaged value $\langle u_i^2 \rangle$, indicating the reduction of the large-scale structures at large yaw angles.

The distributions of $\langle v_i^2 \rangle$ is also symmetric about $y/d = 0$. In agreement with the spectra shown in figure 2a, the contribution from level 7 (organized large structures) dominates (56\%) among all the wavelet levels in figure 3 at $\alpha = 0^\circ$. This is in agreement...
The present study suggests that IP in a square cylinder wake is applicable for $\alpha < 45^\circ$. The contributions from the organized large-scale structures dominate the variations of Reynolds stresses, followed by that from the large-scale structures ($0.75f_0$ and the intermediate-scale structures ($2f_0$). As $\alpha$ increases to $45^\circ$, the contribution from large-scale structures decreases significantly, indicating the dissipation of coherent structures at large yaw angles. For spanwise vorticity variation, the most significant contributions come from the small-scale structures ($16f_0$), which manifest the importance of incoherent structures in the form of spanwise vorticity.

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References


Figure 2. Comparison of energy spectra of the \( u \)-signals measured on the wake centreline and that calculated at various wavelet levels for different inclination angles.

Figure 3. Variances of \( u \), \( v \), \( \omega_z \) and \( \omega_x \) at various wavelet levels for different inclination angles.