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Accuracy of Instantaneous Flow Rate Estimation Using Pressure Measurements

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Abstract

This paper investigates and quantifies the fundamental modelling error in the kinetic differential pressure (KDP) method for measuring unsteady flow rate. The measurement error as functions of measurement location, perturbation size and frequency provides a first step in the validation of this method. Tests are conducted in a numerical reservoir-pipe-reservoir system and the estimated flow rate from the method is compared to that from the Method of Characteristics. The results indicate that the error is unaffected by the position of the KDP units, and under a practical range of system parameters, a maximum error of 2.0 % is observed from the technique when the system is driven to resonance. The study shows that the KDP method can accurately calculate the unsteady flow rate for all system configurations and expected discharge perturbation sizes.

1. Introduction

Accurate measurement of the unsteady flow rate is of great importance in the fuel injection process of internal combustion engines [11], the operation of chemical and pharmaceutical control systems, and the understanding of dynamic characteristics of hydraulic devices and complex unsteady flow phenomena [2,6,13,14]. However, commercial flow meters do not give satisfactory results in unsteady flow where the flow condition varies greatly with time [2,8,9,14].

The kinetic differential pressure (KDP) method for measuring the unsteady flow rate using measured pressures at two points is an attractive alternative to existing methods [2,3,12,13,14]. This method is non-intrusive and can obtain unsteady flow responses at high speeds [3]. Furthermore the KDP model proposed by Washio *et al*. [12] allows the fluctuating flow rate to be obtained at a point away from the measurement section of the pipe and is useful in situations where it is not possible to place a pressure transducer at the location of interest.

The KDP method was shown to accurately measure the fluctuating flow rate in experiments under a limited range of system parameters [3,5,12,13]. To support the use of the KDP method in real systems, the impact of a broader range of system parameters on the prediction accuracy needs to be examined. The present paper aims to assess the validity of the KDP method developed by Washio *et al*. [12] by comparing predictions from the KDP and the Method of Characteristics (MOC) models.

2. Theoretical Background

Equations used in the KDP method are derived from the fundamental momentum and mass equations (equations (1) and (2)) [4]:

$$\frac{\partial(Q_0+q^*)}{\partial t}+gA\frac{\partial(H_0+h^*)}{\partial x}+\frac{f(Q_0+q^*)^2}{2DA}=0 \qquad (1)$$

$$\frac{a^2}{gA}\frac{\partial(Q_0+q^*)}{\partial x}+\frac{\partial(H_0+h^*)}{\partial t}=0 \qquad (2)$$

where a = transient wave propagation speed, x = the distance along the pipeline, t = time, g = the acceleration due to gravity, A = the pipe cross-sectional area, f = the friction factor, D = the pipe diameter, Q_0 = time-averaged mean discharge, q^* = fluctuating flow rate, H_0 = time-averaged mean pressure head and h^* = fluctuating pressure head.

If $q^* << Q_0$ the term $(Q_0 + q^*)^2$ in the last term on the left hand side of equation (1) can be approximated by neglecting higher order term, q^{*2} such that:

$$(Q_0 + q^*)^2 \approx Q_0^2 + 2Q_0q^* \qquad (3)$$

This linear approximation leads to prediction errors in the KDP method and its impact can be identified by comparing the predictions with the nonlinear MOC model.

Incorporation of equation (3) into equation (1) and further manipulation of equations (1) and (2) lead to a set of equations for head and flow responses in the pipeline. These equations are expressed in matrix notation as:

$$\begin{pmatrix} h_2 \\ q_2 \end{pmatrix}=\begin{bmatrix} f_{a11} & f_{a12} \\ f_{a21} & f_{a22} \end{bmatrix}\begin{pmatrix} h_1 \\ q_1 \end{pmatrix} \qquad (4)$$

where $f_{a11} = f_{a22} = cosh(\mu l_a))$, $f_{a12} = -Z_c sinh(\mu l_a)$ and $f_{a21} = -sinh(\mu l_a)/Z_c$ as given in Chaudhry [4]. The subscript 'a' denotes the sub section of the pipe as shown in figure 1 and $\mu = \sqrt{\left(-\omega^2/a^2 + j\,gA\omega R/a^2\right)}$ ω = the frequency of the flow perturbation component in radians per second, $j = \sqrt{-1}$ and $R = fQ_0/gDA^2$ and $Z_c = \mu a^2/j\omega gA$. A similar matrix can be derived for the sub section 'b' in figure 1.

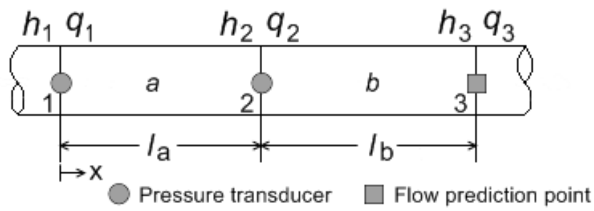

Figure 1. Pipe section

Combining the two matrices, the equation for the flow response at point 3 in figure 1 is:

$$q_3=-\frac{f_{b22}}{f_{a12}}\Delta h+\frac{f_{b21}f_{a12}+f_{b22}f_{a11}-f_{b22}}{f_{a12}}h_2 \qquad (5)$$

where $f_{b11} = f_{b22} = cosh(\mu l_b)$, $f_{b12} = -Z_c sinh(\mu l_b)$ and $f_{b21} = -sinh(\mu l_b)/Z_c$ and $\Delta h = h_2 - h_1$ as shown in Washio *et al*. [12].

Errors in the KDP Flow Estimation

The numerical data is generated by a highly-discretised MOC model and the pressure outputs from two points in the system are used as inputs to the KDP model. The MOC model is a robust, nonlinear 1-D model commonly used for predicting the pressure wave behaviour in pipelines and while outputs from this model can still differ from experimental results—due to deficiencies in current understanding of transient behaviour—predictions from the MOC model are considered as true solutions in this paper. The errors from the linear KDP model identified by this paper are fundamental modelling errors in the approach, and should not be confused with the additional errors caused by a lack of understanding of real transient behaviour which is expected to occur in the application of this technique in reality.

The error from the KDP prediction in this study is the difference in the flow perturbation predicted by the KDP method and that from the MOC model. The MOC model used in the analysis has a computational dimensionless time step of $dx/(aT) = 0.001$, discretising the pipeline into 500 points. dx is the size of space discretisation used in the MOC model and T is the period of the system which is given by $2L/a$, where L is the pipe length. A finer space discretisation of the model was trialled and resulted in no change in the model output, indicating that the MOC model had minimal numerical error at this resolution.

The KDP technique is tested in the elastic single pipeline shown in figure 2. The pipeline with a length of L = 2000 m is bounded by constant head reservoirs. The pipe diameter is 0.3 m and the pipe has a flow rate of 0.173 m^3/s. The pipeline contains a point discharge perturbation introducing unsteadiness into the system which is assumed to be at an unknown location but it is initially placed in the centre of the system to maximise pressure responses.

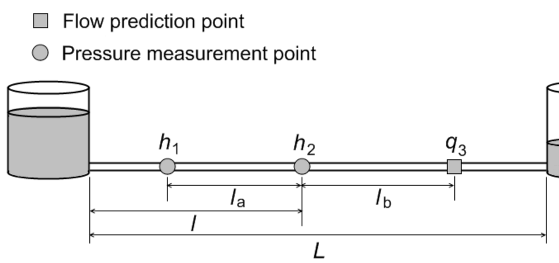

Figure 2. Numerical pipeline system

Any complex signal in time perturbed about a stationary mean can be represented as a summation of continuous sinusoids. The behaviour of this linear system in response to sinusoidal signals can therefore be considered as the building blocks for the system response of more complex signals [7]. In the same way, the KDP prediction error is quantified for continuous sinusoidal system perturbations of various sizes and frequencies. This approach provides a generic quantification of the error in the technique that is representative of any complex perturbation in the system.

The error in the amplitude of the fluctuating flow about the mean state, E, in the KDP prediction is defined as the non-dimensionalised root-mean-square error (RMSE) between the prediction and the output from the MOC model [1,10]:

$$E = \frac{\frac{1}{n}\sqrt{\sum_{n}\left|q_{MOC}(\omega) - q_{KDP}(\omega)\right|^2}}{\frac{1}{n}\sum_{n} q_{MOC}(\omega)} \qquad (6)$$

where $q_{MOC}(\omega)$ and $q_{KDP}(\omega)$ are the magnitudes of the flow fluctuation at a frequency ω calculated from the MOC and KDP methods respectively and n is the number of non-zero frequency components.

The modelling error from the KDP prediction stems from the linearization of the transfer matrix equations (equation (3)). To highlight the nature of this error, other complex phenomena such as unsteady friction and viscoelasticity, which are modelled linearly in both MOC and transfer matrices, are ignored in this study.

Parameters Affecting the Accuracy of the KDP Method

The KDP technique is derived from the linear transfer matrix equations and is bounded by the linear approximation (equation (3)). Parameters expected to affect the accuracy of the KDP method can be divided into two groups, parameters describing the nature of the discharge perturbation being measured and parameters describing the configuration of the KDP system. All of these parameters are expected to affect the importance of the size of the omitted higher order term.

The nature of the discharge perturbation being measured is expected to have a significant impact on the accuracy of the KDP method as it describes the higher order discharge perturbation term ignored in the linear approximation. The study into how the nature of the discharge perturbation affects the accuracy of the KDP method will identify if the technique is best suited for measuring discharge signals with particular properties. A discharge perturbation being measured can be characterised by its size and shape. The dimensionless perturbation size is defined as $q' = q / Q_0$ where q is the maximum peak-to-peak flow perturbation size experienced by the pipeline at a given signal frequency component and Q_0 is the time-averaged flow rate. As the size of the introduced flow perturbation (q) increases, the system is driven further away from the linear bounds and larger errors should result. A limit on the size of the signal relative to the base conditions must therefore exist for the accurate operation of the KDP method and is an important parameter for this study. Apart from the size of the discharge perturbation, the shape of the perturbation is also expected to have an influence on the accuracy of the KDP method. The shape of the discharge perturbation can be defined by the frequency content of the signal and the interaction between the discharge perturbation frequency and the system resonant frequencies is expected to create observable changes in the KDP accuracy.

The operation of the KDP method requires the placement of two pressure transducers and a position of interest for the measurement of the fluctuating flow rate. The study into the impact of the system configuration on the KDP accuracy will determine if an optimum system configuration exists for this technique. The three length parameters to be examined are l, l_a and l_b (figure 2). l is the distance between the upstream reservoir and the most downstream pressure transducer. l_a is the distance between the two pressure transducers and l_b is measured from the most downstream pressure transducer to the point of discharge estimation. All distances are non-dimensionalised by L, the total pipe length.

3. Results and Discussion

The system parameters described in the previous section are examined for their influence on the error in the KDP predictions and the results are presented in this section. The effect of the signal characteristics are first considered followed by the effect of the configuration of the KDP units within the system.

Effect of Signal Characteristics on Error

The behaviour of the KDP error with respect to the signal frequency for three different discharge perturbation sizes is presented in figure 3. In this analysis, the dimensionless perturbation size is defined as $q_{max}' = q_{\max} / Q_0$ in which $q_{\max}$ is the maximum peak-to-peak discharge perturbation experienced in the system for all frequencies. For this test, one pressure transducer is placed next to the upstream reservoir and the other is placed at the first quarter point of the system measured from the upstream boundary. The flow deviation is determined at the middle of the system. The frequency response function (FRF) of the system at the measurement point is also shown in figure 3, which is a representation of the response of the system to sinusoidal inputs at varying frequencies [7] and it illustrates the underlying frequency dependence of the pipeline. The signal frequency, ω is non-dimensionalised by the fundamental frequency of the system, $\omega_{th} = \pi a/L$ and given a symbol ω'.

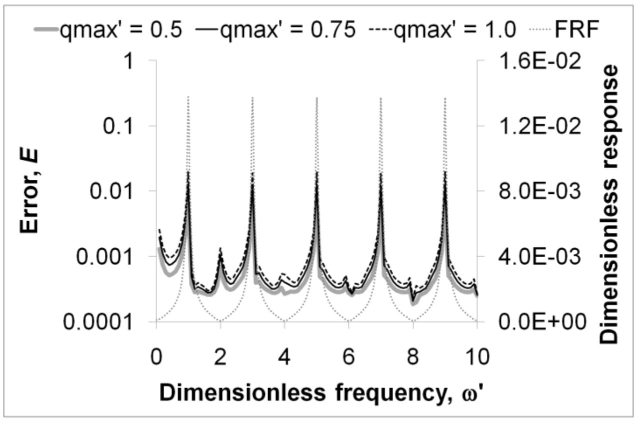

Figure 3. Effect of frequency on the KDP error for different perturbation size

The results in figure 3 indicate that the size of the prediction error increases with perturbation size, with the maximum error observed at the resonant frequencies of the system. With the dimensionless perturbation size of 1.0, the error at a resonant frequency is 2.0 % and is comparable to the known accuracy of other commercial flow measurement devices. The analysis has shown that the KDP method can produce results of an acceptable accuracy with no limit in the size or the shape of the measured signal for the implementation of this method.

Effect of the Configuration of KDP Components on Error

Three key positions in the KDP method—two pressure measurement and a flow estimation points—are termed the KDP units for the following investigations. Due to the complexity of real pipeline systems, the prediction error as a function of the KDP unit placement is of particular interest for the practical application of the technique. The location of access ports for pressure transducers in relation to the system boundaries, as well as the point of discharge measurement, will vary greatly between systems. The variation of the error with respect to the configuration of the KDP components is investigated under three scenarios.

In the first scenario, the transducers and the point of flow estimation are initially grouped together with the dimensionless distance of dx/L = 0.002 which is the finest resolution of the MOC model and shifted along the pipeline as a single unit (figure 4 (a)). This test illustrates the change in error due to the location of these units in the system and determines whether there is a particular placement location for optimum accuracy. In the second and third scenarios, the distances between the three units are changed to determine whether the spacing between the transducers or the distance to the discharge prediction point is more critical for the accuracy of the technique (figures 4 (b) and (c)). In all test scenarios, the discharge perturbation being measured has a dimensionless frequency ω' = 4.5. The dimensionless perturbation size, q' is 0.006 to minimise the error induced by the perturbation size and to highlight the effect of the KDP configuration on the flow prediction error. The results from these cases are presented in figure 5.

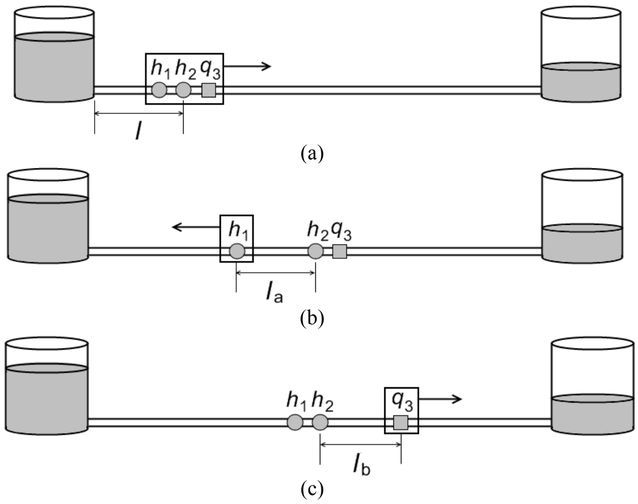

Figure 4. KDP configurations investigated in the analysis, showing the distances involved in each scenario

Figure 5 (a) shows the prediction error as a function of the position of the three KDP units from the upstream reservoir when all three units are shifted in unison. The magnitude error is approximately constant for all positions of the units with the exception of small error spikes occurring at the positions of minimum flow response—caused by machine error at these small values. The average change in the prediction error as a result of the location of the units is found to be 1.7×10^{-4} % and is negligible. The finding indicates that the location of the KDP units in the system has no effect on the accuracy of the technique and no general optimum locations of the units exist.

Figures 5 (b) and (c) show the effect of the individual spacing between the units on the KDP prediction error. The error is plotted as a function of the transducer spacing (l_a) and distance between the transducer and the flow prediction point (l_b) in figure 4 (b) and (c) respectively. Figures 5 (b) and (c) show that the KDP prediction error remains unchanged with the two distances except for minor error spikes of a maximum size of 3.0×10^{-2} %. In figure 5 (b) the spikes are observed when the measured unsteady pressure perturbation magnitudes at the two transducers are the same. The difference in the pressure perturbation magnitude (labelled as dh in figure 5 (b)) for different transducer spacing is also plotted in figure 5 (b) for illustration. This finding is consistent with the remark made in Chen [5] which states that the small differential pressure perturbation magnitudes can cause error in the KDP method. Similar to figure 5 (a), the spikes in figure 5 (c) are located at the minimum flow response and are likely due to machine error at these small values. Despite the error spikes, the average changes in errors as a result of the changes in the two distances are 8.0×10^{-4} % and 7.0×10^{-4} % respectively.

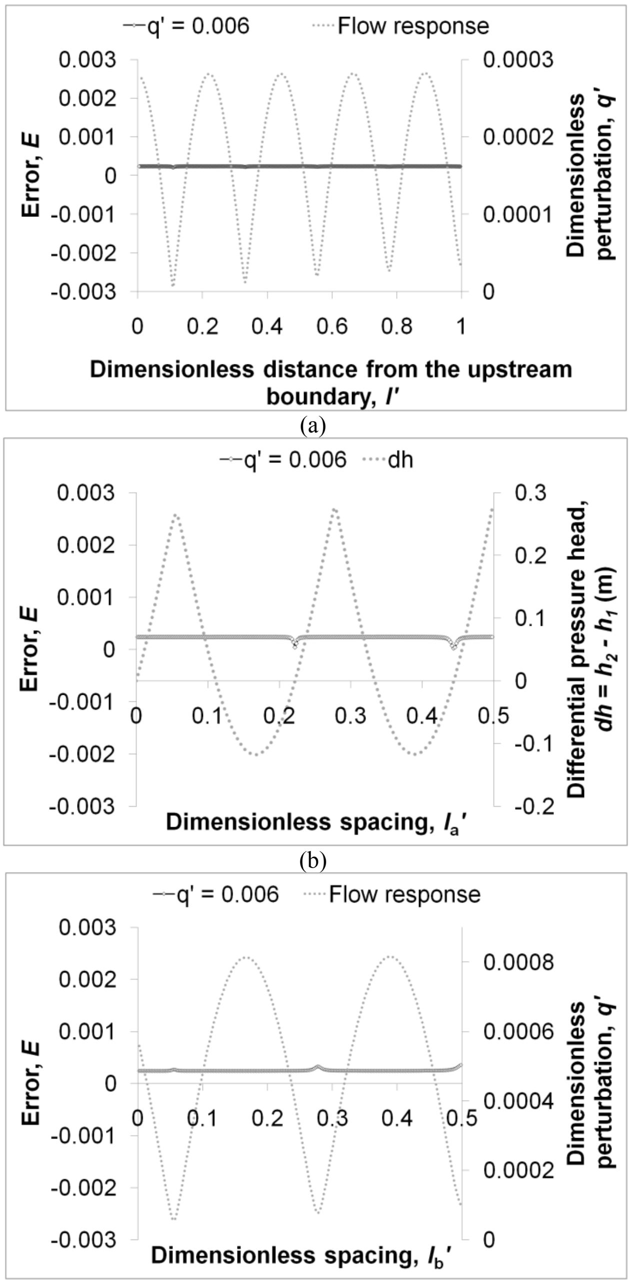

Figure 5. Effect of the configuration of the KDP unit on the error: (a) magnitude error in the case in figure 4 (a), (b) magnitude error in the case in figure 4 (b), (c) magnitude error in the case in figure 4 (c)

An additional scenario was tested where the spacing between the three units is varied uniformly such that the upstream transducer and the discharge prediction points are equidistant from the centre unit. The average change in error due to different spacing was found to be 1.8×10^{-3} %. The size of the errors seen in this study indicates that the KDP method provides excellent prediction accuracy regardless of the placement of the KDP units.

4. Conclusions

Errors in the KDP predictions were found to arise from the linear approximation of the friction term in the governing 1-D unsteady momentum equation. The error was observed to increase with the perturbation size and a maximum error of 2.0 % was observed at the harmonic peak frequencies of the pipe system with the dimensionless perturbation size of 1.0 of the steady flow rate. The size of the error suggests that there is no practical limit on the signal size and frequency that can be measured by the KDP method.

Analysis of the location of the KDP components showed that the size of the error is nearly constant for any arrangement of the KDP units. The magnitude error changes by 9.0×10^{-4} % on average due to the position of the KDP units. The results suggest that the arrangement of the KDP units plays a very minor role in the accuracy of the technique and the signal characteristics have a greater influence on the KDP prediction error.

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