

## Wave run-up investigation on a square cylinder

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### Abstract

As the demand for oil and gas is increasing, the need for fixed and floating structures is gaining importance in the form of offshore facilities. The decks of these offshore facilities are supported by structures made of circular or square columns. During strong wave action, considerable water could run up along the columns hitting the underside of horizontal deck. Hence determining the minimum distance from the water surface to deck, also known as air gap, is vital.

The paper examines the run-up on a square cylinder using experimental and numerical methods. Experiments were performed in a wave tank on a 1:66 scaled model of a square column. Numerical simulations were conducted using the commercial software FLUENT. It was found that the results for wave run-up from simulations and experiments were in good agreement, and were consistently greater than linear diffraction theory. Numerical comparison between circular and square cylinders showed that the run-up was higher for square geometry for all conditions simulated. An attempt has been made to understand the various nonlinearities in the run-up profile.

### Introduction

The wave run-up phenomenon can be described as the vertical up rush of water that is the result of an incident wave train breaking on a partially immersed body. Estimation of maximum wave run-up on fixed and floating offshore structures such as Tension leg Platform (TLP), spar and semi submersible platform is necessary to determine whether underside structural damage may occur in extreme design conditions. Steep waves show significant non-linear behavior and wave amplification due to interaction with the platform legs that must be accounted for in air gap design. Accurate prediction of deck clearance from the free surface is critical for predicting the performance of these structures in harsh environmental conditions. Swan et al. [1] documented the potentially destructive nature of wave run-up. Their study focused on the Brent Bravo concrete gravity structure, whose design was based on insufficient air gap leading to severe underside structural damage.

Experimental investigations in plane progressive waves undertaken by Kriebel [2] show that wave run-up in extreme conditions was more than 1.5 times the incident wave amplitude. Issacson & Cheung [3] proposed a sinusoidal wave theory to estimate wave run-up. Although the sinusoidal wave theory was found to underestimate the experimental data, it was more accurate than the results calculated from applying the linear diffraction theory. The expressions proposed in [3] for predicting maximum wave run-up at the front of a cylinder was considered reasonable in a later paper of Niedzwecki & Duggal [4]. However the proposed theories in [3] and [4] were inaccurate in predicting the wave run-up and corresponding free-surface elevations around a cylinder model.

The first study that considered the harmonic components of wave run-up was by Mercier & Niedzwecki [5]. Their experiments were conducted in a wave basin in which monochromatic waves were generated towards a fixed truncated cylinder. The results of the study noted an increasing modulus of each harmonic with increasing scattering parameter  $ka$  (incident wave non linearity), where  $a$  is the characteristic length which in this case is the diameter of the cylinder and  $k$  is wave number defined as

$$k = \frac{2\pi}{\text{wave length}} \quad (1)$$

The study did not include the effect of  $ka$  on the measured harmonics. The inherent scatter in the measured results had diluted the direct conclusions that were made from this study. In recent times Computational Fluid Dynamics (CFD) has been employed to extensively to model multiphase flows. A review of application of CFD to model wave run-up on circular cylinders is presented in Repalle et al. [6]. In the present work the dynamics of wave run-up on a square column is analysed and the results are compared with the run-up on a circular cylinder of similar characteristic length to estimate the variation in run-up due to a change in geometry.

### Experimental Setup

The experiments were carried out by one of the authors [7], in a wave flume of dimensions 50 m long, 1.5 m wide and an operating depth of 1.5 m. The waves in the flume were generated with piston type wave maker. The wave flume was equipped with a sloped artificial beach at the far end to reduce wave reflections. The model was located 35 m from the beach to minimize the influence of reflected waves. A schematic diagram of the side view of the test setup is shown in figure 1.

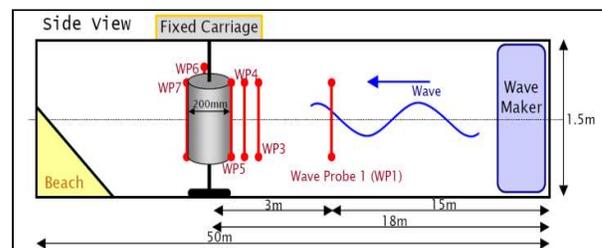


Figure 1. Schematic view of experimental setup [7]

The cylinder model was scaled from the Visund platform which was previously employed by Morris-Thomas & Thiagarajan [8] in their research. A scaling ratio of 1:66 was used to determine the cylindrical model dimensions. A square cylinder was constructed using poly plastic with 0.76 m height and 0.2 m side as shown in figure 2.



Figure 2. Cylinder model with probes attached [7]



Figure 3. Wave flume with cylinder attached to the fixed carriage [7]

The wave run-up experiments were conducted on fixed vertical cylinders in a water height of 1.2m subjected to plane progressive waves. The free surface elevation and wave run-up were recorded with capacitance type wire probes. The surge motion of the cylinder was negligible as this motion was restricted by horizontal stiffness provided by the pin connection to the vertical pole shown in figure 3. The wave run-up was studied for three different scatter parameter values of 0.208, 0.283 and 0.358. The process was repeated for different amplitudes to quantify the effect of wave steepness ( $ka$ ) on wave run-up phenomenon.

### Computational Model

A three dimensional numerical model of the flow around a circular and square cylinder was developed using the commercial CFD code FLUENT. The computational domain was modelled as shown in figure 4. The dimensions of the numerical wave tank were 20 m length, 2 m width and 2 m height. The cylinder used here was 0.76 m high and 0.2 m wide. The draft of the cylinder was 0.38 m while the position of the cylinder centre was 6.0 m away from the inlet and 1.0 m from the side walls.

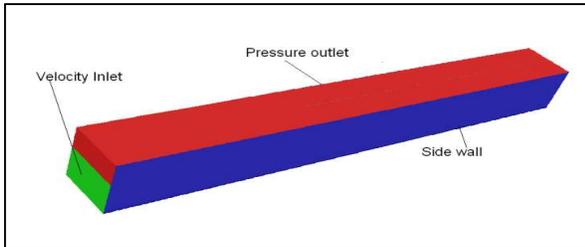


Figure 4. Computational model of the wave run-up experiment

The Volume of Fluid (VOF) method was adopted to capture the free surface. A single momentum equation was solved throughout the domain and the resulting velocity field was shared among the phases.

A velocity inlet boundary condition was used to generate waves from left wall as shown in figure 4. Second order waves were generated using equation 2 and 3 (see Chakrabarti [9]) with the help of user defined functions (UDF). The resultant second order wave elevation profile is given in equation 4.

$$u = Hgk \frac{\cosh[k(h+z)]}{2\omega \cosh(kh)} \cos(kx - \omega t) + \frac{1}{4} H^2 \omega k \frac{\cosh[2k(h+z)]}{\sinh^4(kh)} \cos 2(kx - \omega t) \quad (2)$$

$$v = Hgk \frac{\sinh[k(h+z)]}{2\omega \sinh(kh)} \sin(kx - \omega t) + \frac{1}{4} H^2 \omega k \frac{\sinh[2k(h+z)]}{\cosh^4(kh)} \sin 2(kx - \omega t) \quad (3)$$

$$\eta = \frac{H}{2} \cos(kx - \omega t) + \frac{H^2 k}{16} \frac{\cosh(kh)}{\sinh^3(kh)} [2 + \cosh(2kh)] \cos 2(kx - \omega t) \quad (4)$$

Here,  $H$ ,  $k$ ,  $g$ ,  $\omega$ ,  $t$ ,  $h$ ,  $z$  are wave height, wave number, gravity, frequency, wave amplitude and water depth respectively. The pressure velocity coupling was solved using Pressure-Implicit with Splitting of Operators (PISO) scheme. A modified high-resolution interface capturing (Modified HRIC) scheme was employed for accurate representation of interface. A sample of wave profile around the cylinder is shown in figure 5.

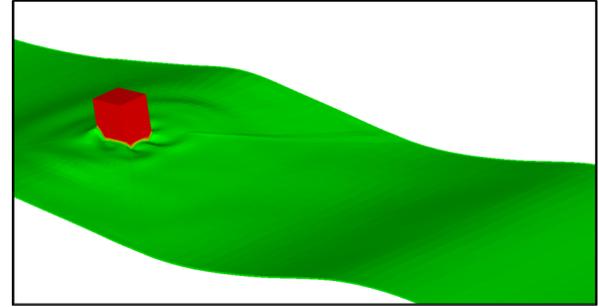


Figure 5. Representation of the free surface and cylinder grid during simulation

The computational model was validated using the wave elevation profile recorded at a distance of 3 m from the cylinder. Waves of time period 1.20 s with amplitude of 0.05 m were used for this comparison. Wave propagation history is shown in figure 6. The results show that the wave trains are well established and the numerical results compare reasonably well with the theoretical results. A slight loss in amplitude is observed due to numerical diffusion in the computational domain.

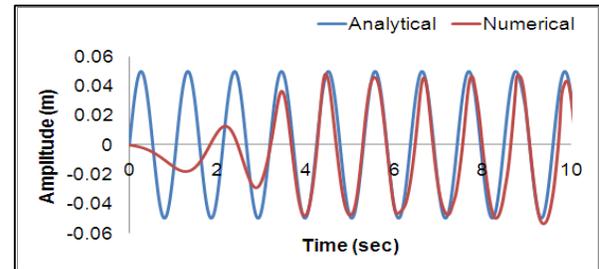
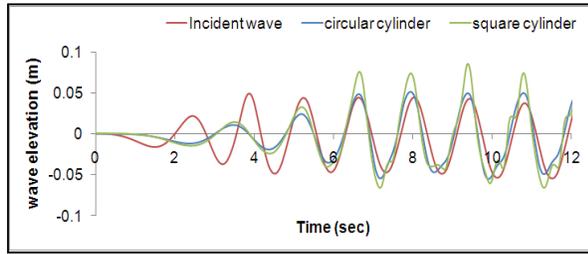


Figure 6. Wave elevation profile comparison of the numerical model with the analytical solution for  $T=1.06$  s and  $A=0.05$  m

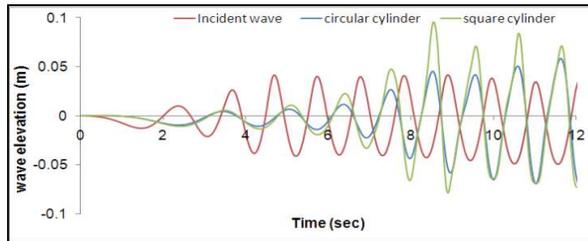
### Results and Discussion

Experiments were carried out by parametric variation of the scattering parameter  $ka$  and wave steepness  $kA$ . To make comparison with linear diffraction theory, a wave input height of 0.1m was chosen. The input frequency ranged from 0.3 to 0.9 Hz, as only a limited frequency range produced acceptable sinusoidal waves. The theoretical run-up is defined in this analysis as a non dimensional parameter  $R/H$  which corresponds to the ratio of the wave height at the cylinder measured at (WP5) to the wave height of the incident wave measured at (WP1) (see figure 1).

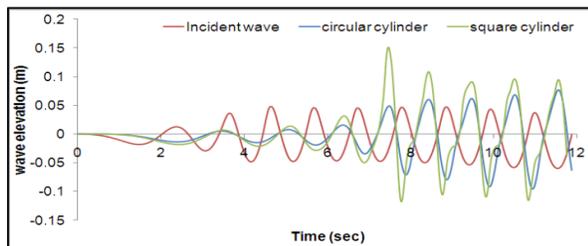
The graphs in figure 7(a)-(c) shows the wave profiles over time for three different  $ka$  values of 0.208, 0.283 and 0.36 considered for the present computational simulation.



(a)  $ka=0.208$  and  $kA=0.145$



(b)  $ka=0.283$  and  $kA=0.227$



(c)  $ka=0.36$  and  $kA=0.359$

Figure 7. Wave profiles of incident wave and run-up for square and circular cylinder obtained from computational model.

We can clearly see from figure 7 that the incident wave builds up first as this position is closer to the velocity inlet. The run-up at the cylinder takes 2-3 cycles more to build up as it is further from the inlet. It is evident that the run-up is higher for a square cylinder compared to a circular cylinder for all the cases considered. This difference can be directly attributed to the geometry of the respective cylinders. The resistance to flow in the case of square geometry is much higher than the circular cylinder and this additional resistance is realised in the form of higher run-up. It is also observed from the figure 7 that the run-up amplitude increases with the increase in  $ka$  values.

Another feature noticed from figure 7 is the increasing appearance of the secondary crest or kink in the run-up profile at lower  $ka$ . This highly non-linear effect has also been observed by others including Danmeier et al. [10]. A detailed view of the run-up is shown in figure 8 for  $ka$  values of 0.208. Free surface profile and the velocity field along the free surface are shown in figure 9 corresponding to the locations mentioned in figure 8.

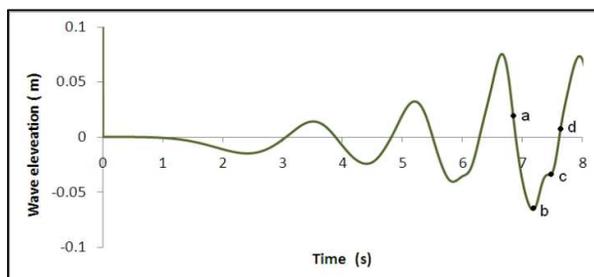
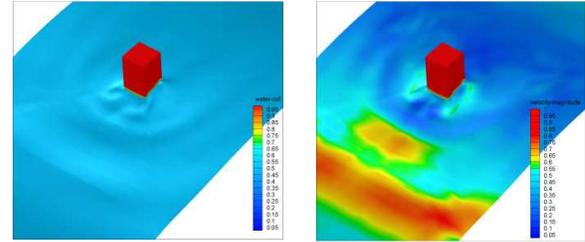
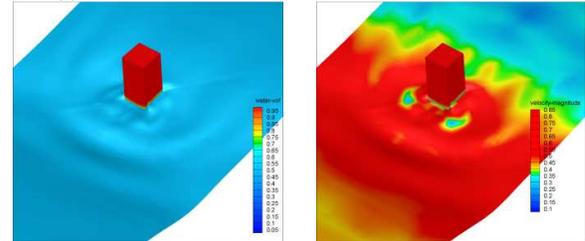


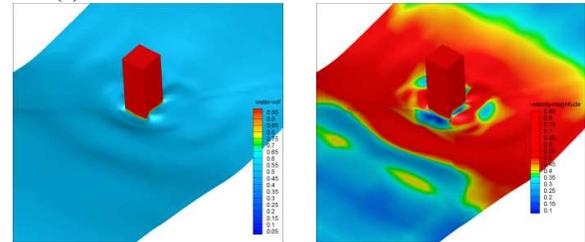
Figure 8. Closer view at the run-up profile at  $ka=0.208$



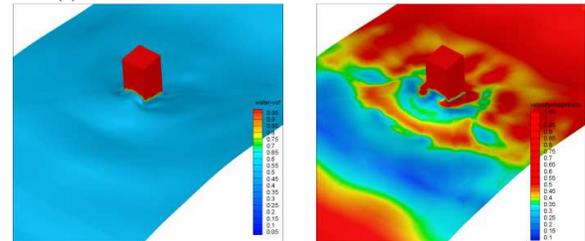
(a)  $T=6.82$  sec



(b)  $T=7.10$  sec



(c)  $T=7.39$  sec



(d)  $T=7.62$  sec

Figure 9. Free surface position and velocity distribution at locations shown in figure 8

The appearance of the secondary crest could be due to interaction of the incoming waves and the retreating water flow from behind the cylinder. The sequence of free surface images suggests that a new crest originates with relatively small amplitude from the rear of cylinder and moves towards the cylinder front (figure 9a). Additionally the vertical flat surface at the front of the cylinder obstructs the flow and builds up the water head (figure 9b). When the flow has a longer wavelengths (lower  $ka$  values) it seems that the combined effect of crest travelling to the front face from the rear of the cylinder, the retreating water head at the front face and relative longer time between peaks of the incident waves allows for the formation of the secondary crest (figure 9c). At some point the amplification of this secondary crest dissipates and with this the run-up profile returns to normal (figure 9d).

#### Run-up Ratio comparison: Wave Scatter against Steepness

The experimental results and the numerical results have been compared with the run-up predicted from the linear diffraction theory and are shown in figure 10. The linear diffraction results are obtained from a boundary element program analysis [11]. It is observed that the experimental results and the numerical results are in good agreement with one another. To calculate the run-up

from the simulations, we used the wave amplitude measured at probe 3 *m* before the cylinder and not the amplitude measured at 6 *m*. This is done in order to be consistent with the experiments and we neglect the small attenuation of wave amplitude that is observed as the wave progresses.

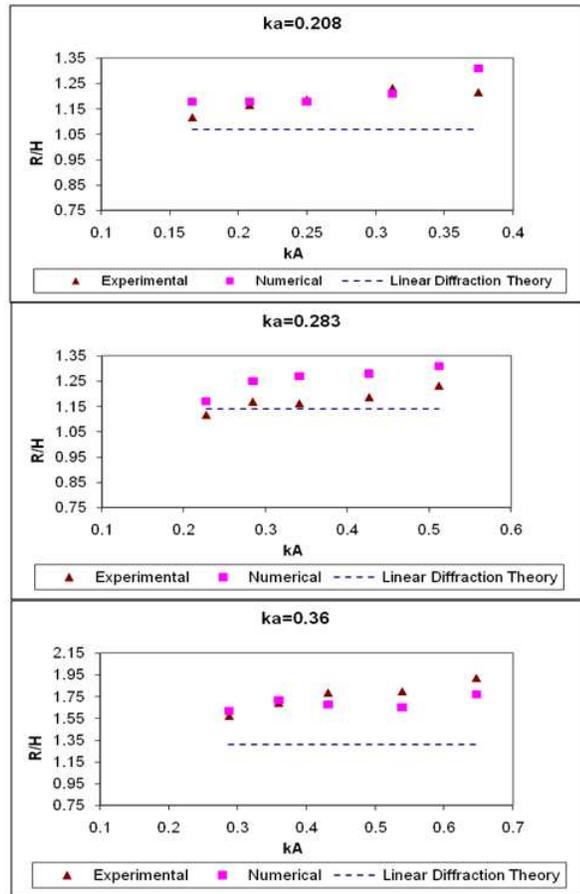


Figure 10. Comparison between experimental and numerical results with the linear diffraction theory

For  $ka$  value of 0.208 the results obtained follows linear diffraction theory (LDT) closely at lower  $ka$  values and show variation at the higher  $ka$  values. A gradual increase is noted in the wave run-up as the wave steepness increases with a maximum variation of about 8%. For  $ka$  value of 0.283 the experimental results have better agreement with the theory and the numerical results follow the experimental results with an offset of about 6%. For  $ka$  value of 0.358 the run-up results obtained are significantly greater than the run-up from linear diffraction theory. In this region the under prediction of the run-up by linear diffraction theory might be due to the fact that there are strong nonlinearities, and this region is well represented by a Stokes second order formulation.

## Conclusion

An attempt has been made to understand the run-up phenomenon on a vertical square cylinder utilizing experiments and numerical techniques. The wave run-up on the square cylinder was found to be significantly greater than the run-up predicted from the linear diffraction theory. The results diverged from linear diffraction theory increasingly with  $ka$  and  $kA$  values.

Further comparison of the free surface profile shows that the CFD code is able to reproduce with reasonable accuracy of the free surface profile around the cylinder. Numerical results also

show that the run-up is higher for a square cylinder compared to circular cylinder of same characteristic length for all the cases considered for the present study. It is also observed that the numerical results are in good agreement with the experimental results and is able to capture non linear phenomenon like secondary kink. A thorough understanding of physics of the kink is necessary for better realization of the run-up phenomenon for future study.

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