19th Australasian Fluid Mechanics Conference
Melbourne, Australia
8-11 December 2014

Laboratory Experiments on Symmetric and Asymmetric Mode-2 Internal Solitary Waves in a Three-layer Fluid

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Abstract

Numerous observational examples of Mode-2 internal solitary waves (ISWs) in the ocean (*e.g.* Yang *et al*.[12], Schroyer *et al*.[8]) have motivated recent modelling studies. The present laboratory investigation has been undertaken to investigate aspects of the behaviour of Mode-2 ISWs predicted by numerical model results obtained by Salloum *et al*. [6] and Olsthoorn *et al*.[5]; in particular, the dependence of the structure and stability of such waves upon asymmetrical initial conditions.

As in the numerical models, asymmetry in the initial conditions is imposed by a prescribed offset between the mid-depth of the undisturbed 3-layer stratified fluid and the mid-level of the middle layer. For stable mode-2 waves, the experiments demonstrate that observed flow patterns (and their time development) agree well with numerical model predictions. For symmetrical initial conditions (0% offset), the amplitudes of the dominant and secondary mode-2 waves (and their phase velocities) are seen to agree well with the values obtained numerically by Olsthoorn *et al* [5] for identical conditions. The critical amplitude required for mode-2 ISW instability compares well with the predicted value for symmetrical initial conditions but the critical amplitude is shown to decrease for increasing degrees of asymmetry in the initial conditions. As the initial condition asymmetry (*i.e* the % offset) increases, the top-bottom asymmetry of the manifestation of instability is shown to increase.

Introduction

Nonlinear internal solitary waves (ISWs) attract interest because many aspects of their behaviour require further theoretical analysis to describe them fully and because they occur (and are important dynamically) in many oceanic contexts (*e.g.* Vlasenko *et al*., [11]; Apel *et al*., [2]). The focus of the present study is the generation and development of mode-2 ISWs, a class of waves that has been studied extensively by theoretical and laboratory modellers (*e.g.* Maxworthy [5], Akylas & Grimshaw [1], Stamp & Jacka, [10]; Terez & Knio, [11]; Schmidt & Spigel, [8]; Gavrilova & Lyapidevskii, [4]; Brandt & Shipley, [3])) and for which there are many observational examples in the ocean (*e.g.* Yang *et al*., [13]; Schroyer *et al*., [9]). The study has been inspired particularly by the recent numerical and laboratory modelling studies of (i) Salloum *et al*. [7] and Olsthoorn *et al* [6] and (ii) Brandt & Shipley [3], respectively. These studies have focussed on, *inter alia,* the stability and the tendency to asymmetry of mode-2 waves for symmetrical and asymmetrical initial conditions. It is with these properties of mode-2 internal solitary waves that the present laboratory study is primarily concerned, not least because of the relevance of asymmetrical Mode-2 waves in ocean observations in locations with seasonal offset pycnoclines (Yang *et al*., [14]).

The Experiments

Figure 1 shows a side view of the experimental arrangement; a tank of length 6.4 m, width 0.4 m and rectangular cross section was divided into 2 sections by a vertical gate situated a distance L_G from one end. Within the main part of the tank, a quiescent, 3-layer, stably-stratified fluid system was established by filling sequentially with homogeneous saline layers of density ρ_1 and ρ_3 and respective thickness h_1 and h_2 separated by an interfacial layer of thickness h_3 and density $\Delta\rho$ (z), where z is the coordinate in the vertical direction.

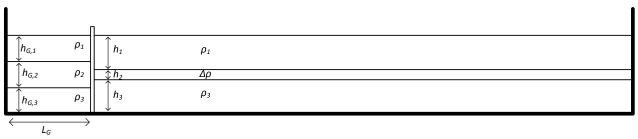


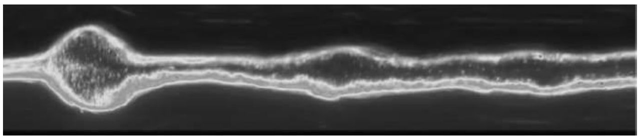
Figure 1. Schematic view of the experimental arrangement –see text for definitions

Behind the gate were layers of density $\rho_{1,2,3}$ and thicknesses $h_{G,1}$, $h_{G,2}$ and $h_{G,3}$, with ρ_2 (= $(\rho_1 + \rho_3)/2$). The elevation of the central level of the middle layers at both sides of the gate was aligned always to be coincident. The total depth was H (=0.4 m) on both sides of the gate. Cases were run in which the mid-plane of the middle layer was located either at mid depth $H/2$ (the so-called 0%-offset condition) or displaced upwards from the mid-depth by a fraction H/n (n = 5, 10, 20) - henceforth delineated respectively as 20%, 10% and 5% offset conditions. For one set of experiments, the conditions replicated exactly those adopted by Olsthoorn *et al* [6]. The experiment was initiated by the vertical removal of the gate and the ISW was recorded by an array of digital video cameras placed along the outside of the main portion of the tank. The fluid in the main tank only was seeded with tracer particles and the *DigiFlow* PIV system was used to process the recorded images. A fixed array of 4 conductivity sensors was programmed to acquire vertical density profiles through the wave as it passed.

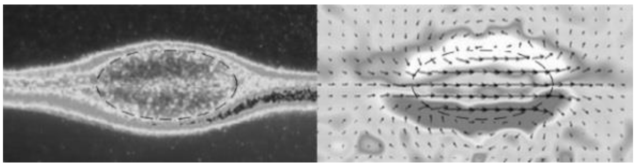
Results

Zero-offset Cases

Figure 2 shows examples of symmetric, stable waves generated for 0% offset cases; such images are directly comparable with those of Salloum *et al* [7] and Olsthoorn *et al* [6]. The time series shows the characteristic bulge and symmetric core of the dominant mode 2 wave, with two additional mode 2 waves of decreasing amplitude. As the forcing increases (by increasing either $h_{G,2}$ or L_G, or both, for otherwise identical conditions) shear-induced instability (see below) within the core results in (i) overturning at the rear of the bulge and (ii) entrainment from the external fluid. The results also show evidence for the occurrence of the so-called PAC-man bulge feature within the parameter range for which the feature was observed by Salloum *et al* [7] and Brandt & Shipley [3]. Comparisons can be made (Table 1) between the laboratory data and the numerical results of Olsthoorn *et al* [6] with regard to (i) the amplitudes $a_{1,2,3}$ of 3 successive mode-2 waves and (ii) the distances $x_{1,2,3}$ travelled by each wave in the reference elapsed time of t = 100s. The comparisons show good agreement between model prediction and laboratory measurement.



(a)



(b) (c)

Figure 2. Stable mode-2 wave, 0% offset: (a) typical time series at fixed location, (b) interior core with (c) associated velocity and vorticity fields.

	x_1 (m)	x_2 (m)	x_3 (m)	a_1 (m)	a_2 (m)	a_3 (m)
Numerical	5.40	4.42	3.75	0.0346	0.0213	0.0106
Laboratory	5.40	4.60	4.10	0.0330	0.0170	0.0060

Table1. Comparisons of lab measurements with numerical model results of Olsthoorn *et al* [6] – see text.

Non-zero-Offset Cases

For cases of non-zero offset, the results show many of the features described by Olsthoorn *et al* [6], namely (i) the appearance of mode-1 waves in the tail of the leading mode-2 wave and (ii) top and bottom and fore-aft asymmetry in the shape of the leading mode-2 wave (see Figure 3). Increasing the offset value leads to increases in the latter asymmetry and the appearance of localised shear-induced overturning at the rear of the wave (as observed by Olsthoorn *et al.*, [6]) – see Figure 4. As the forcing increases, the vigour of the overturning is seen to increase, with strong top-bottom asymmetry in the overturning. The experiments show a tendency for the top and bottom asymmetry to increase with increasing degree of offset; this finding is in partial accord with the numerical model predictions of Olsthoorn *et al* [6], who were able to identify this effect when comparing the 0% and 5% offset cases but did not detect increasing asymmetry when increasing the offset to 20%.

In all cases, the overturning observed in regions at the rear of the leading bulge of the Mode-2 flow pattern is ascribed to the occurrence of shear (Kelvin –Helmholtz) instability. This conclusion is relevant to the observations made for zero-offset cases by Salloum *et al* [7]. They observed a phenomenon they characterised as "vortex shedding" in the wake of the primary

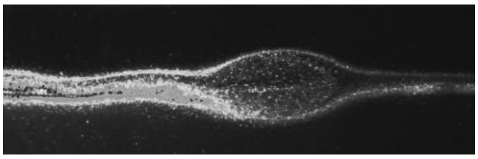
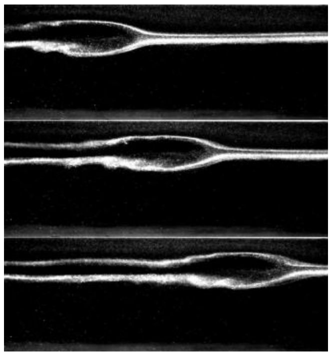
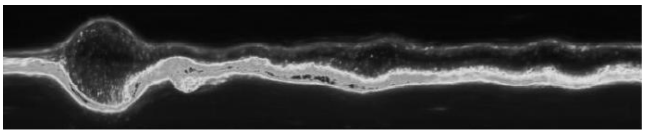


Figure 3. Image from a non-zero offset case, showing the leading Mode-2 ISW bulge and the tail of Mode-1 ISWs

bulge of the Mode-2 pattern at large wave amplitudes, conjecturing that when the appropriate value of the minimum gradient Richardson number Ri_{min} of the flow dropped below 1/4, the wave became unstable and "vortex shedding" resulted (with dramatic impact on mass transport within the wave). In order to investigate the critical conditions for instability, they determined the conditions on the wave amplitude for the critical condition Ri_{min} = ¼ for linear 2D Kelvin-Helmholtz instability to be achieved and found that such instability occurred for wave amplitudes $2a/h_2 > 2.75$. For the present experiments, the critical value of $2a/h_2$ for the occurrence of overturning billows in zero offset cases was found to be $2a/h_2 = 2.36 \pm 7\%$, in good agreement with the value obtained by Salloum *et al* [7] and an inferred value of about 2.2 from the study by Maxworthy [5].



(a)



(b)

Figure 4. Non-zero offset cases: (a) sequence of images illustrating asymmetrical shear instability and (b) time series of unstable Mode-2 wave.

For cases of non-zero offset, the present experiments demonstrate a dependence of the critical amplitude upon the degree of offset. This property is illustrated on Figure 5.

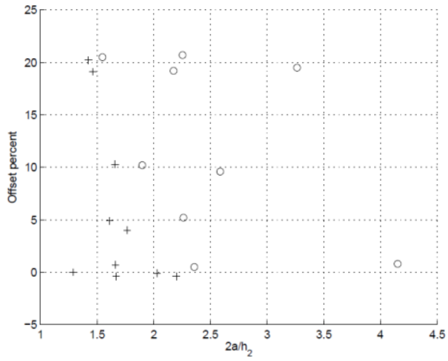


Figure 5. Flow regime classification of unstable (o) and stable (+) Mode 2 ISWs in terms of degree of offset (%) and dimensionless amplitude $2a/h_2$

Conclusions

For stable mode-2 waves, the observed flow patterns (and their time development) agree well with relevant numerical model results of Salloum *et al* [7] and Olsthoorn *et al* [6]. For 0% offset cases, the amplitudes of the dominant and secondary mode-2 waves (and their phase velocities) agree very well with the values obtained numerically by Olsthoorn *et al* [6] for identical conditions.

The critical amplitude required for instability compares well with the value obtained by Salloum *et al* [7] for 0% offset cases but the critical amplitude is shown to decreases for increasing offset values. Furthermore, as the offset increases, the top-bottom asymmetry of the manifestation of instability increases.

For unstable 0% offset cases, the strength of the shear-induced billows decreases with elapsed time as more secondary mode-2 waves form in the tail.

Acknowledgments

One of the authors (RH) was supported for a summer internship by a small research award from the University of Dundee.

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